

Drill problems 2.

Problem 1. Let A, B be sets. Prove that $A \Delta B = (A \cup B) \setminus (A \cap B)$.

Problem 2. Is the statement below true or false? Write its negation. Is the negation true or false? Why?

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|---|---|
| a. $\forall x \in \mathbb{Z} \exists y \in \mathbb{N} \quad y = x^2,$ | b. $\forall x \in \mathbb{Z} \exists y \in \mathbb{N} \quad y = 5 - x^2,$ |
| c. $\forall x \in \mathbb{Z} \exists y \in \mathbb{N} \quad y = 3,$ | d. $\forall x \in \mathbb{Z} \exists y \in \mathbb{N} \quad x = 0,$ |
| e. $\forall x \in \mathbb{N} \exists y \in \mathbb{Z} \quad y^2 - x = 0,$ | f. $\forall x \in \mathbb{Z} \exists y \in \mathbb{Z} \quad y + x \neq 0,$ |
| g. $\exists x \in \mathbb{N} \forall y \in \mathbb{Q} \quad x + y \in \mathbb{N},$ | h. $\exists x \in \mathbb{N} \forall y \in \mathbb{Q} \quad x + y \neq 0,$ |
| i. $\exists x \in \mathbb{N} \forall y \in \mathbb{Q} \quad x = 10,$ | j. $\exists x \in \mathbb{N} \forall y \in \mathbb{Q} \quad y = 5,$ |
| k. $\exists x \in \mathbb{N} \forall y \in \mathbb{Z} \quad y = x^2,$ | l. $\exists x \in \mathbb{Z} \forall y \in \mathbb{Z} \quad x + y = 0.$ |

Remarks.

1. Compare statements **a** and **k**.
2. Can you simplify statements **c**, **d**, **i**, **j**?