MATH 201—Final Examination

Date: April 26, 2014 *Time*: 2 hours

Surname: ____________________ **Given name(s):** ____________________

(Please print.)

ID#: __________________ **Signature:** ________________________________

Please check your section/instructor!

Section	*Instructor*	✓
Q1	P. Minev	
R1	H. van Roessel	
S1	C. Quigley	
T1	V. Yaskin	
U1	P. Minev	
V1	V. Bouchard	

	Multiple Choice	*Long Answer Questions*			
		1	2	3	**Total**
Maximum	20	20	20	30	**90**
Mark					

Instructions

1. This is a closed book exam. ***No books, notes, or calculators are allowed!***

2. There are ***four multiple choice questions*** and ***three long answer questions***; for the long answer questions you must show all your work.

3. ***Do the multiple choice questions first; the answer sheet will be collected after 90 minutes.***

4. ***A mark of zero will be given for multiple choice questions with more than one circled answer.***

5. ***A Laplace Transform Table is attached.***

Multiple Choice Questions

Mark your answers ***on the official answer sheet on the last page*** and ***detach it***. It will be collected ***after 90 minutes***.

1. Let $f(t)$ be a periodic function with a period of 2 and let its windowed version be

$$f_2(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & 1 \le t \le 2. \end{cases}$$

The Laplace transform of $f(t)$ is

(a) $1/s$ (b) 0 (c) $\dfrac{1-e^{-s}}{s(1+e^{-2s})}$ (d) $\dfrac{1-e^{-s}}{s(1-e^{-2s})}$ (e) None of these.

2. The Laplace transform of $f(t) = t^2\delta(t-3)$, where δ is the Dirac Delta function, is

(a) $9e^{-3s}$ (b) e^{-3s} (c) $2/s^3$ (d) $2e^{-3s}/s^3$ (e) None of these.

3. Consider the initial value problem

$$x^2y'' + xy' - y = 0, \quad y(1) = 1, \quad y'(1) = 0.$$

Then at $x = 2$, y equals

(a) $5/4$ (b) $1/2$ (c) 1 (d) $3/2$ (e) 0.

4. If

$$f(x) = |x| + 1, \quad -1 \le x \le 1,$$

and f is expanded in a Fourier series

$$a_0/2 + \sum_{n=1}^{\infty} \left[a_n \cos(n\pi x) + b_n \sin(n\pi x)\right],$$

then a_0, a_2, b_5 are (in this order)

(a) $(1, 0, 1)$ (b) $(3, -1/\pi^2, 0)$ (c) $(3, 0, 0)$ (d) $(1, -1/\pi^2, 0)$ (e) None of these.

Long Answer Questions

You must show all your work.

1. Consider the following problem 20 marks

$$y' + 4\int_0^t y(v)dv = f(t), \qquad y(0) = 0,$$

where

$$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & 1 \le t. \end{cases}$$

(a) Write $f(t)$ in terms of unit step functions.

(b) Find the solution, $y(t)$, of the problem.

2. Find the first three nonzero coefficients of the power series expansion about $x_0 = 0$ for the solution of the initial value problem 20 marks

$$5y'' - (x-1)y' = x^2 + x, \qquad y(0) = 1, \quad y'(0) = 0.$$

3. Use the method of separation of variables to find the formal solution of the problem 30 marks

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} + x, \quad 0 < x < \pi, \quad t > 0$$

$$u(0,t) = 0, \quad u(\pi,t) = \pi^3/6, \quad t > 0,$$
$$u(x,0) = -x^3/6 + \pi^2 x/3 + 1, \quad 0 < x < \pi.$$

You must:

(a) formulate and solve the corresponding eigenvalue problem,

(b) find the series expansion for the solution,

(c) compute the coefficients of the expansion.

Multiple Choice Answer Sheet

Surname: ____________________ **Given name(s):** ____________________
(Please print.)

ID#: ____________________ **Signature:** ____________________

Please, check your section/instructor!

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Q1	P. Minev	
R1	H. van Roessel	
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This is the ***official*** answer sheet for multiple choice questions.
A mark of ***zero*** will be given for questions with more than one circled answer.

Question	*Answer*				
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e

Detach this page!
It will be collected after 90 minutes.

A TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s) = \mathscr{L}\{f\}(s)$	$f(t)$	$F(s) = \mathscr{L}\{f\}(s)$
1. $f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$	**20.** $\frac{1}{\sqrt{t}}$	$\frac{\sqrt{\pi}}{\sqrt{s}}$
2. $e^{at}f(t)$	$F(s-a)$	**21.** $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
3. $f'(t)$	$sF(s) - f(0)$	**22.** $t^{n-(1/2)}$, $\quad n = 1, 2, \ldots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+(1/2)}}$
4. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0)$	**23.** t^r, $\quad r > -1$	$\frac{\Gamma(r+1)}{s^{r+1}}$
	$- \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$	**24.** $\sin bt$	$\frac{b}{s^2+b^2}$
5. $t^nf(t)$	$(-1)^nF^{(n)}(s)$	**25.** $\cos bt$	$\frac{s}{s^2+b^2}$
6. $\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	**26.** $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
7. $\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$	**27.** $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
8. $(f * g)(t)$	$F(s)G(s)$	**28.** $\sinh bt$	$\frac{b}{s^2-b^2}$
9. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$	**29.** $\cosh bt$	$\frac{s}{s^2-b^2}$
10. $f(t-a)u(t-a)$, $\quad a \geq 0$	$e^{-as}F(s)$	**30.** $\sin bt - bt\cos bt$	$\frac{2b^3}{(s^2+b^2)^2}$
11. $g(t)u(t-a)$, $\quad a \geq 0$	$e^{-as}\mathscr{L}\{g(t+a)\}(s)$	**31.** $t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
12. $u(t-a)$, $\quad a \geq 0$	$\frac{e^{-as}}{s}$	**32.** $\sin bt + bt\cos bt$	$\frac{2bs^2}{(s^2+b^2)^2}$
13. $\Pi_{a,b}(t)$, $\quad 0 < a < b$	$\frac{e^{-sa} - e^{-sb}}{s}$	**33.** $t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
14. $\delta(t-a)$, $\quad a \geq 0$	e^{-as}	**34.** $\sin bt\cosh bt - \cos bt\sinh bt$	$\frac{4b^3}{s^4+4b^4}$
15. e^{at}	$\frac{1}{s-a}$	**35.** $\sin bt\sinh bt$	$\frac{2b^2s}{s^4+4b^4}$
16. t^n, $\quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$	**36.** $\sinh bt - \sin bt$	$\frac{2b^3}{s^4-b^4}$
17. $e^{at}t^n$, $\quad n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}$	**37.** $\cosh bt - \cos bt$	$\frac{2b^2s}{s^4-b^4}$
18. $e^{at} - e^{bt}$	$\frac{(a-b)}{(s-a)(s-b)}$	**38.** $J_\nu(bt)$, $\nu > -1$	$\frac{(\sqrt{s^2+b^2}-s)^\nu}{b^\nu\sqrt{s^2+b^2}}$
19. $ae^{at} - be^{bt}$	$\frac{(a-b)s}{(s-a)(s-b)}$		