Assignment # 3.

Due Oct. 2, 13:00

Problem 1. Using the induction principle prove that for every $x > 0$ and every $n \in \mathbb{N}$ one has

$$(1+x)^n \geq 1 + x^n.$$

Problem 2. Let a, b be non-zero real numbers. Prove the following statements using only definitions and axioms (you may use also that $1 > 0$ and that for every x one has $x \cdot 0 = 0$).

a. $(a^{-1})^{-1} = a$; **b.** $\frac{1}{a} = a^{-1}$; **c.** $(ab)^{-1} = a^{-1}b^{-1}$;

d. $a > 0$ if and only if $a^{-1} > 0$; **e.** $a > 1$ if and only if $0 < a^{-1} < 1$.

Recall order axioms

(for field axioms see the solutions of assignment 2.)

O1. $\forall a \in \mathbb{R}$ $a \leq a$;
O2. $\forall a, b \in \mathbb{R}$ if $a \leq b$ and $b \leq a$ then $a = b$;
O3. $\forall a, b, c \in \mathbb{R}$ if $a \leq b$ and $b \leq c$ then $a \leq c$;
O4. $\forall a, b \in \mathbb{R}$ $a \leq b$ or $b \leq a$;
O5. $\forall a, b, c \in \mathbb{R}$ if $a \leq b$ then $a + c \leq b + c$;
O6. $\forall a, b, c \in \mathbb{R}$ if $a \leq b$ and $0 \leq c$ then $ac \leq bc$.