Assignment # 7.

Due Oct. 30, 13:00

Problem 1. Let $\ell \in \mathbb{R}$, $c \in \mathbb{R}$, $c > 0$. Show that the following two statements are equivalent.

a. $\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \forall n \geq N \quad |x_n - \ell| < \varepsilon$;

b. $\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \forall n \geq N \quad |x_n - \ell| < c\varepsilon$.

Problem 2. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence convergent to 0. Let $\{y_n\}_{n=1}^{\infty}$ be a bounded sequence. Show that

$$\lim_{n\to\infty} (x_n y_n) = 0.$$

Problem 3. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence convergent to 0. Is it true that for every sequence $\{y_n\}_{n=1}^{\infty}$ one has

$$\lim_{n\to\infty} (x_n y_n) = 0$$

(prove if YES; provide a counterexample if NO).

Problem 4. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence convergent to ℓ.
a. Prove that

$$\lim_{n\to\infty} x_n^2 = \ell^2.$$

b. Assuming that for every $n \in \mathbb{N} \quad x_n \geq 0$ and $\ell \geq 0$, prove that

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{\ell}.$$

Problem 5. Find limits of the following sequences (you can use facts proved in the class).

a. $\left\{\dfrac{2n^2 + n + \sqrt{n} - 3}{n^2 - 5n + 7}\right\}_{n=1}^{\infty}$, **b.** $\left\{\dfrac{1 + 2 + 3 + \ldots + n}{n^2}\right\}_{n=1}^{\infty}$.