CURRICULUM VITAE

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Date and place of birth: April 28, 1953, Vancouver, B.C., KANADA

Nationality: Canadian Hobbies: Wine making

Education

Ph.D.	1981	University of British Columbia (Algebraic Geometry)
B.Sc.	1976	University of British Columbia (Pure Mathematics)
Dipl. T.	1973	BCIT, Burnaby, B.C. (BioMedical Electronics,)

Employment History

University of Alberta	Prof. & Assoc. Chair	2006 - Present
University of Alberta	Full Professor	1994 - 2006
University of Alberta	Associate Professor	1991 - 1994

Mathematical Specialization

Main field: (Transcendental) Algebraic Geometry – Algebraic Cycles.
Other fields: Hodge theory, Arithmetic Geometry, Algebraic K-theory.
Current research interests: Regulators on algebraic cycles.

Honours, Distinctions, Fellowships, Memberships

(i) NATO ASI Directorship. Was awarded BF$3,300,000 (BF = Belgian Francs) for a NATO ASI conference on the Arithmetic and Geometry of Algebraic Cycles, as well as CDN$60,000 from CRM to run a concurrent CRM Summer School, held in Banff, Alberta, June 7 - 19, 1998.
(ii) Delta Chi Fraternity Teaching Award, March 2002.
(iii) Membership, IAS at Princeton, fall term 1986.
(iv) Post-Doctoral Fellow, Harvard University, 1980 - 1981.

Grant Information *Although I have been additionally supported periodically by a number of agencies in the past, my main research support has always been from NSERC. This is my most recent:*

- NSERC Discovery Research Grant (5 year period beginning April 1, 2008), $24,000 per year. (Previous NSERC grant was $20,000 per year.)

Articles submitted

- (Co-authored with Xi Chen) Real regulators on self-products of $K3$ surfaces. 27 pages. (Subject to the referee's approval of our revisions and the approval from the JAG editorial board, this paper will appear in the Journal of Algebraic Geometry (JAG).)

Articles in refereed journals

- (Co-authored with Ali Sinan Sertöz) Motives of some Fano varieties. To appear in Math. Zeit., 14 pages. (Online version available.)
- (Co-authored with Shuji Saito) Algebraic cycles and Mumford-Griffiths invariants. Amer. J. Math. **129** (2007), no. 6, 1449-1499.
- (Co-authored with Matt Kerr) The Abel-Jacobi map for higher Chow groups II, Invent. Math. **170** (2007), no. 2, 355-420.
- (Co-authored with Matt Kerr and Stefan Müller-Stach) The Abel-Jacobi map for higher Chow groups, Compositio Mathematica **142** (2006), 374-396.
- (Co-authored with Xi Chen) The Hodge-$\mathcal{D}$-conjecture for $K3$ and Abelian surfaces, Journal of Algebraic Geometry **14** (2004), 213-240.
- (Co-authored with Xi Chen) Noether-Lefschetz for K_1 of a certain class of surfaces, Bol. Soc. Mat. Mexicana (3) Vol. **10**, (2004), 29–41.
- Real regulators on Milnor complexes, K-Theory **25**, (2002), 277–298.
- Lectures on algebraic cycles, Bol. Soc. Mat. Mexicana (3) 7 (2001), no. 2, 137–192.
- A filtration on the Chow groups of a complex projective variety, Compositio Math. **128** (2001), no. 3, 299–322.
- A duality pairing between cohomology and higher Chow groups, J. Reine Angew. Math. 504 (1998), 177–193.
- Co-authored with Gordon, B. Brent. Indecomposable higher Chow cycles on products of elliptic curves, J. Algebraic Geom. 8 (1999), no. 3, 543–567.
- A note on indecomposable motivic cohomology classes, J. Reine Angew. Math. 485 (1997), 161–172.
- A result on cycles algebraically equivalent to zero, Illinois J. Math. 41 (1997), no. 1, 16–22.
- The Hodge conjecture for a certain class of singular varieties, Math. Z. 224 (1997), no. 1, 25–31.
- Higher Chow groups and the Hodge-$\mathcal{D}$-conjecture, Duke Math. J. 85 (1996), no. 1, 183–207.
- Generalized Poincaré classes and cubic equivalences. Math. Nachr. 178 (1996), 249–269.
- A generalization of Mumford's theorem II, Illinois J. Math., No. 2 **39**, (1995), 288–304.
- Cylinder homomorphisms and Chow groups, Math. Nachr. **160**, (1993), 205–221.
- Some boundedness results for zero-cycles on surfaces, Illinois J. Math., No. 2 **35**, (1991), 269–285.
- Towards a generalization of Mumford's theorem, J. Math. Kyoto Univ., no. 2 **29**, (1989), 267–272.

