The Alberta High School Mathematics Competition
Part II, February 6, 2013.

Problem 1.
Determine all pairs of positive integers (a, b) with $a \le b$ such that

$$\left(a + \frac{6}{b}\right)\left(b + \frac{6}{a}\right) = 25.$$

Problem 2.
A set S of positive integers is called *perfect* if any two integers in S have no common divisors greater than 1. Candy wants to build a perfect set of numbers between 1 and 20 inclusive, in such a way that her set contains as many numbers as possible.

(a) How many elements will her set have?

(b) How many different such sets can she build?

Problem 3.
Randy plots a point A. Then he starts drawing some rays starting at A, so that all the angles he gets are integral multiples of 10°. What is the largest number of rays he can draw so that all the angles at A between the rays are unequal, including all angles between non-adjacent rays?

Problem 4.
In a convex pentagon of perimeter 10, each diagonal is parallel to one of the sides. Find the sum of the lengths of its diagonals.

Problem 5.
Find all integers $r > s > t$ and all quadratic polynomials of the form $f(x) = x^2 + bx + c$ such that b and c are integers, $r + t = 2s$, $f(r) = 1$, $f(s) = b$ and $f(t) = c$.