Alberta High School Mathematics Competition Newsletter

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The Second Round of the 51st Alberta High School Mathematics Competition was written on February 7, 2007 by 65 students representing 13 schools. Here is the list of fellowship winners and top performers.

ConocoPhillips Canada Fellow, First Place:
Jeffrey Mo, William Aberhart High School, Calgary.

Pacific Institute for the Mathematical Sciences Special Prize, Second Place:
Jerry Lo, Ross Sheppard High School, Edmonton.

Peter H. Denham Memorial Fellow, Third Place:
Boris Braverman, Sir Winston Churchill High School, Calgary.

Canadian Mathematics Society Fellow, Fourth Place:
Jarno Sun, Western Canada High School, Calgary, **Grade 10**.

Alberta Teachers' Association Grade XI Fellow, Seventh Place:
Brett Baek, Western Canada High School, Calgary.

Alberta Teachers' Association Grade X Fellow, Sixth Place:
Danny Shi, Sir Winston Churchill High School, Calgary.

Honorable Mentions:

5	Linda Zhang	Western Canada High School, Calgary.
7	Tony Zhao	Sir Winston Churchill High School, Calgary, Grade 10.
9	Sherwin Ghafouri	Western Canada High School, Calgary.
	Dustin Styner	Queen Elizabeth Junior/Senior High School, Calgary.
11	Matthew Wang	Western Canada High School, Calgary, Grade 11.
	Darren Xu	Sir Winston Churchill High School, Calgary, Grade 11.
	Simon Sun	Sir Winston Churchill High School, Calgary.
14	Kyle Boone	Western Canada High School, Calgary, Grade 11.
	Annie Xu	Old Scona Academic High School, Edmonton, Grade 11.
16	Yu Xiang Liu	Western Canada High School, Calgary, Grade 11.
	Chong Shen	Sir Winston Churchill High School, Calgary, Grade 11.
	Michael Wong	Tempo School, Edmonton, Grade 11.
	Graham Hill	Sir Winston Churchill High School, Calgary.
	Stephanie Li	Sir Winston Churchill High School, Calgary.
	Cindy Qian	Harry Ainlay High School, Edmonton.
	David Ting	William Aberhart High School, Calgary.

We offer our congratulations to the above students, their schools and their teachers.

2006 Canadian Mathematical Olympiad

1. Let $f(n,k)$ be the number of ways of distributing k candies to n children so that each child receives at most 2 candies. Determine the value of

$$f(2006,1)+f(2006,4)+f(2006,7)+\cdots+f(2006,4009)+f(2006,4012).$$

2. Let ABC be an acute triangle. $DEFG$ is a variable rectangle with D on AB, E on AC and both F and G on BC. Determine the locus of the point of intersection of DF and EG.

3. In a rectangular array of non-negative real numbers with m rows and n columns, each row and each column contains at least one positive element. Moreover, if a row and a column intersect in a positive element, then the sums of their elements are the same. Prove that $m=n$.

4. Consider a tournament with $2n+1$ teams, where each team plays each other team exactly once. We say that three teams X, Y and Z form a *cyclic triple* if X beats Y, Y beats Z and Z beats X. There are no ties.

   (a) Determine the minimum number of cyclic triples possible.

   (b) Determine the maximum number of cyclic triples possible.

5. A is a point on a circle with diameter BC. D is a point on the circle on the opposite side of BC to A, E is a point on the minor arc CA and F is a point on the minor arc AB. The tangents to the circle at D, E and F intersect the line AB at D', E' and F', and the line AC at D'', E'' and F'' respectively. If $DD'=DD'', EE'=EE''$ and $FF'=FF''$, prove that DEF is an equilateral triangle.

Answers:
1. 3^{2005}.
2. the segment joining the midpoint of BC and the midpoint of the altitude from A to BC.
4. (a) 0; (b) $\frac{n(n+1)(2n+1)}{6}$.

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News Item

Jeffrey Mo, the winner of the Second Round (as well as being in a three-way tie for first in the First Round), won the same contest six years ago as a Grade 10 student. He finished third the year before when he was still in Grade 5. His experience in between the two winning performances was most unusual. He enrolled in the University of Calgary as a regular student and studied there for some time before resuming his junior high school education in Grade 9, moving through Grades 10 (again) and 11 and is currently in Grade 12. This unfortunately means that he is no longer eligible for the International Mathematical Olympiad.

Jerry Lo, who finished second to Jeffrey, is a visiting student from Taiwan. His first trip to Edmonton was seven years ago, and Jeffrey was in town for a summer camp. They met, but probably neither can remember much about it. The following year, Jerry spent the entire school year here. Three years ago, he won this contest as a Grade 9 student. Like last time, he is not eligible for any of the fellowships because he is not even a landed immigrant of Canada.

2006 International Mathematical Olympiad

1. Let ABC be a triangle with incentre I. A point P in the interior of the triangle satisfies $\angle PBA + \angle PCA = \angle PBC + \angle PCB$. Prove that $AP \geq AI$, and that equality holds if and only if $P = I$.

2. Let P be a regular 2006-gon. A diagonal of P is called *good* if its endpoints divide the boundary of P into two parts, each composed of an odd number of sides of P. The sides of P are also called good. Suppose P has been dissected into triangles by 2003 diagonals, no two of which have a common point in the interior of P. Determine the maximum number of isosceles triangles having two good sides that could appear in such a configuration.

3. Determine the least real number M such that for all real numbers a, b and C, the inequality $|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| \leq M(a^2 + b^2 + c^2)^2$ holds.

4. Determine all pairs (x, y) of integers such that $1 + 2^2 + 2^{2x+1} = y^2$.

5. Let $P(x)$ be a polynomial of degree $n > 1$ with integer coefficients and let k be a positive integer. Consider the polynomial $Q(x) = P(P(\cdots P(P(x))\cdots))$, where P occurs k times. Prove that there are at most n integers t such that $Q(t) = t$.

6. Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Prove that the sum of the areas assigned to the sides of P is at least twice the area of P.

Answers:
2. 1003. **3.** $\frac{9\sqrt{2}}{32}$. **4.** (0,2), $(0, -2)$, (4,23), $(4, -23)$.

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- **The University of Alberta.**