- The cylinder correspondence for hypersurfaces of degree n in $\mathbb{P}^n$, Amer. J. Math. **110**, (1988), 77–114.
- On hypersurfaces admitting a covering by rational curves, Communications in Algebra **13**(9), (1985), 1917–1930.
- The cylinder homomorphism for quintic fourfolds, Compositio Math. **56** (2001), 315–329.
- The Hodge conjecture for a certain class of fourfolds, Math. Ann. (no. 1) **268**, (1984), 85–90.

Articles in refereed proceedings

- (Co-authored with Su-Jeong Kang) Beilinson's Hodge Conjecture for K_1 Revisited. 15 pages. To appear in the Proceedings for the Colloquium "Cycles, Motives and Shimura , Varieties (TIFR, Jan. 2008)".
- Cycles on Varieties Over Subfields of $\mathbb{C}$ and Cubic Equivalence. To appear in the Bloch Proceedings. 15 pages.
- Real regulators on Milnor complexes, II, Real regulators on Milnor complexes. II. Algebraic cycles and motives. Vol. 2, 214-240, London Math. Soc. Lecture Note Ser., **344**, Cambridge Univ. Press, Cambridge, 2007.
- Regulators of Chow cycles on Calabi-Yau varieties, Proceedings of the Field's Institute Workshop, "Arithmetic, Geometry and Physics around Calabi-Yau Varieties and Mirror Symmetry", July 22-29, 2001. Edited by Noriko Yui and James D. Lewis. Fields Institute Communications, Vol. **88**, (2003), 87–117.
- The real regulator for a product of K3 surfaces, Accepted for publication on Feb. 17, 2005. To appear in *Mirror Symmetry V,* the Proceedings of the Banff Dec. 2003 Conference on Calabi-Yau Geometry and Mirror Symmetry, Edited by James D. Lewis (Alberta), Shing-Tung Yau (Harvard), Noriko Yui (Queen's), 12 pages. Advanced Studies in Mathematics Series, International Press.
- Three lectures on the Hodge conjecture, Transcendental Aspects of Algebraic Cycles, Proceedings of the Grenoble Summer School, 2001. Edited by Stefan Müller-Stach and Chris Peters, London Mathematical Society Lecture Note Series **313**, Cambridge University Press, (2004), 199–234.
- Coauthored with Gordon, B. Brent. Indecomposable higher Chow cycles. The arithmetic and geometry of algebraic cycles (Banff, AB, 1998), 193–224, NATO Sci. Ser. C Math. Phys. Sci., 548, Kluwer Acad. Publ., Dordrecht, 2000.

Books - Research

1. A Survey of the Hodge Conjecture, **CRM** Monograph series. [`Zbl 0778.14002`]
2. A Survey of the Hodge conjecture. Second edition. [This is an updated version of 1. above, with additional appendices.] Appendix B by B. Brent Gordon. **CRM** Monograph Series, 10. American Mathematical Society, Providence, RI, 1999. xvi+368 pp. [`Zbl 0922.14004`]

Editorial work

- Co-editor (with Rob de Jeu) on the Proceedings of the Conference on Motives and Algebraic Cycles, March 2007 at Fields. A conference in honor of the mathematical heritage of Spencer J. Bloch.

