Assignment # 5.

Due Oct. 16, 13:00

Problem 1. Let $r > 0$. Prove that

$$|x| < r \;\Leftrightarrow\; -r < x < r.$$

Problem 2. Solve and write the answer in the interval notation.

a. $\left|\dfrac{3}{2x-1}\right| < 1,$ **b.** $\left|\dfrac{1}{1-3x}\right| > 4.$

Problem 3. Let $S \neq \emptyset$ be a bonded (below and above) subset of $\mathbb{R}$. Denote $a = \inf S$, $b = \sup S$. Is it true that $\forall n \in \mathbb{N}$ $\exists x \in S$ such that

a. $a > x - \frac{1}{n}$; **b.** $a > x + \frac{1}{n}$; **c.** $b < x - \frac{1}{n}$; **d.** $b < x + \frac{1}{n}$.

(Explain your answer.)

Problem 4. For a given set S write $\sup S$, $\inf S$, $\max S$, $\min S$ (if exist).

a. $S = \{-1, 2, 5\}$; **b.** $S = [1, 2]$; **c.** $S = (2, 3]$;
d. $S = [0, 4)$; **e.** $S = (-\infty, 3]$; **f.** $S = (1, \infty)$;
g. $S = \mathbb{N}$; **h.** $S = \mathbb{Z}$; **i.** $S = \emptyset$.

(Write answers only.)