Drill problems 3.

Problem 1. Using mathematical induction prove the following statements.
a. $\forall n \in \mathbb{N} \quad 1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$,
b. $\forall n \in \mathbb{N} \quad 1 + 4 + 9 + 16 + ... + n^2 = n(n + 1)(2n + 1)/6$,
c. $\forall n \in \mathbb{N} \quad 5^n - 2^n$ is divisible by 3.

Problem 2. Find an error in the following argument. The proof of the statement "every human is a genius". Let $P(n)$ states that in every group of n people everyone is a genius. We will prove by induction that $\forall n \in \mathbb{N} \quad P(n)$. Indeed, take an arbitrary $n \in \mathbb{N}$ and suppose that $P(n)$ is true. Now we show $P(n + 1)$. Consider an arbitrary group of $n + 1$ humans and arrange their names in a list. By the induction hypothesis, the first n of them are geniuses. Also, the last n of them are geniuses. Hence, all the $n + 1$ are geniuses. Therefore, by induction, $\forall n \in \mathbb{N} \quad P(n)$. In particular, $P(n)$ is true where n is the total number of people.

Problem 3. Find an error in the following argument: The proof of the statement "all people have the same hair color". Again, we will prove by induction that for every $n \in \mathbb{N}$, all people in any group of n have the same hair color. For the base, trivially, every single person has the same hair color (as himself). Now let n be any natural number, and suppose that every group of n people has the same hair color. Take any group of $n + 1$ persons. Then the first n persons in the group have the same hair color. Also, the last n persons have the same hair color. Hence, all $n + 1$ have the same hair color.