- Co-Editor-in-Chief for the Canadian Mathematical Bulletin (ended 2006).
- CMS Notes Research Editor. 2001-2002.
- Co-Editor (and co-organizer, along with N. Yui, S.-T. Yau) on Mirror Symmetry V, Proceedings of the Banff Dec. 2003 Conference on Calabi-Yau Geometry and Mirror Symmetry.
- Co-Editor (and co-organizer, along with N. Yui, V. Batyrev, S. Hosono, B. H. Lian, S.-T. Yau) of the Field's conference on the "Arithmetic, Geometry and Physics around Calabi-Yau Varieties and Mirror Symmetry", at the Field's Institute, July 22-29, 2001.
- Co-editor (and co-organizer, along with N. Yui, B. Gordon, S. Müller-Stach, Shuji Saito) of the NATO ASI Conference/CRM Summer School on the Arithmetic and Geometry of Algebraic Cycles, in Banff Alberta, July 1998.

Other professional activities

- Translated (AMS) a book for Astérisque (Vol. 256) by Fabien Morel, entitled Théorie Homotopique des Schémes, Société Mathématique de France, 1999.
- Translated (AMS) two chapters of the book by Bertin, José; Demailly, Jean-Pierre; Illusie, Luc; Peters, Chris; entitled, Introduction to Hodge Theory. Translated from the 1996 French original by James Lewis and Peters. SMF/AMS Texts and Monographs, 8. American Mathematical Society, Providence, RI; Société Mathématique de France, Paris, 2002. x+232 pp.

- Referee for NSF and NSA proposals, as well as for some major journals (e.g. J. Algebraic Geometry, K-Theory, etc.).

Selected Conferences organized

Twice every year I organize (along with Noriko Yui) a workshop at the Fields Institute in Toronto on Algebraic Geometry with an Emphasis on Calabi-Yau Manifolds.

- [Future] Regulators IV. (Co-organizing with Rob de Jeu.) Amsterdam. Probably the summer of 2013.
- [Future] Regulators III. (Co-organizing with Rob de Jeu and José Ignacio Burgos Gil.) Summer of 2010 (somewhere in Spain).
- [Future] Antalya Algebra Days XI (Co-organizer, with a special emphasis on Algebraic Geometry), Antalya, Turkey, Conference, May 2009.
- Regulators and Heights in Algebraic Geometry, (Edmonton), April 12-17, 2008 (Co-organized with M. Lalin and X. Chen.)
- Motives and Algebraic Cycles, Fields Institute (Toronto), March 19-23, 2007. (Co-organizer along with Rob de Jeu). A conference in honor of the mathematical heritage of Spencer J. Bloch.
- Motives and Periods, supported by PIMS. University of British Columbia, Vancouver, June 5-12, 2006. (Co-organizer along with Jim Carrell (UBC), Stefan Müller-Stach (Universität Mainz), Andreas Rosenschon, (University at Buffalo) and Pramath Sastry (University of Toronto)).
- Regulators II workshop (co-organized with V. Snaith). BIRS, December 2005.
- Co-organizer and speaker, Antalya Algebra Days, Antalya, Turkey, May 2005.
- Co-organizer (along with N. Yui, V. Batyrev, S. Hosono, B. H. Lian, S.-T. Yau, D. Zagier), on the BIRS workshop on "Calabi-Yau Varieties and Mirror Symmetry", December 2003.

- Co-organizer [and speaker] (along with C. Weibel, V. Srinivas, E. J. Elizondo, P. Luis d'Angel), on The Arithmetic, Geometry and Topology of Algebraic Cycles, June 15-July 4/03, Morelia, Mexico (`www.math.unam.mx/cycles`). [Funding provided by NSF (US$30K), Clay Institute (US$25K), IMU (US$5K) and other sources.]
- Co-organizer [and speaker], (along with N. Yui, V. Batyrev, S. Hosono, B. H. Lian, S.-T. Yau) of the Field's conference on the "Arithmetic, Geometry and Physics around Calabi-Yau Varieties and Mirror Symmetry", at the Field's Institute, July 22-29, 2001.
- Co-organizer [and speaker], (along with M. Levine, C. Weibel, J. Elizondo, P. Luis d'Angel) on a joint AMS/SMM session and extended workshop on Algebraic Geometry, in Morelia, Mexico (May 23-30/01).
- NATO ASI Director [and speaker] (and co-organizer, along with N. Yui, B. Gordon, S. Müller-Stach, Shuji Saito) of the NATO ASI Conference/CRM Summer School on the Arithmetic and Geometry of Algebraic Cycles, in Banff Alberta, July 1998.

