Assignment # 7

(1) Find the Laplace transform of the following 2π-periodic functions:

(i) $f_1(t) = t$ if $0 \le t < 2\pi$ ("sawtooth wave");

(ii) $f_2(t) = \begin{cases} 1 & \text{if } 0 \le t < \pi \\ 0 & \text{if } \pi \le t < 2\pi \end{cases}$ ("square wave");

(iii) $f_3(t) = \left|\sin\left(\frac{1}{2}t\right)\right|$ ("rectified sine wave").

(2) Consider the first-order ODE for $y = y(t)$,

$$y' + y = f_1\,, \tag{7.1}$$

where f_1 is the sawtooth wave of Problem 1(i).

(i) Using Laplace transform, find the general solution of (7.1).

(ii) Find a 2π-periodic function y^* with the property that $\lim_{t\to+\infty} |y(t) - y^*(t)| = 0$ for *every* solution y of (7.1).

(3) Find the unique solution $y = y(t)$ of the IVP

$$y'' + y = f\,, \quad y(0) = y'(0) = 0\,,$$

with the 2π-periodic function $f = f(t)$ given by

(i) $f(t) = 2\pi \sin t$;

(ii) $f = f_1$, the sawtooth wave of Problem 1(i).

(4) Determine a function $y = y(t)$ such that

$$y(t) + e^{-t}\int_0^t e^v y(v)\,\mathrm{d}v = \sin t \quad \text{for all } t \ge 0\,. \tag{7.2}$$

Hint: Laplace transform (7.2); think convolution.

Please deposit your assignment into the MATH 334 box on the third floor of CAB. Make sure all sheets of your submission are stapled together and your name and student ID number is clearly written on the front page.

This assignment is due **Thursday, 17 March 2016, at 4:00 pm**.

No late submissions !!