Book Review

Tomasz Kaczynski, Konstantin Mischaikow, and Marian Mrozek, Computational Homology. In: S. S. Antman, J. E. Marsden, and L. Sirovich (eds.) Applied Mathematical Sciences, Vol. 157, Springer-Verlag Berlin Heidelberg New York. 2004, XVII, 480 pp. Hardcover EUR 79.95, SFR 135.50, GBP 61.50, US $ 69.95, ISBN 0-387-40853-3

Drawing on sophisticated devices such as powerful computers, high-resolution tomographic scanners and remote sensors, scientists are able to model and analyse real-world objects and processes ever more accurately, thereby usually generating enormous amounts of numerical data. Though welcome from a statistical point of view, such an abundance of data can be a mixed blessing: for a concrete problem it may become increasingly difficult to separate relevant from redundant or irrelevant information. This problem is particularly manifest wherever essentially topological information is to be extracted from the data, e.g., if the number and structure of cavities in human tissue is to be determined from an MRI scan, or if a hand gesture is to be recognised from a photograph or video. The central theme of *Computational Homology* is, informally put, that firstly an elaborate mathematical machinery has long been developed to help addressing these topological challenges (essentially by algebraising them), and that secondly and perhaps more practically relevant this mathematical machinery can in fact be turned into a versatile and applicable tool. The latter aspect has only recently been the focus of intense research, and this is the first textbook on what is necessarily a mixture of classical mathematics, computer science, and applications.

Part I, the core section of the book, develops homology, one of the fundamental tools provided by algebraic topology. That the latter, pure mathematical subject may play an important role in the global and qualitative study of objects represented (or representable) by spaces and maps is not too much of a surprise, especially if one is interested in dynamics. Indeed, algebraic topology and the qualitative analysis of dynamical systems have been linked closely ever since both disciplines were founded by Poincaré more than a century ago. Homology is probably the algebraic-topological tool most accessible also to non-mathematicians. This is because homology simply counts objects which are – at least on an elementary level – easy to grasp intuitively: pairs of points that are not endpoints of continuous paths, closed loops that do not bound a disc, etc. The homology actually developed in this book is *cubical* homology, a homology theory not usually found in standard texts on algebraic topology which typically present the simplicial, cellular, and singular theories instead. There are, however, good reasons to utilise cubical homology: it is conceptually very simple and thus accessible to a wide audience; the handling of maps is close to rigorous numerics; and, perhaps most importantly, the theory offers itself for a direct and transparent algorithmisation. In fact, it is a unique feature of *Computational Homology* that every geometric step, however conceptually simple, is broken down into elementary operations. Thus the reader really gets a hands-on experience how topology is turned into algebra and how, in the context of cubical homology, the whole process can be carried out by a computer. The latter is crucial because the necessary computations already become prohibitively cumbersome for objects as simple as a torus (surface of a doughnut). Moreover, for more complicated spaces it is imperative not only to have algorithms available but to find *efficient* ones. Consequently, the book first develops algorithms showing that all quantities

of interest can be computed, and then explains how these algorithms' efficiency can (and for all practical purposes has to) be increased. Throughout, therefore, the reader will have the reassuring feeling that all the mathematical objects involved (homology groups, boundary and homology maps) are actually within reach.

Anyone who, having followed the development of the computational machinery of cubical homology in Part I, may be eager to apply this tool, is in for a slight disappointment: there may still be a long way to go before concrete problems can be tackled, and homology alone is an unlikely panacea. Applications, however, do exist or are emerging, and Part II discusses some of them. The analysis of patterns in human tissue, numerical solutions of PDE, or sign language offer fascinating prospects of algebraic topology being applied to real-world problems. As the authors point out repeatedly, this subject is quite new and developing rapidly. Not surprisingly (given their common origin alluded to earlier), applications of (computational) algebraic topology are already more established in the analysis of dynamical systems. Accordingly, the largest chapter in the book illustrates how computational homology can be used to analyse dynamical systems, e.g., by tracking down periodic orbits or trajectories, and by detecting chaotic dynamics. Basically, this can be achieved by making advanced tools computationally accessible, such as the Lefschetz fixed point theorem and Conley index theory, a field in which the authors are renowned experts. While this section of the book should not be considered a crash course in nonlinear dynamics, it gives the persistent reader an inspiring impression of what can be achieved by means of the computational tools developed earlier – provided that they are combined with powerful mathematical results. Other topics covered (rather informally) by Part II concern the categorical character of cubical homology, its finer internal structure (relative homology, Mayer–Vietoris) and its relations to other homology theories. The book concludes with Part III which contains some preparatory material from topology and algebra as well as a summary of the syntax of algorithms used throughout the main part of the text.

This is the first authoritative textbook in a rapidly emerging field, aimed at advanced undergraduates and beginning graduates in mathematics, computer science and engineering. In view of this mixed audience, mathematical prerequisites have been kept minimal. For a mathematics student interested in algebraic topology, *Computational Homology* could be an ideal text complementing (but not replacing!) a more traditional, less computational course. Computer science students may find many challenges in terms of the efficiency and complexity of algorithms. The book offers a reliable yet practical introduction to (cubical) homology, with a strong emphasis on computational aspects. Hands-on experience can be gained through the many problems within the book and also by means of the software packages of the computational homology program CHomP, developed largely by the authors. Naturally, there is a price to pay for this level of concreteness: making mathematics computer-ready can be uninspiring at times and does not foster a clear, reliable geometric intuition. In a sense, this applies to every computationally oriented mathematics text and it is, in the present case and the

reviewer's opinion, more than compensated for by the easy accessibility and the careful, pedagogical presentation. However, a critical reader might find some of the definitions and notations more complicated than necessary, and also sense a certain lack of elegance throughout.

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