Alberta High School Mathematics Competition Newsletter

Volume 19, Number 3 **March 15, 2010**

The second part of the 54th Alberta High School Mathematics Competition was written on February 3, 2010 by 59 students representing 15 schools. Here is the list of fellowship winners and top performers.

ConocoPhillips Canada Fellow, First Place:
Mariya Sardarli, Strathcona High School, Edmonton (Grade X).

Peter H. Denham Memorial Fellow, Second Place:
Hunter Spink, Western Canada High School, Calgary **Grade XI**.

Canadian Mathematical Society Fellow, Third Place:
Soroosh Hemmati, Western Canada High School, Calgary.

Alberta Teachers' Association Grade XI Fellow, Fourth Place:
Kaiven Zhou, Strathcona High School, Edmonton.

Alberta Teachers' Association Grade X Fellow, Seventeenth Place:
HanHyung Lee, Old Scona Academic High School, Edmonton.

Robert Barrington Leigh Memorial Fellow, Thirteenth Place:
Weilian Chu, Grandview Heights Junior High School, Edmonton (Grade IX).

Honorable Mentions:

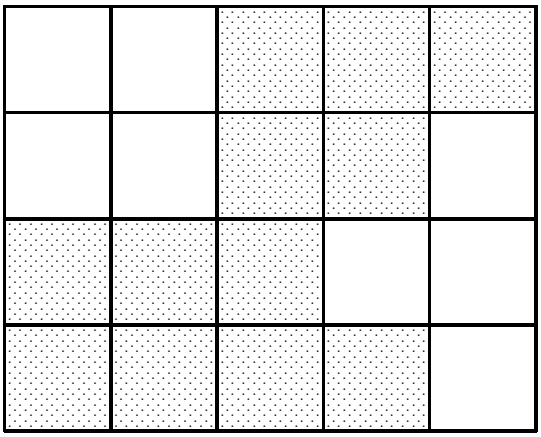
5	Yaroslav Babich	Sir Winston Churchill High School, Calgary (Grade XI).
6	Justine Zhang	Sir Winston Churchill High School, Calgary (Grade XI).
7	Tim He	Henry Wise Wood High School, Calgary (Grade XI).
8	Yifan Wang	Western Canada High School, Calgary.
9	Meng Zhou	Western Canada High School, Calgary (Grade XI).
10	Jaclyn Chang	Western Canada High School, Calgary.
11	Yuri Delanghe	Harry Ainlay High School, Edmonton.
12	Calvin Tseng	Western Canada High School, Calgary.
14	Stephanie Bohaichuk	Harry Ainlay High School, Edmonton.
	Myfan Li	Western Canada High School, Calgary.
	Michael Yang	Henry Wise Wood High School, Calgary.
18	Fernando Lopas	Sir Winston Churchill High School, Calgary.
	Nam Song	Sir Winston Churchill High School, Calgary (Grade XI).
20	Isaac Lin	Sir Winston Churchill High School, Calgary (Grade XI).

We offer our congratulations to the above students, their schools and their teachers.

2009 Canadian Mathematical Olympiad

1. Given an $m \times n$ grid with squares coloured either black or white, we say that a black square is *stranded* if there is some square to its left in the same row that is white, and there is some square above it in the same column that is white. Find a closed formula for the number of $2 \times n$ grids with no stranded black squares.

   The diagram below shows a 4×5 grid with no stranded black squares.



2. Two circles of different radii are cut out of cardboard. Each circle is subdivded into 200 equal sectors. On each circle 100 sectors are painted white and the other 100 are painted black. The smaller circle is then placed on top of the larger circle so that the centres coincide. Show that one can rotate the small circle so that the sectors on the two circles line up and at least 100 sectors on the small circle lie over sectors of the same colour on the big circle.

