Assignment # 2.

Due Sept. 25, 13:00

Problem 1. Using the induction principle prove that
a. for every integer $n \geq 1$ one has

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2;$$

b. $8^n - 3^n$ is divisible by 5 for every integer $n \geq 1$;
c. for every integer $n \geq 2$ one has

$$1 \cdot 3 \cdot 5 \cdot ... \cdot (2n+1) < n^{2n}.$$

Problem 2. Using only definitions and axioms A1 – A4 prove that
a. $-0 = 0$; **b.** $0 - 1 = -1$

Problem 3. Using only definitions and axioms A1 – A9 prove that
a. for every $a, b \in \mathbb{R}$ one has $(-a)b = -(ab)$;
b. for every $a \in \mathbb{R}$ one has $(-1)a = -a$.
(In this problem you may also use the fact saying that $x0 = 0$ for every $x \in \mathbb{R}$).