Selected invited presentations

- [Future] (Plenary) Antalya Algebra Days XI, May 2009.
- [Future] Vrije Universiteit, Amsterdam, Geometry Seminar, November 20, 2008.
- [Future] University of Amsterdam, Geometry Seminar, November 17, 2008.
- [Future] Barcelona, Algebraic Geometry Seminar, November 14, 2008.
- Buenos Aires Conference on Algebraic Geometry, July 20-27, 2008.
- Antalya Algebra Days X, Antalya, Turkey, Conference, May 2008.
- BIRS, Banff, Workshop on Hodge Theory, Apr 2008.
- Ohio State University, Columbus, Ohio, Conference on Algebraic Cycles, Mar 2008.
- Fields Institute, Toronto, Workshop on Algebraic Geometry, Mar 2008.
- Michigan State University, Lancing, Michigan, Seminar speaker, Mar 2008
- Conference on Algebraic Cycles, TIFR (Bombay), January 2008.
- Fields Institute, Toronto, Workshop on Algebraic Geometry, Nov 2007.
- BIRS, Banff, Workshop on Low Dimensional Topology, Oct 2007
- Algebra Summer School Conference, University of Alberta, August 2007.
- Osaka University, Osaka, Japan, Seminar speaker, Oct 2007
- Mini-symposium speaker at Hiroshima University, Hiroshima, Japan, October 2007.
- Algebraic cycle minicourse seminar speaker, University of Tokyo, October 2007.
- Antalya Algebra Days, Antalya, Turkey, May 2007.
- Vrije Universiteit, Amsterdam, Geometry Seminar, Jul 2007
- Invited Algebraic Geometry seminar speaker, Caltech, May 2007.
- Algebraic Geometry Seminar Speaker, April 17-21/07, University of Chicago.
- ABC Conference, Edmonton, Alberta. April 14-15/07.
- Algebraic Cycle minicourse speaker, University of Leiden and Utrecht, October 2006.
- Hodge Theory, Venice, Italy. June 2005.
- Algebraic Geometry Seminar Speaker, March 1-3/06, University of Chicago.
- Algebraic Geometry Workshop in CIMAT, Guanajuato, Mexico, Feb. 6 - 12/06.
- Variations on Mahler's Measure, Luminy, 30 May - 3 June 2005.

- Antalya Algebra Days, Antalya, Turkey, May 2006.
- Conference on Hodge Theory and Log Geometry, week of March 14/05. Japanese American Mathematics Institute (JAMI), Johns Hopkins University.
- Motives, K-theory and Arithmetical Geometry, Genova Italy, June 28 - July 2, 2004.
- The Many Aspects of Mahler's Measure, April 26 - May 1/03, BIRS Workshop.
- Conference on K-theory and Algebraic Cycles, Miami University, Oxford, Ohio, April 5-6, 2003. `http://calico.mth.muohio.edu/reza/conf.html`
- Transcendental Aspects of Algebraic Cycles, Summer School, June 18-July 6, 2001, Institut Fourier, Grenoble, France.
- Conference on Algebraic Cycles, Columbus, Ohio, December 2000. `http://www.math.ohio-state.edu/conferences/algcycles/`
- Geometría Algebraica y Conmutativa, 20-24 August 2000, at IMUNAM Cuernavaca and UNAM in Mexico City.
- Workshop on Arithmetical Algebraic Geometry, CRM, Montréal, May 14-19, 1999.
- Regulators, Oberwolfach Germany, May 1998.

Administrative, supervisory and related experience

Over the years, I have served on a number of key departmental and administrative committees, such as executive, promotion and tenure, appointments committees; Academic VP search committee (Univ. of Sask.), Dean's advisory committee, General Faculties Council, and a Chair selection committee for our department (2001), where the [then] Dean of Science requested at that time that I allow myself to be a candidate for the chairmanship. I am currently the Associate Chair of Undergraduate Studies in the Department of Mathematics.

