Book Review

Geon Ho Choe, Computational Ergodic Theory. In: M. Bronstein, A. M. Cohen, H. Cohen, D. Eisenbud, and B. Sturmfels (eds.), Algorithms and Computation in Mathematics, Vol. 13, Springer-Verlag Berlin Heidelberg New York. 2005, XIX, 453 pp. Hardcover EUR 69.95*, SFR 123.50, £ 54.50, US $ 89.95 ISBN 3-540-23121-8 (* gültig in Deutschland incl. MWSt.)

Informally put, ergodic theory constitutes the probabilistic study of dynamics. While some important incitements trace back to Boltzmann and Gibbs in the late 19th century, the field really took form in the 1930s. Since then, ergodic theory has undergone enormous development. Topics which nowadays attract much attention include limit theorems, the study of recurrence properties, and the theory of entropy. Fruitful and deep interactions with other disciplines abound, notably with number and probability theory, and functional analysis. While the field's multifaceted nature fascinates the expert, it makes the subject notoriously difficult for students to familiarise with and get enthused by. A computational approach, as advocated by the book under review, can help to make ergodic theory appealing to a wide range of science students by providing some hands-on experience. Such an approach appears to be profoundly different from the one found in standard textbooks such as e.g., Walters, P.: *An Introduction to Ergodic Theory* (Springer, 1982) or Petersen, K.: *Ergodic Theory* (Cambridge University Press, 1983). If carefully executed, however, it definitely has its merits.

Following the brief introduction of a variety of tools, including a short course in Maple, *Computational Ergodic Theory* first studies invariant measures, the most fundamental objects in ergodic theory. The material presented is standard except perhaps a section on so-called coding maps, the usefulness of which, however, does not seem to extend beyond the generation of a few entertaining pictures. The concept of ergodicity is then explored through irrational rotations, continued fractions, and the Hénon and standard mapping families. Stronger signatures of stochasticity, e.g. rapid decay of correlations, conformance with the Central Limit Theorem, and the statistics of return times, have been studied extensively in recent years. In Chaps. 4 and 5 the reader can gain computational acquaintance with these topics. The more classical theme of circle homeomorphisms and skew-products over ergodic circle maps is studied in Chaps. 6 and 7. One of the main objectives of ergodic theory is to distinguish and classify different types of dynamical behaviour. A natural step towards this goal is to find invariants (numbers, sets, sequences of groups, etc.) such that equivalent systems yield the same value for the respective invariant while – ideally – non-equivalent systems yield different values. Entropy, a non-negative number or $+\infty$ and perhaps the most important of all invariants in ergodic theory, is discussed in Chap. 8. Quite often, the entropy of a system is intimately related to other characteristics such as dimension and average return rates. These fascinating topics are explored numerically in Chaps. 12 and 13. Other important tools in the analysis of dynamical systems are Lyapunov exponents and invariant (stable and unstable) manifolds which pertain in particular to smooth hyperbolic systems like Anosov diffeomorphisms and solenoid maps. The text concludes with a short discussion of data compression (Huffman, Lempel–Ziv, arithmetic coding) in terms of symbolic dynamics and entropy introduced earlier.

An attentive reader of *Computational Ergodic Theory* cannot fail to notice that the word *computational* is used quite differently here than it is e.g. in computational homology, linear algebra, or fluid dynamics. For the latter, well-established disciplines of applied mathematics the rigorous analysis of algorithms, as well as their efficiency, plays an all-important role and in fact determines the flavour of the whole discipline. (Think for instance of the numerical eigenvalue problem.) Contrary to this, the discipline called computational ergodic theory does not yet exist, and for reasons explicated below it seems questionable whether it ever will. Correspondingly, the computations in *Computational Ergodic Theory*, carried out by means of symbolic mathematical software, are of no substantial interest in themselves. Rather they are used to underpin abstract results with concrete numbers. Thus a perhaps more precise (yet certainly less appealing) title for the book could have been *Numerical Experiments in Ergodic Theory*. Numerical evidence is indeed invaluable in developing a clear feeling for the various concepts of the theory. Even the more numerically inclined student, however, will hardly be challenged by computations whose sole purpose is to illustrate proven facts and which do not lead to new stimulating speculations about dynamics. The rigorous analysis of errors, which inevitably arise e.g. in approximating invariant densities or generating uniformly distributed data, might involve more sophisticated tools or might even be prohibitively hard at the present time. Without such an analysis, however, statements like "If n is large, say $n = 10$", which permeate the book, are likely to leave the reader with the uncomfortable feeling of not being told the full story. While this remark is not at all meant to disqualify the efforts made in *Computational Ergodic Theory*, it explains why a computational approach may, after all, not be too useful for making ergodic theory appealing to a wide audience. This said, it is necessary to point out that some aspects of the subject are nevertheless highly computational. The accurate determination of Lyapunov exponents, for instance, requires sophisticated methods, as does the computation of invariant manifolds. The latter has lately become a real testing ground for cutting-edge computational techniques in dynamics. Unfortunately though, this is not reflected by the rather unsophisticated (and thus ultimately non-computational) way by which Lyapunov exponents and invariant manifolds are presented in the text.

Overall, with its mixture of theoretical and experimental parts, *Computational Ergodic Theory* constitutes an interesting addition to the textbook literature on the subject. The expert reader will no doubt find several less widely known examples together with ideas for future courses on dynamical systems. Contrary to what the information on the back cover seems to suggest, familiarity with measure theory is indispensable in order to benefit from the text (and to appreciate the numerical experiments). Any student seriously interested in ergodic theory is thus well-advised to at least complement this book with one of the classical texts mentioned earlier.

A final editorial remark may be in order. The Maple codes and the output they produce (altogether more than 200 figures and diagrams) obviously constitute a central part of the text. One might have expected that this vital part would be presented with utmost care and diligence. This, alas, is not the case. Many of the Maple codes are unstructured and marred by obscure choices of parameters. Numerous figures thereby produced are poorly scaled and labelled and convey only exiguous information. Rigorous editing standards could have helped to make this vital part of the book more informative and enjoyable.

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