Alberta High School Mathematics Competition Newsletter

Volume 16, Number 1 **October 1, 2006**

As announced last year, the **Nickle Family Foundation** of Calgary has decided to part amicably with us after a good working relationship which has lasted half a century. The **Alberta High School Mathematics Competition Board** is pleased to announce that **ConocoPhillips Canada**, an oil company based in Calgary, will replace the Foundation as our major sponsor. Henceforth, the first prize in the Second Round of our contest will be renamed the **ConocoPhillips Canada Fellowship**.

The next item of news is a tragic one. **Robert Barrington Leigh**, formerly of Old Scona Academic High School in Edmonton, died in the river valley of his home town in the evening of August 13, 2006. In our AHSMC, Part I, he finished second in 2001 and first in 2002. In Part II, he finished second in 2000 and 2001, and first in 2002 and 2003. In the Canadian Mathematical Olympiad, he had an Honorable Mention in 2000 and in 2002, plus a third place finish in 2003.

Robert had represented Canada twice in the International Mathematical Olympiad, winning a Bronze Medal each time. He also won a Silver Medal in the International Physics Olympiad. He had just completed three years of undergraduate studies at the University of Toronto. In each year, he placed in the top fifteen in the prestigious William Lowell Putnam Mathematics Competition for university students across North America. The AHSMC will be establishing a prize to commemorate Robert.

On a happier note, the 2006 International Mathematical Olympiad was held in Slovenia. **David Rhee** of McNally High School, Edmonton, represented Canada for the third year in a row, earning a Silver Medal this time. The overall team standing (unofficial) was fifteenth place. The other five National Team members all won medals, four Silver and a Bronze.

David is entering the University of Waterloo. Based on the work done at last year's International Mathematics Tournament of the Towns Summer Seminar, David and Jerry Lo of Taiwan will have a joint paper, *Non-transitivity in Tournaments*, appearing in the September issue of *Crux Mathematicorum*, a publication of the Canadian Mathematical Society. Jerry has come to Edmonton to finish his high school studies in Ross Sheppard High School.

Students aspiring to get on the national IMO team should go to the website http://www.cms.math.ca and click on "Mathematics Competitions". There are many useful pieces of information. In particular, go to the "Mathematical Olympiad Correspondence Program". This is the first step in the selection process, and it provides valuable training too.

A contest open to all Canadian students is the "Canadian Open Mathematics Challenge". This is usually written in November, and students register through their own schools. Top performers are invited to participate in the Canadian Mathematical Olympiad, usually written the following March.

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On the following pages, we reproduce the paper of last year's AHSMC, Part I.

The answers are *acae ccdd abce ccdc*.

Alberta High School Mathematics Competition
Part I November 16, 2005.

1. The value of $2005 \times 20042004 - 2004 \times 20052005$ is

   (a) 0 (b) 10000 (c) 2003000 (d) 2005000 (e) none of these

2. An ice cream store has 20 kinds of ice cream. A customer may get one scoop or two scoops of ice cream, and if she gets two scoops they can be the same or different, but the order of the scoops does not matter. The number of different cones possible is

   (a) 210 (b) 220 (c) 230 (d) 420 (e) none of these

3. In triangle ABC, let D be the midpoint of BC and let E be on AD such that $ED = 2AE$. If the area of triangle ABC is 150, then the area of triangle ABE is

   (a) 25 (b) 32.5 (c) 50 (d) 75 (e) none of these

4. Let a, b and c be real numbers and $x = 11c - a - b$. If $b - a - 3c \leq -2$ and $b - 2a + c \geq 3$, then

   (a) $x \in [0, 1]$ (b) $x \in [2, 5]$ (c) $x \in [6, 9]$ (d) $x \in [10, 11]$ (e) $x \in [12, \infty)$

5. Penelope has 3 red socks, 3 yellow socks and 3 blue socks. If she picks them without looking, the smallest number she must take in order to guarantee that she has four socks that form two pairs of socks of matching colours is

   (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

6. A gazebo consists of a square based pyramid on top of a square based prism. All vertical edges of the prisms have lengths 2 metres. All other edges have lengths 3 metres. The height, in metres, of the top of this gazebo from the ground is

   (a) $2 + \frac{\sqrt{2}}{2}$ (b) $\sqrt{\frac{17}{2}}$ (c) $2 + \frac{3\sqrt{2}}{2}$ (d) $\sqrt{22}$ (e) none of these

7. If f is a function defined on the positive real axis and $f(x) + 3f(\frac{1}{x}) = x - \frac{5}{x} + 4$ for all $x > 0$, then $f(\frac{1}{2})$ is

   (a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2 (e) 4

8. In the sequence obtained by omitting the squares and the cubes from the sequence of positive integers, 2005 sits on the position

   (a) 1949 (b) 1950 (c) 1951 (d) 1952 (e) 1953

9. The positive integer a is such that the inequality $2a + 3x \leq 101$ has exactly six solutions in positive integers x. The number of possible values of a is

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

10. P is a point inside a parallelogram $ABCD$. If the area of triangle PAD is one-third that of $ABCD$, and the area of triangle PCB is 6 square centimetres, then the area, in square centimetres, of the parallelogram is

    (a) 24 (b) 36 (c) 48 (d) 60 (e) 72

11. There are three problems in a contest. Students win bronze, silver or gold medals if they solve 1, 2 or 3 problems respectively. Each problem is solved by exactly 60 students, and there are exactly 100 medalists. The difference between the number of bronze medalists and the number of gold medalists is

    (a) 0 (b) 10 (c) 20 (d) 30
    (e) not uniquely determined

12. A tile is obtained from a 3×3 square by first removing two squares in opposite corners, and then one square adjacent to each of the two squares removed, so that the remaining five squares form a connected piece. The tile is placed on an infinite piece of graph paper so that it covers exactly five squares. It may be turned over or rotated. We wish to paint the squares of the graph paper in such a way that no matter where the tile is placed, it never covers two squares of the same colour. The smallest number of colours needed is

    (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

13. The positive numbers $a,\ b$ and c are such that $a+b+c=7$ and $\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}=\frac{10}{7}$. The value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is

    (a) 1 (b) 3 (c) 7 (d) 10
    (e) dependent on $a,\ b$ and c

14. The number of ordered pairs of integers (x, y) such that $xy = x^2 + y^2 + x + y$ is

    (a) 3 (b) 4 (c) 6 (d) 8 (e) none of these

15. The positive integer n that satisfies $\log_2 3 \log_3 4 \cdots \log_n(n+1) = 2005$ is a multiple of

    (a) 2 (b) 3 (c) 5 (d) 31 (e) none of these

16. Three spheres of unit radius sit on a plane and they are tangent to one another. A large sphere with its centre in the plane contains all three unit spheres and is tangent to them. The radius of the large sphere is

    (a) $\sqrt{2}+1$ (b) $\sqrt{\frac{5}{3}}+1$ (c) $\sqrt{\frac{7}{3}}+1$ (d) $\sqrt{3}+1$ (e) none of these