Assignment # 10.
Due Nov. 20, 13:00

Problem 1.   Let $k, \ell \in \mathbb{N}$ be such that $k > \ell > 2$. Find the following limits.

a. $\displaystyle\lim_{n\to\infty} \frac{n^k + 1}{10n^\ell + 5n^2 + n}$,
b. $\displaystyle\lim_{n\to\infty} \frac{n^{k+\ell} + n^k + n^\ell}{2^n}$,
c. $\displaystyle\lim_{n\to\infty} n^{\frac{1}{\sqrt{n}}}$.

Problem 2.   Find the domain and sketch the graph of the following functions

a. $|x-1|-|x+1|$,
b. $\begin{cases} x, & \text{if } x = 1/n \text{ for some } n \in \mathbb{N}, \\ 0, & \text{otherwise}, \end{cases}$

c. $\dfrac{x^4}{x^2}$
d. $\sqrt{1 - x^2}$.

Problem 3.   Does the limit of function f at the point a exist? If YES, find the limit and prove your answer using (ε/δ)-definition of the limit. If NO, prove it.

a. $a = 9$, $f(x) = \sqrt{x}$,
b. $a = 1$, $f(x) = \sqrt{1 - x^2}$,

c. $a = 2$, $f(x) = \begin{cases} 2x, & \text{if } x > 2, \\ 5, & \text{if } x = 2, \\ 6 - x, & \text{if } x < 2, \end{cases}$

d. $a = -1$, $f(x) = \begin{cases} |x|, & \text{if } x > -1, \\ x^2, & \text{if } x < -1, \end{cases}$
e. $a = 0$, $f(x) = \begin{cases} \sqrt{x}, & \text{if } x \geq 0, \\ 1 - x, & \text{if } x < 0. \end{cases}$