Also, over the years I have been very active in organizing both small and large scale conferences in Algebraic Geometry. Having edited a number of Proceedings, I have a great deal of experience in this direction. Along with this, I have been very successful in soliciting conference grant funds as a main organizer, from NATO, CMI (Clay Mathematical Institute), CRM, Fields, PIMS (and BIRS), and the IMU (International Mathematical Union), as well as a coapplicant with others in soliciting funds from the ICTP, NSF, and NSA. Two countries where I have played (and continue to play) an active role in conference planning, offering minicourses and the soliciting of graduate students are Mexico and Turkey. I currently have one post-doctoral fellow and five PhD graduate students (3 Turkish, 1 Mexican, 1 Dutch-Canadian) under my direct supervision.

My future ambitions are aimed towards the development of collaborative Institutes of Research with partnership countries. For example, during a recent trip to Japan, I suggested to my colleagues there the idea of forming a joint Japanese-Canadian Institute along the lines that was (albeit no longer) in place in the form of the JAMI at Johns Hopkins University. Such an Institute could conceivable operate under joint financial support from PIMS and the Japanese government.

Current Research Interests

My research program can best be described as [Hodge theoretic] Regulators of Algebraic Cycles on algebraic varieties over subfields of the complex numbers. In

laypersons terms, a regulator is a generalization of the logarithm. To be more precise, Dirichlet used the logarithm to define a map from the multiplicative group of a ring of algebraic integers to a real vector space. Then Dirichlet proved the celebrated analytic class number formula which relates all the important number theoretic invariants of the number field to the covolume of the Dirichlet regulator. Since the 1960's Dirichlet's fundamental discovery has been found potentially to occur elsewhere in number theory, in algebraic geometry, in class field theory, in algebraic K-theory, in the theory of algebraic cycles and motives, and in Hodge theory. In fact, the central conjectures in each of these disciplines (the Birch-Swinnerton-Dyer conjecture, the Lichtenbaum conjecture, the Beilinson-Soulé conjecture, the Bloch-Kato conjecture, the Stark conjecture etc.) are all outgrowths of Dirichlet's formula and each in its own way involves a regulator. Regulators come in many different forms, according to the context. For instance, the Borel regulator is the higher-dimensional analogue of the Dirichlet regulator, considered as a map on algebraic K-theory in dimension one. On the other hand, in Riemann surface theory, the regulators might involve abelian integrals and jacobians, extending the ideas of the 19th century analytic number theorists and geometers. Generally speaking, in its current incarnation, a regulator is a map from the algebraic K-theory of an algebraic variety to a suitable cohomology theory such as étale cohomology, Deligne or absolute Hodge cohomology. One of the imports of the works of A. Grothendieck and S. Bloch is a cycle theoretic interpretation of the K-theory of an algebraic variety.

In general, my research involves a multifaceted approach to algebraic cycles. This includes the detecting of interesting [higher] algebraic cycles via regulators and height pairings, filtrations on Chow groups, arithmetical aspects of algebraic cycles and arithmetic normal functions, and so forth. Rather than explain all of these directions and developments, I'll focus on one particular project that I am currently working on with my collaborator (Rob de Jeu) in Holland.

Title: Beilinson's Hodge Conjecture

(Supported in part by the Dutch NSF.)

Motivation. It is common knowledge among experts working on Hodge theory applications to algebraic cycles that there are counterexamples to Beilinson's formulation of the Hodge conjecture for the higher K-groups of smooth complex quasiprojective varieties (see [Be]). With $\mathbb{Q}$-coefficients, these higher K-groups are the same objects as S. Bloch's higher Chow groups. Our goal is to provide a comprehensive treatment of this problem, and show how it relates to currents works by S. Saito and M. Asakura, as well as to other conjectures due to Jannsen, Voisin and Bloch-Kato.

