The Alberta High School Mathematics Competition

Part II, February 4th, 2015.

Problem 1.
Find the number of isosceles triangles of perimeter 2015 such that all three sides are **odd** integers.

Problem 2.
Find all pairs (m, n) of positive integers such that $m^3 - n^3 = 5mn + 43$.

Problem 3.
Let $f : [0, 4] \to [0, \infty)$ be such that $f(4) = 2$ and $f(x + y) \geq f(x) + f(y)$ for any real numbers x and y in the closed interval [0,4] such that $x + y \leq 4$.

(a) Suppose that $0 \leq x \leq y \leq 4$. Show that $f(y) \geq f(x)$.

(b) Show that $f(x) \leq x$ for any x in [0,4].

Problem 4.
E and F are points on the sides CA and AB, respectively, of an equilateral triangle ABC such that EF is parallel to BC. G is the intersection point of medians in triangle AEF and M a point on the segment BE. Prove that $\angle MGC = 60^\circ$ if and only if M is the midpoint of BE.

Problem 5.
Karys is helping her father move basketballs from his car to the gymnasium. She carries either 3 or 4 basketballs each trip, while her father carries 6 or 7 basketballs each trip. Altogether Karys makes 15 more trips and carries 15 fewer basketballs than her father.

(a) Determine the minimum number of basketballs that Karys carries.

(b) Determine the maximum number of basketballs that Karys carries.