The Alberta High School Mathematics Competition

November 18, 2014.

Question 1.
When the repeating decimal $0.\overline{6}$ is divided by the repeating decimal $0.\overline{3}$, the quotient is

(a) $0.\overline{2}$ (b) 2 (c) $0.\overline{5}$ (d) 0.5 (e) none of these

Question 2.
The number of two-digit positive integers such that the sum of the two digits is 12 is

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

Question 3.
Let $a > b > 0$ be prime numbers. Of the following five numbers, the one which cannot be equal to $a - b$ is

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

Question 4.
Chau and Matt are picking berries. On Monday Matt picks twice as many kilograms of berries as Chau does, and on Tuesday Chau picks twice as many kilograms of berries as Matt does. Between them, they pick a total of 30 kilograms of berries over the two days. The number of kilograms of berries Chau picks over the two days is

(a) 14 (b) 15 (c) 16 (d) some number greater than 16

(e) not uniquely determined

Question 5.
For every set of five of the numbers 1, 2, . . . , 2014, Lac writes down the smallest of the five numbers. The largest number she writes down is

(a) 5 (b) 2009 (c) 2010 (d) 2014 (e) none of these

Question 6.
The sum of four consecutive integers is n. The largest of these four numbers is

(a) n (b) $\frac{n}{4}$ (c) $\frac{n-2}{2}$ (d) $\frac{n+6}{4}$ (e) none of these

Question 7.
Let $f(n) = 2n^3$ for any positive integer n. For any odd number $m \geq 3$, the largest positive integer k such that 2^k divides $f(f(f(m)))$ is

(a) 4 (b) 8 (c) 12 (d) 13 (e) not uniquely determined

Question 8.
Let f be a function such that $f(x) + 3f(\frac{1}{x}) = x^2$ for any nonzero real number x. Then the value of $f(-3)$ is

(a) $-\frac{77}{72}$ (b) $-\frac{37}{36}$ (c) $-\frac{25}{24}$ (d) $-\frac{13}{12}$ (e) none of these

Question 9.
The number of integer pairs (m, n) such that $mn = m + n$ is

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

Question 10.
The number of five-digit positive integers such that the digits are alternately odd and even (either odd-even-odd-even-odd or even-odd-even-odd-even) is

(a) 10×5^5 (b) 2×5^5 (c) 5^5 (d) 9×5^4 (e) none of these

Question 11.
Let a, b and c be real numbers such that $a + b + c = 5$ and $\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} = 6$. The value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is

(a) 21 (b) 23 (c) 25 (d) 27 (e) not uniquely determined

Question 12.
P is a point inside an acute triangle ABC. D, E and F are the feet of the perpendiculars from P on BC, CA and AB respectively. If $BD = 1$, $DC = 10$, $CE = 6$, $EA = 9$ and $AF = 13$, the length of FB is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Question 13.
Three candles which can burn for 30, 40 and 50 minutes respectively are lit at different times. All three candles are burning simultaneously for 10 minutes, and there is a total of 20 minutes in which exactly one of them is burning. The number of minutes in which exactly two of them are burning is

(a) 35 (b) 45 (c) 70 (d) 90 (e) none of these

Question 14.
P is a point inside a convex quadrilateral $ABCD$ of area 168 such that $PA = 9$, $PB = PD = 12$ and $PC = 5$. The perimeter of the quadrilateral is

(a) 38 (b) 56 (c) 58 (d) 60 (e) 62

Question 15.
For any real numbers x and y, the minimum value of $x^4 - 5x^2 + y^2 + 2x + 2y + 2xy + 6$ is

(a) -5 (b) -4 (c) -1 (d) 0 (e) none of these

Question 16.
Four cages are arranged in a 2×2 formation. Each cage contains some chickens and some rabbits (some cages may contain only chickens or only rabbits). The total number of heads in the two cages in the first row is 60. The total number of legs in the two cages in the second row is 240. The total number of heads in the two cages in the first column is 70. The total number of legs in the two cages in the second column is 230. The minimum number of animals in all four cages is

(a) 120 (b) 128 (c) 145 (d) 180 (e) none of these