# The Alberta High School Mathematics Competition
## Part II, February 3, 2010

1. Of Melissa's ducks, $x\%$ have 11 ducklings each, $y\%$ have 5 ducklings each and the rest have 3 ducklings each. The average number of ducklings per duck is 10. Determine all possible *integer* values of $x$ and $y$.

2. (a) Find all real numbers $t \neq 0$ such that $tx^2 - (2t-1)x + (5t-1) \geq 0$ for all real numbers $x$.

   (b) Find all real numbers $t \neq 0$ such that $tx^2 - (2t-1)x + (5t-1) \geq 0$ for all $x \geq 0$.

3. Points $A$, $B$, $C$ and $D$ lie on a circle in that order, so that $AB = BC$ and $AD = BC + CD$. Determine $\angle BAD$.

4. Let $n$ be a positive integer. A $2^n \times 2^n$ board, missing a $1 \times 1$ square anywhere, is to be partitioned into rectangles whose side lengths are integral powers of 2. Determine in terms of $n$ the smallest number of rectangles among all such partitions, wherever the missing square may be.

5. Let $f$ be a non-constant polynomial with non-negative integer coefficients.

   (a) Prove that if $M$ and $m$ are positive integers such that $M$ is divisible by $f(m)$, then $f(M+m)$ is also divisible by $f(m)$.

   (b) Prove that there exists a positive integer $n$ such that each of $f(n)$ and $f(n+1)$ is a composite number.