1. Of Melissa’s ducks, \( x\% \) have 11 ducklings each, \( y\% \) have 5 ducklings each and the rest have 3 ducklings each. The average number of ducklings per duck is 10. Determine all possible \emph{integer} values of \( x \) and \( y \).

2. (a) Find all real numbers \( t \neq 0 \) such that \( tx^2 - (2t - 1)x + (5t - 1) \geq 0 \) for all real numbers \( x \).

   (b) Find all real numbers \( t \neq 0 \) such that \( tx^2 - (2t - 1)x + (5t - 1) \geq 0 \) for all \( x \geq 0 \).

3. Points \( A, B, C \) and \( D \) lie on a circle in that order, so that \( AB = BC \) and \( AD = BC + CD \). Determine \( \angle BAD \).

4. Let \( n \) be a positive integer. A \( 2^n \times 2^n \) board, missing a \( 1 \times 1 \) square anywhere, is to be partitioned into rectangles whose side lengths are integral powers of 2. Determine in terms of \( n \) the smallest number of rectangles among all such partitions, wherever the missing square may be.

5. Let \( f \) be a non-constant polynomial with non-negative integer coefficients.

   (a) Prove that if \( M \) and \( m \) are positive integers such that \( M \) is divisible by \( f(m) \), then \( f(M + m) \) is also divisible by \( f(m) \).

   (b) Prove that there exists a positive integer \( n \) such that each of \( f(n) \) and \( f(n + 1) \) is a composite number.