Assignment # 8.

Due Nov. 6, 13:00

Problem 1. Is the following statement true or false? As usual, EXPLAIN your answer: if you claim "TRUE" then prove it; if you claim "FALSE" then provide a counterexample.

a. If both $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are divergent sequences then $\{x_n + y_n\}_{n=1}^{\infty}$ is also divergent.

b. If both $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are divergent sequences then $\{x_n\ y_n\}_{n=1}^{\infty}$ is also divergent.

c. If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence and $\{y_n\}_{n=1}^{\infty}$ is a divergent sequence then $\{x_n + y_n\}_{n=1}^{\infty}$ is divergent.

d. If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence and $\{y_n\}_{n=1}^{\infty}$ is a divergent sequence then $\{x_n y_n\}_{n=1}^{\infty}$ is divergent.

Problem 2. Let $\{x_n\}_{n=1}^{\infty}$ be defined as follows: $x_1 = 0$ and $x_{n+1} = \sqrt{2 + x_n}$ (for every $n \geq 1$). Prove that the sequence is convergent and find its limit.

Problem 3. Let $\{x_n\}_{n=1}^{\infty}$ be defined by $x_n = \frac{\cos n}{n}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes find the limit. Justify your answer.

Problem 4. Let $0 \leq a \leq b$. Let $\{x_n\}_{n=1}^{\infty}$ be defined by $x_n = (a^n + b^n)^{1/n}$. Prove that $\{x_n\}_{n=1}^{\infty}$ is convergent to b.

Problem 5. Let $x_n = (1 + \frac{1}{n})^{n+1}$. Is $\{x_n\}_{n=1}^{\infty}$ convergent? If yes, find its limit. Justify your answer.