Alberta High School Mathematics Competition

1. A positive integer has 1001 digits all of which are 1s. When this number is divided by 1001, the remainder is

   (a) 1 (b) 10 (c) 11 (d) 100 (e) none of these

2. Some cats have got into the pigeon loft because the total head count is 34 but the total leg count is 80. The number of cats among the pigeons is

   (a) 6 (b) 12 (c) 17 (d) 22 (e) 28

3. In triangle ABC, $AB \le 1 \le BC \le 2 \le CA \le 3$. The maximum area of triangle ABC is

   (a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$ (e) none of these

4. The number of ways in which five As and six Bs can be arranged in a row which reads the same backwards and forwards is

   (a) 1 (b) 5 (c) 10 (d) 15 (e) none of these

5. Among twenty consecutive integers each at least 9, the maximum number of them that can be prime is

   (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

6. The non-negative numbers x and y are such that $2x+y=5$. The sum of the maximum value of $x+y$ and the minimum value of $x+y$ is

   (a) 0 (b) $\frac{5}{2}$ (c) 5 (d) $\frac{15}{2}$ (e) none of these

7. We wish to choose some of the positive integers from 1 to 1000 inclusive, such that no two differ by 3 or 5. The maximum number of positive integers we can choose is

   (a) 200 (b) 300 (c) 333 (d) 500 (e) none of these

8. The number of polynomials p with integral coefficients such that $p(9)=13$ and $p(13)=20$ is

   (a) 0 (b) 1 (c) 2 (d) 3 (e) infinitely many

9. In the quadrilateral $ABCD$, $AB=CD$, $AD=2$ and $BC=6$. AD and BC are parallel lines at a distance 8 apart. The radius of the smallest circle which can cover $ABCD$ is

   (a) $\sqrt{18}$ (b) $\sqrt{20}$ (c) $\frac{\sqrt{85}}{2}$ (d) 5 (e) none of these

10. The number of pairs (a, b) of positive integers such that all three roots of the cubic equation $x^3 - 10x^2 + ax - b = 0$ are positive integers is

(a) 3 (b) 8 (c) 10 (d) 66 (e) none of these

11. The real numbers x and y are such that $x + \frac{2}{y} = \frac{8}{3}$ and $y + \frac{2}{x} = 3$. The value of xy is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) 2 (d) 4

(e) not uniquely determined

12. Let θ be an acute angle such that $\sec^2\theta + \tan^2\theta = 2$. The value of $\csc^2\theta + \cot^2\theta$ is

(a) 2 (b) 3 (c) 4 (d) 5 (e) none of these

13. The diameter AC divides a circle into two semicircular arcs. B is the midpoint of one these arcs, and D is any point on the other arc. If the area of $ABCD$ is 16 square centimetres, the distance, in centimetres, from B to AD is

(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $4\sqrt{2}$

(e) dependent on the radius of the circle

14. Five students took part in a contest consisting of six true-or-false questions. Student $\#i$ gave the answer T to question $\#j$ if and only if $i < j$. The total number of incorrect answers is 8 or 9, and there are more incorrect answers of T than incorrect answers of F. The student who has both an incorrect answer of T and an incorrect answer of F is

(a) #1 (b) #2 (c) #3 (d) #4 (e) #5

15. An integer n is randomly chosen from 10^{99} to $10^{100} - 1$ inclusive. The real number m is defined by $m = \frac{9n}{5}$. Of the following five numbers, the one closest to the probability that $10^{99} \leq m \leq 10^{100} - 1$ is

(a) $\frac{1}{3}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{5}{9}$ (e) $\frac{2}{3}$

16. The smallest value of the real number k such that $(x^2 + y^2 + z^2)^2 \leq k(x^4 + y^4 + z^4)$ holds for all real numbers x, y and z is

(a) 1 (b) 2 (c) 3 (d) 6 (e) 9