Statement of the problem. Let $U/\mathbb{C}$ be a smooth quasiprojective variety of dimension d, $\mathrm{CH}^r(U,m)$ Bloch's higher Chow group, and

$$\mathrm{cl}_{r,m} : \mathrm{CH}^r(U,m) \otimes \mathbb{Q} \to \Gamma\big(H^{2r-m}(U,\mathbb{Q}(r))\big),$$

the cycle class map, where

$$\Gamma\big(H^{2r-m}(U,\mathbb{Q}(r))\big) := \hom_{\mathrm{MHS}}\big(\mathbb{Q}(0), H^{2r-m}(U,\mathbb{Q}(r))\big).$$

In the case $m = 0$, the conjectured surjectivity of $\mathrm{cl}_{r,0}$ is equivalent to the classical Hodge conjecture. Beilinson ([Be]) once conjectured that $\mathrm{cl}_{r,m}$ is always surjective. We now know that this statement is *false* in its present form, a first counterexample provided by Jannsen ([Ja-1]) in the case $m = 1$, making use of the lack of injectivity of the Abel-Jacobi for complex varieties. One possible amended version of this conjecture is for surjectivity of $\mathrm{cl}_{m,m}$, viz., the case $r = m$, as considered by S. Saito and M. Asakura ([A-S]). Their evidence thus far suggests that this version may be true. A first goal (which is now mostly realized), is to provide a comprehensive explanation of this map $\mathrm{cl}_{r,m}$, in terms of kernels of higher regulators, as first initiated in [dJ] (as well as others) in the special case $r = m = 2$.

Our approach. ([dJ-L]) One approach to studying $\mathrm{cl}_{r,m}$ is in terms of a spectral sequence relating $\mathrm{CH}^r(U, m)$ in terms of residues of algebraic cycles lying in the normal crossing divisor of a good compactification of U, as carried out in [K-L]. The nature then of this map $\mathrm{cl}_{r,m}$ can then be discussed integrally as well in terms of kernels of a collection of regulators (in this case higher Abel-Jacobi maps). It is from this approach that we begin to see why the surjectivity in the case $r = m$ could hold, and not only over $\mathbb{Q}$, but even integrally as well. For instance, the case when $r = m = 2$ is related to a conjecture of Voisin (on the countability of the space of indecomposables $\mathrm{CH}^2_{\mathrm{ind}}(X, 1)$ for smooth projective X), and there are no obstructions to surjectivity via kernels of Abel-Jacobi maps, as one is only dealing with the Abel-Jacobi map associated to the Picard variety, which has zero kernel. Further, our approach allows us to determine the following:

• Counterexamples to the surjectivity of $\mathrm{cl}_{r,m}$ can be found for all $r,\ m$ in the region $m < r \leq \dim U$ ([dJ-L]). Then $r = m$ is already a conjecture, and $m = 0$ is the classical Hodge conjecture. All other cases of r and m imply that $\Gamma\big(H^{2r-m}(U, \mathbb{Q}(r))\big) = 0$ by a weight argument.

• If one assumes the classical Hodge conjecture, together with the conjectures of Bloch and Beilinson on the injectivity of the Abel-Jacobi map for smooth projective varieties over $\overline{\mathbb{Q}}$, then $\mathrm{cl}_{r,m}$ is surjective for all $U/\overline{\mathbb{Q}}$, for all r and m. This was first established by M. Saito, and proven as well from a different point of view in [K-L].

• We propose the following generalization of the classical Hodge conjecture (and amendment of Beilinson's version):

Conjecture Let $X/\mathbb{C}$ be a projective variety. Then

$$\mathrm{cl}^{\lim}_{r,m} : \mathrm{CH}^r\big(\mathrm{Spec}(\mathbb{C}(X)), m; \mathbb{Q}\big) \twoheadrightarrow \Gamma\big(H^{2r-m}(\mathbb{C}(X), \mathbb{Q}(r))\big),$$

is surjective for all r and m.

One can think of this as the "generic point" version of the Beilinson-Hodge conjecture. In the case $r = \dim X,\ m = 1$, it can be proven as a theorem. A conjecture of Jannsen ([Ja-2]) implies this conjecture in the case $m = 1$ with r arbitrary (see [Ka-L]). As for an explanation as to why the generic point version should be true, keep in mind that the definition of higher Chow groups involves numerator conditions. For instance, a higher precycle on a quasiprojective variety U can become a cycle after shrinking U. Another reason in support of this conjecture, is that it should follow from the surjectivity of $\mathrm{cl}_{r,m}$ for all smooth quasiprojective

varieties $U/\overline{\mathbb{Q}}$. The essential idea is to generalize Deligne's notion of absolute Hodge classes to the higher cycle group situation, and to apply some recent ideas due to Voisin ([V]). Much of the work in this direction is close to completion, and parallels Voisin's work.