3. Define $f(x,y,z) = \dfrac{(xy+yz+zx)(x+y+z)}{(x+y)(y+z)(z+x)}$. Determine the set of real numbers r for which there exists a triple (x,y,z) of positive real numbers satisfying $f(x,y,z) = r$.

4. Find all ordered pairs (a,b) such that a and b are integers and $3^a + 7^b$ is a perfect square.

5. A set of points in marked on the plane, with the property that any three marked points can be covered with a disk of radius 1. Prove that the set of all marked points can be covered with a disk of radius 1.

Answers:
1. $2 \cdot 3^n - 2^n$. **3.** $(1, \frac{9}{8}]$. **4.** (1,0) and (2,1).

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Götz Balance

You are allowed to use twice a Götz balance which has three pans. The one containing the lightest weight will go up. If two pans contain the same weight and the weight of the third one is not less than this, then no pan goes up. You are given 7 coins one of which is slightly heavier. The weight of a normal coin is much greater than any variation in weight. In other words, a larger number of coins always outweigh a smaller number of coins. How can you identify the fake coin? What is the total number of coins you can handle if the fake coin is slightly lighter?

2009 International Mathematical Olympiad

1. Let n be a positive integer and let $a_1, a_2, \ldots, a_k$, $k > 2$ be distinct integers in the set $\{1, 2, \ldots, n\}$ such that n divides $a_i(a_{i+1} - 1)$ for $i = 1, 2, \ldots, k-1$. Prove that n does not divide $a_k(a_1 - 1)$.

2. Let ABC be a triangle with circumcentre O. The points P and Q are interior points on the sides CA and AB respectively. Let K, L and M be the midpints of the segments BP, CQ and PQ, respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ$.

3. Suppose that $s_1, s_2, s_3, \ldots$ is a strictly increasing sequence of positive integers such that the subsequences $s_{s_1}.s_{s_2}, s_{s_3}, \ldots$ and s_{s_1+1}, $s_{s_2+1}, s_{s_3+1}, \ldots$ are both arithmetic progressions. Prove that the sequence $s_1, s_2, s_3, \ldots$ is itself an arithmetic progression.

4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E, respectively. Let K be the incentre of triangle ACD. Suppose that $\angle BEK = 45^\circ$. Find all possible values of $\angle CAB$.

5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b, there exists a triangle with non-collinear vertices, with sides of lengths a, $f(b)$ and $f(b + f(a) - 1)$.

6. Let $a_1, a_2, \ldots, a_n$ be distinct positive integers and let M be a set of $n - 1$ positive integers not containing $s = a_1 + a_2 + \cdots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths $a_1, a_2, \ldots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.

Answers:
4. 60° and 90°. **5.** $f(n) = n$.

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News Bulletin

The Alberta High School Math Competition Book: 1957–2006, ISBN 0-88385-830-1, has been published jointly by the Canadian Mathematical Society and the Mathematical Association of America. It is edited by AHSMC Board member **Andy Liu**. It contains a brief history of the first fifty years of our competition, question papers for all fifty years, answers only to the papers up to 1983, and complete solutions, lists of winners and First Round statistics to the papers after 1984.

The companion volume, *Problems from Murray Klamkin*, ISBN 0-88385-828-8, has also been published jointly by the CMS and the MAA. It is jointly edited by **Bruce Shawyer**, former editor of *Crux Mathematicorum* (now *Crux Mathematicorum with Mathematical Mayhem*), and AHSMC Board member **Andy Liu**. It contains all problems in *Crux Mathematicorum* which were proposed by **Murray Klamkin**, 1921–2004, long time member of the AHSMC Board. Solutions by various people, including Murray himself, are included. The problems are at a higher level than those found in the AHSMC.

These two books will be used as prizes in the First Round of the AHSMC. Additional copies are available for sale from the AHSMC Board, to schools participating in the AHSMC. The price per copy is a very favourable Can$60.00, which includes GST and shipping. The list price is US$61.50 when purchased directly from the publishers, with GST and shipping to be added.