The Alberta High School Mathematics Competition

Part I — November 19, 2013.

1. Of the first 100 positive integers 1, 2, . . . , 100, the number of those not divisible by 7 is

   (a) 85 (b) 86 (c) 87 (d) 88 (e) 89

2. The total score of four students in a test is 2013. Ace scores 1 point more than Bea, Bea scores 3 points more than Cec, and Cec scores 2 points more than Dee. None of their scores is divisible by

   (a) 3 (b) 4 (c) 5 (d) 11 (e) 23

3. Of the following five fractions, the largest one is

   (a) $\frac{1}{75}$ (b) $\frac{2}{149}$ (c) $\frac{3}{224}$ (d) $\frac{4}{299}$ (e) $\frac{6}{449}$

4. Two teams A and B played a soccer game on each of seven days. On each day, the first team to score seven goals won. There were no ties. Over the seven days, A won more games than B, but B scored more goals than A overall. The difference in the total numbers of goals scored by B and A is at most

   (a) 17 (b) 18 (c) 19 (d) 20 (e) none of these

5. $ABCD$ is a quadrilateral with $AB = 12$ and $CD = 18$. Moreover, AB is parallel to CD and both $\angle ADC$ and $\angle BCD$ are less than $90°$. P and Q are points on side CD such that $AD = AP$ and $BC = BQ$. The length of PQ is

   (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

6. Each of four cows is either normal or mutant. A normal cow has 4 legs and always lies. A mutant cow has either 3 or 5 legs and always tells the truth. When asked how many legs they have among them, their respective responses are 13, 14, 15 and 16. The total number of legs among these four cows is

   (a) 13 (b) 14 (c) 15 (d) 16 (e) none of these

7. Let a and b be positive integers such that $ab < 100$ and $\frac{a}{b} > 2$. Denote the minimum possible value of $\frac{a}{b}$ by m. Then we have

   (a) $m \leq 2.15$ (b) $2.15 < m < 2.2$ (c) $m = 2.2$

   (d) $2.2 < m < 2.25$ (e) $m \geq 2.25$

8. Let $ABCD$ be a quadrilateral with $\angle DAB = \angle CBA = 90°$. Suppose there is a point P on side AB such that $\angle ADP = \angle CDP$ and $\angle BCP = \angle DCP$. If $AD = 8$ and $BC = 18$, the perimeter of the quadrilateral $ABCD$ is

(a) 70 (b) 72 (c) 74 (d) 76 (e) 78

9. Two bus routes stop at a certain bus stop. Then A bus comes in one-hour intervals while the B bus also comes in regular intervals of a different length. When grandma rests on the bench by the bus stop, one A bus and two B buses come by. Later, grandpa rests on the same bench and eight A buses come by. The *minimum* number of B buses that must have come by during that time is

(a) less than 4 (b) 4 or 5 (c) 6 or 7 (d) 8 or 9 (e) more than 9

10. Suppose that $16^{2013} = a^b$, where a and b are positive integers. The number of possible values of a is

(a) 2 (b) 8 (c) 11 (d) 16 (e) 24

11. The following statements are made about the integers $a,\ b,\ c,\ d$ and e: (i) ab is even and c is odd; (ii) bc is even and d is odd; (iii) cd is even and e is odd; (iv) de is even and a is odd; (v) ea is even and b is odd. The maximum number of these statements which may be correct is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

12. A very small cinema has only one row of five seats numbered 1 to 5. Five movie goers arrive one at a time. Each takes a seat not next to any occupied seat if this is possible. If not, then any seat will do. The number of different orders in which the seats may be taken is

(a) 24 (b) 32 (c) 48 (d) 64 (e) 72

13. Let $f(x) = x^2 + x + 1$. Let n be the positive integer such that $f(n) = f(20)f(21)$. Then the number of distinct prime divisors of n is

(a) 1 (b) 2 (c) 3 (d) 4 (e) more than 4

14. The number of pairs (x, y) of integers satisfying the equation $x^2 + y^2 + xy - x + y = 2$ is

(a) 3 (b) 4 (c) 5 (d) 6 (e) none of these

15. A triangle ABC with $AB = 7,\ BC = 8$ and $CA = 10$ has a interior point P such that $\angle APB = \angle BPC = \angle CPA = 120°$. Let $r_1,\ r_2$ and r_3 be the radii of the circles passing through the vertices of triangles $PAB,\ PBC$ and PCA respectively. The value of $r_1^2 + r_2^2 + r_3^2$ is

(a) 71 (b) 72 (c) 73 (d) 74 (e) 75

16. The list 1, 3, 4, 9, 10, 12, 13, ... contains in increasing order all positive integers which can be expressed as sums of one or more distinct integral powers of 3. The 100-th number in this list is

(a) 981 (b) 982 (c) 984 (d) 985 (e) 999