Assignment # 9.

Due Nov. 13, 13:00

Problem 1. Is the following definition equivalent to the definition of a Cauchy (fundamental) sequence? As usual, prove if YES, provide a counterexample if NO.

$$\forall \varepsilon > 0 \ \exists N \in \mathbb{N} \ \forall n \geq N \quad |x_n - x_{n+1}| < \varepsilon.$$

Problem 2. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be two convergent (in $\mathbb{R}$) sequences such that $x_n \geq y_n$ for all $n \in \mathbb{N}$. Show that

$$\lim_{n\to\infty} x_n \geq \lim_{n\to\infty} y_n.$$

Problem 3. Using only the definition, prove that

$$\lim_{n\to\infty} (-n^2 + 10n + 100) = -\infty.$$

Problem 4. Let $x_n = \sqrt{n^2 + 4n + 5} - n$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If YES find the limit, if NOT explain why.

Problem 5. Let $x_n \geq 0$ for all $n \in \mathbb{N}$. Assume that

$$\sum_{n=1}^{\infty} x_n < \infty.$$

Prove that

$$\lim_{n\to\infty} x_n = 0.$$