REVIEW INFO

The Review will be held in ETLE1-001 on Thursday Oct. 14 from 5pm. The questions will be drawn from recent midterms, and are attached.

The Review is FREE and all students in the course are welcome. However, donations to Engineers Without Borders will be gratefully accepted

Multiple Choice

1. The change in $f = z + z\cos x - y\sin x + y$ as we move from $P_0 = (0,-1,2)$ towards $(2,1,0)$ a distance of 0.2 is estimated by differentials. The estimated value of the change in f is:

   (a) .3; (b) .2; (c) .1; (d) 0; (e) -1.

2. At what point on the surface $z = x^2 + y^2$ is the tangent plane parallel to the plane $x + 3y + 2z = 1$?

   (a) $(\frac{-1}{4}, \frac{-3}{4}, \frac{5}{8})$, (b) $(0, \frac{-1}{4}, \frac{1}{16})$, (c) $(-2, 3, 13)$, (d) $(\frac{-1}{4}, 0, \frac{1}{16})$, (e) $(\frac{-1}{4}, \frac{3}{2}, \frac{37}{16})$.

3. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$ equals:

   (a) does not exist; (b) 0; (c) $\frac{1}{5}$; (d) $\frac{1}{2}$; (e) 1.

4. If f, g are twice differentiable functions of a single variable, then $u(x,y) = xf(x+y) + yg(x+y)$ satisfies

$$au_{xx} - 2u_{xy} + u_{yy} = 0$$

   provided that the constant a equals:

   (a) -2; (b) -1; (c) 0; (d) 1; (e) 2.

Long Answers

1. Let $f(x,y,z) = ze^{zy+x^2}$. You are at the point $P = (2,-4,1)$.

   (a) Find the directional derivative of f as you move from P towards the point $(0,2,-2)$.

   (b) Find the maximum rate of change of f at P and the direction in which it occurs.

   (c) In which direction(s) should you head from P for the rate of change of f to equal 0?

2. Find all local maximum and minimum values and saddle point(s) of the function

$$f(x,y) = 30x - 2x^3 - 12xy + 3y^2.$$

You must identify every critical point you find as a maximum, a minimum, or a saddle point.

3. Use Lagrange Multipliers to find the maximum and minimum values of $f(x,y,z) = 3x - y - 3z$ subject to the constraints: $x + y - z = 0$, $x^2 + 2z^2 = 1$.

MULTIPLE CHOICE

1. If z is defined implicitly as a function of x and y by the equation

$$z\sin(\ln(xy)) + yz^3 + xy^2 + 7 = 0,$$

then $\frac{\partial z}{\partial x}$ evaluated at the point $x = 1$, $y = 1$ with $z = -2$ equals

(a) $-\frac{1}{12}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) 0 (e) $\frac{1}{12}$

2. The limit of the function $f(x,y) = \dfrac{x^2 + \sin^2 y}{2x^2 + y^2}$ as $(x,y) \to (0,0)$ is

(a) $+\infty$ (b) 1 (c) 0 (d) non-existent (e) $\frac{1}{2}$

3. Which of the following is a saddle point of $f(x,y) = \cos x \cos y$?

(a) $(\frac{\pi}{2}, \frac{\pi}{4})$ (b) $(\frac{\pi}{2}, \frac{\pi}{2})$ (c) $(0, \pi)$ (d) $(0,0)$ (e) $(\frac{\pi}{4}, -\frac{\pi}{2})$

4. Suppose that $f(x,y,z) = xe^{y-z^2}$, where $x = 2uv$, $y = u - v$, and $z = u + v$. Then the value of $\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}$ at $u = 3$ and $v = -1$ is

(a) 44 (b) -48 (c) 52 (d) 36 (e) -40

Long Answer

1. Find the maximum and the minimum values of the function $f(x,y,z) = x(x-1) + yz$ and the points at which they are attained on the ellipsoid given by $(x-1)^2 + \frac{y^2}{2} + 2z^2 = 1$.

2. Let $f(x,y,z) = \ln(x^2 + z^2 - 1) + 6y + z$.
   (a) Find the direction in which $f(x,y,z)$ increases most rapidly at $(1,0,1)$.
   (b) Find the directional derivative of $f(x,y,z)$ at $(1,0,1)$ in the direction found in (a).
   (c) Find also the direction in which $f(x,y,z)$ decreases most rapidly at $(1,0,1)$.

3. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 50 cm, respectively, with a possible error of 2 mm in each dimension. Use differentials to approximate the maximum error in calculating the surface area of the box.