Drill Problems # 5.

Problem 1. Let $\{x_n\}_{n=1}^{\infty}$ be defined as follows: $x_1 = 0$ and $x_{n+1} = \sqrt{3 + 2x_n}$ (for every $n \geq 1$). Prove that the sequence is convergent and find its limit.

Problem 2. Let $r \in \mathbb{R}$. Let $\{x_n\}_{n=1}^{\infty}$ be defined by

$$x_n = 1 + r + r^2 + ... + r^n = \sum_{k=1}^{n} r^k.$$

Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes find the limit. Justify your answer (hint: consider 2 cases, $|r| < 1$ and $|r| \geq 1$).

Problem 3. Let $x_n = (1 + \frac{1}{n})^{n+5}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes, find its limit. Justify your answer.

Problem 4. Let $x_n = (1 + \frac{1}{n})^{3n}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes, find its limit. Justify your answer.

Problem 5. Let $x_n = (1 + \frac{1}{n^3})^{n^3+7}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes, find its limit. Justify your answer.

Problem 6. Let $x_n = (1 + \frac{1}{n^2})^{3n^2}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes, find its limit. Justify your answer.

Problem 7. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence convergent to ℓ. Prove that

$$\lim_{n\to\infty} x_n^{1/3} = \ell^{1/3}.$$