Alberta High School Mathematics Competition

Part I November 18, 2008.

1. The numbers m and n are related by $m = \frac{9n}{5} + 32$. If n is an integer between 1 and 99, the number of corresponding values of m which are also integers is

   (a) 1 (b) 19 (c) 20 (d) 50 (e) 99

2. When the positive integers a, b and c are divided by 13, the respective remainders are 9, 7 and 10. When $a + 2b + 3c$ is divided by 13, the remainder is

   (a) 1 (b) 3 (c) 4 (d) 5 (e) 7

3. A polygon has area greater than 0. All side lengths are integers, the largest of which is 10. The smallest possible perimeter of this polygon is

   (a) 10 (b) 11 (c) 20 (d) 21 (e) none of these

4. Let f and g be functions such that $f(x) = g(2x)$ and $g(x) = 2f(x)$ for all real numbers x. If $g(2) = 3$, the value of $f(\frac{1}{2})$ is

   (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3 (e) 6

5. If a certain positive integer is divided by 7, the remainder is 6. If it is divided by 4, the remainder is 1. The remainder when it is divided by 28 is

   (a) 5 (b) 6 (c) 7 (d) 13 (e) 17

6. A building is in the shape of a right circular cylinder, with mirrors for its interior wall. A horizontal ray starts from a point on the wall, reflects off the wall exactly 11 times and returns to its starting point for the first time. The number of possible directions along which it may have started is

   (a) 4 (b) 8 (c) 10 (d) 11 (e) infinite

7. For a real number $t \geq 0$, $|t| = t$, and for a real number $t < 0$, $|t| = -t$. The number of real numbers x satisfying $|||||x-1|-2|-3|-4|-5| = 0$ is

   (a) 0 (b) 1 (c) 2 (d) 5 (e) 32

8. A rectangular box has integral dimensions and volume 2008 cubic centimetres. In square centimetres, the minimum value of its surface area is

   (a) 3028 (b) 4024 (c) 4534 (d) 5028 (e) none of these

9. The positive integers a and b are such that there are exactly 10 integers greater than a and less than b, and exactly 1000 integers greater than a^2 and less than b^2. The value of b is

   (a) 50 (b) 51 (c) 52 (d) 55 (e) 102

10. Linda is walking at constant speed halfway between two parallel train tracks. On each track is a train of the same length. They are approaching Linda from different directions, both at V kilometres per hour. The one going the same way as Linda takes t_1 seconds to pass her, while the other takes t_2 seconds to pass her. In kilometres per hour, Linda's walking speed is

    (a) $\frac{t_1-t_2}{t_1+t_2}V$ (b) $\frac{t_2-t_1}{t_2+t_1}V$ (c) $\frac{t_1+t_2}{t_1-t_2}V$ (d) $\frac{t_2+t_1}{t_2-t_1}V$

    (e) dependent on the length of the trains

11. In base 10, the positive integer a has 2 digits, the positive integer b has a digits and the positive integer c has b digits. The smallest possible value for c is

    (a) 10^{9^9} (b) 10^{10^9-1} (c) 10^{10^9} (d) $10^{10^{10}-1}$ (e) $10^{10^{10}}$.

12. Of the eight members of a Sports Club, two are senior boys, two are senior girls, two are junior girls and two are junior boys. An official team is of size four, and must consist of two seniors and two juniors, as well as two girls and two boys. The number of different official teams is

    (a) 2 (b) 8 (c) 10 (d) 16 (e) 18

13. On a vertical wall are drawn four geometric figures with equal area. The first is an equilateral triangle with a side along the floor. The second is an isosceles right triangle with one of the short sides along the floor. The third is a square with a side along the floor. The fourth is a circle tangent to the floor. The figure which reaches the highest point above the floor is

    (a) only the equilateral triangle (b) only the isosceles right triangle
    (c) either triangle (d) only the square (e) only the circle

14. The sum of the fifth powers of the roots of $x^3 - 3x + 1 = 0$ is

    (a) -15 (b) -9 (c) 0 (d) 6 (e) none of these

15. A country has 100 cities numbered from 1 to 100. For all $m < n$, there is a road linking the m-th and the n-th city if and only if $\frac{n}{m}$ is a prime number. One can travel in either direction on a road. The minimum number of roads one needs to use to go from the 99th city to the 100th city is

    (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

16. In each lottery ticket, you choose two of the numbers 1, 2, 3, 4 and 5. Eventually, two of these five numbers will be drawn. Your ticket wins if neither of the numbers drawn are chosen on your ticket. The smallest number of tickets you must have in order to guarantee that at least one of them wins is

    (a) 3 (b) 4 (c) 5 (d) 6 (e) 7