The Alberta High School Mathematics Competition
Part I, November 16, 2010

1. The number of positive integers n such that the number $4n$ has exactly two digits is

   (a) 21 (b) 22 (c) 23 (d) 24 (e) 25

2. A 4×6 plot of land is divided into 1×1 lots by fences parallel to the edges of the plot, with fences along the edges as well. The total length of fences is:

   (a) 58 (b) 62 (c) 68 (d) 72 (e) 96

3. The greatest common divisor and least common multiple of two positive integers are 1 and 10 respectively. If neither of them is equal to 10, their sum is equal to

   (a) 3 (b) 6 (c) 7 (d) 11 (e) none of these

4. The number of pairs (x, y) of non-negative integers such that $3x + 2y = 27$ is

   (a) 4 (b) 5 (c) 8 (d) 9 (e) 10

5. In the sequence 1, 2, 3, 4, 6, 7, 8, 9, ..., obtained by deleting the multiples of 5 from the sequence of the positive integers, the 2010th term is

   (a) 2511 (b) 2512 (c) 2513 (d) 2514 (e) none of these

6. Alice, Brian, Colin, Debra and Ethel are in a hotel. Their rooms are on floors 1, 2, 3, 21 and 40 respectively. In order to minimize the total number of floors they have to cover to get together, the floor on which the get-together should be is

   (a) 18 (b) 19 (c) 20 (d) 21 (e) none of these

7. A square pigeon coop is divided by interior walls into 9 square pigeonholes in a 3×3 configuration. Each of two pigeons chooses a pigeonhole at random, possibly the same one. The probability that they choose two holes on the opposite sides of an interior wall is

   (a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{4}{27}$ (d) $\frac{8}{27}$ (e) $\frac{1}{3}$

8. The set of all values of the real number x such that $\dfrac{1}{x} \leq -3 \leq x$ is

   (a) $\{x \leq -1/3\}$ (b) $\{-3 \leq x \leq -1/3\}$ (c) $\{-3 \leq x < 0\}$

   (d) $\{-1/3 \leq x < 0\}$ (e) none of these

9. In the quadrilateral $ABCD$, AB is parallel to DC, $DC = 2AB$, $\angle ADC = 30°$ and $\angle BCD = 50°$. Let M be the midpoint of CD. The measure of $\angle AMB$ is

(a) $80°$ (b) $90°$ (c) $100°$ (d) $110°$ (e) $120°$

10. We are constructing isosceles but non-equilateral triangles with positive areas and integral side lengths between 1 and 9 inclusive. The number of such triangles which are non-congruent is

(a) 16 (b) 36 (c) 52 (d) 61 (e) none of these

11. In each of the following numbers, the exponents are to be evaluated from top down. For instance, $a^{b^c} = a^{(b^c)}$. The largest one of these five numbers is

(a) $2^{2^{2^{2^3}}}$ (b) $2^{2^{2^{3^2}}}$ (c) $2^{2^{3^{2^2}}}$ (d) $2^{3^{2^{2^2}}}$ (e) $3^{2^{2^{2^2}}}$

12. A gold number is a positive integer which can be expressed in the form $ab + a + b$, where a and b are positive integers. The number of gold numbers between 1 and 20 inclusive is

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12

13. The edges DA, DB and DC of a tetrahedron $ABCD$ are perpendicular to one another. If the length of DA is 1 cm and the length of each of DB and DC is 2 cm, the radius, in cm, of the sphere passing through A, B, C and D is

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{5}+1}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{2} + \frac{1}{2}$ (e) none of these

14. Let $f(x) = x^2$ and $g(x) = x^4$. We apply f and g alternatively to form

$$f(x) = x^2,\ g(f(x)) = g(x^2) = (x^2)^4 = x^8,\ f(g(f(x)) = f(x^8) = (x^8)^2 = x^{16},$$

and so on. After we have applied f 50 times and g 49 times, the answer is x^n where n is

(a) 148 (b) 296 (c) 2^{148} (d) 2^{296} (e) none of these

15. Triangle ABC has area 1. X, Y are points on the side AB and Z a point on the side AC such that $XY = 2AX$, XZ is parallel to YC and YZ is parallel to BC. The area of XYZ is

(a) $\frac{1}{27}$ (b) $\frac{2}{27}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$ (e) $\frac{1}{3}$

16. The number of integers n for which $n^3 - 3n + 2$ is divisible by $2n + 1$ is

(a) 3 (b) 4 (c) 5 (d) 6 (e) 8