A new direction. Perhaps the most intriguing of all, is an idea due to Spencer Bloch. First of all, as mentioned above, there are good reasons to anticipate the surjectivity of the corresponding integrally defined groups:

$$(*) \qquad \mathrm{cl}^{\lim}_{r,m} : \mathrm{CH}^m(\mathbb{C}(X), m) \twoheadrightarrow \Gamma\big(H^m(\mathbb{C}(X), \mathbb{Z}(m))\big).$$

Now suppose that $H^m(\mathbb{C}(X), \mathbb{Z}(m))/\Gamma\big(H^m(\mathbb{C}(X), \mathbb{Z}(m))\big)$ is divisible (which is a easy check in the case $m = 1$). Then $(*)$ implies that for any nonzero integer N:

$$(**) \qquad \mathrm{CH}^m(\mathbb{C}(X), m)/N \twoheadrightarrow H^m(\mathbb{C}(X), \mathbb{Z}(m)/N \cdot \mathbb{Z}(m)),$$

is surjective. Attempts to establish divisibility, as well as injectivity of the above map, are currently underway, as a means of relating this version of the "Milnor"-Hodge conjecture (viz., $(*)$) to the Bloch-Kato conjectural statement in $(**)$. For instance, it is not difficult to reduce the question of divisibility to varieties $X/\overline{\mathbb{Q}}$. But notice that the Bloch-Beilinson conjecture implies that

$$\mathrm{cl}^{\lim}_{m,m} : \mathrm{CH}^m(\mathrm{Spec}(\overline{\mathbb{Q}}(X)), m) \to H^m_{\mathcal{H}}(\overline{\mathbb{Q}}(X), \mathbb{Z}(m)),$$

is injective modulo torsion, where $H^m_{\mathcal{H}}(\overline{\mathbb{Q}}(X), \mathbb{Z}(m))$ is Beilinson's absolute Hodge cohomology, which is an extension of $\Gamma\big(H^m(\mathbb{C}(X), \mathbb{Z}(m))\big)$ by a divisible group. Working modulo N, does this lead to to an isomorphism in $(**)$?

References

[A-S] M. Asakura & S. Saito, Beilinson's Hodge conjecture with coefficients, in Algebraic Cycles and Motives, Vol. 2, Edited by J. Nagel and C. Peters, LNS **344**, London Math. Soc., (2007), 3-37.

[Be] A. Beilinson, Notes on absolute Hodge cohomology, Contemporary Mathematics **55**, Part I, 1986, 35–68.

[dJ] R. de Jeu, Notes on Beilinson's Hodge conjecture for surfaces.

[dJ-L] R. de Jeu and J. D. Lewis, Beilinson's Hodge Conjecture for Smooth Varieties. In progress.

[Ja-1] U. Jannsen, Mixed Motives and Algebraic K-Theory, Lecture Notes in Mathematics **1400**, (1988).

[Ja-2] U. Jannsen, Equivalence relations on algebraic cycles, in: The Arithmetic and Geometry of Algebraic Cycles, Proceedings of the NATO Advanced Study Institute on The Arithmetic and Geometry of Algebraic Cycles, Banff, Alberta, Canada, 1998, (Editors: B. Gordon, J. Lewis, S. Müller-Stach, S. Saito and N. Yui), Kluwer Academic Publishers, Dordrecht, The Netherlands **548**, (2000), 225-260.

[Ka-L] S. J. Kang and J. D. Lewis, Beilinson's Hodge Conjecture for K_1 Revisited. 15 pages. To appear in the Proceedings for the Colloquium "Cycles, Motives and Shimura , Varieties (TIFR, Jan. 2008)".

[K-L] M. Kerr, J. D. Lewis, The Abel-Jacobi map for higher Chow groups II, Inventiones Math. **170** (2), (2007), 355-420.

[V] C. Voisin, Hodge loci and absolute Hodge classes, Compositio Math. **143** (2007), 945-958.