A MODELING PERSPECTIVE OF JUVENILE CRIMES

YOUNG LEE AND TAE SUG DO

Abstract. Youth gang activity is on the rise again since declining from its peak in the early 2000s. We focus on contagion of youth gang membership and delinquency among adolescents who are at risk to peer pressure by creating an epidemiological model with differential equations. The model seeks to examine dynamics of the system through stability analysis. A tipping point that spreads youth gang activity is identified, and sensitivity analysis on the threshold condition is performed to discuss the prevention strategies and the effectiveness of juvenile arrest and sanction. All parameters are approximated and results are also exploited by simulations. Analysis indicates that the system is most sensitive to prevention, early intervention and an effective juvenile system with treatment and rehabilitation.

Key words. Epidemiological model, Peer pressure, Stability analysis, Reproductive number, Juvenile crimes

1. Introduction

Youth gang activity is a widespread problem across the world. Youth gangs are self-identified, organized groups of adolescents, banded together under common interests and a common leader in activities that typically are regarded as menacing to society or illegal[10]. The Office of Juvenile Justice and Delinquency Prevention (OJJDP) in the U.S. Department of Justice[9] reports that the prevalence rate of youth gangs has been significantly elevated across the United States since declining from its peak in the early 2000s. The National Gang Center estimates that the number of gangs increased by 28 percent, and the number of gang members increased by 6 percent from 2002 to 2008[9].

We define juvenile crimes as assault, robbery, breaking and entering, felony theft, marijuana use, or drug selling. Many researchers have shown that delinquents who are repeat offenders are likely to continue to commit crime into adulthood. Therefore, preventing delinquency is crucial to reducing adult criminality. In this article, the term gangs refers to youth gangs unless otherwise specified, and our adolescents are assumed to be ages 13-18.

It is well known that peers play a central role in adolescence. Many sociologists have found that peer pressure is the most significant predictor of delinquent behavior in early adolescence [6][19][26]. According to Brown[5], susceptibility to peer pressure reaches its highest level in the younger population and in people with low confidence and lack of social interaction skills. "At-risk" youth in this study is defined as adolescents who are in the environment of poverty or ghettos, who have low academic achievement or low school commitment, or who are children of illegal immigrants. There is overwhelming evidence that the prevalence of delinquency among gang youth is much higher than one among non-gang youth[12][18][20]. Troubled youth join a gang for a variety of reasons. Some do so hoping that the gang can protect them[22]. The Associated Press[23] has said, "loosely organized gangs present the biggest concern for law enforcement officials because they are hard to investigate and their members often commit random acts.
of crime out of self-protection.” Siegel and Welsh[16] note that as youths move through adolescence, they gravitate toward cliques that provide support, assurance, protection and direction, and when a group provides the social and emotional basis for antisocial behaviors, the clique is transformed into a gang. Some youths join a gang by thinking that the gang represents their disadvantaged structural position in their community or society, and they justify their delinquent activities as a resolution of their problems[1]. Johnson[15] summarizes the research of Cohen, Ohlin, Miller, Sutherland and Cressey on the effect of delinquent associates: peer approval of the delinquent others is a strong motivation getting into delinquency, which strengthens the belonging and status; the greater the association with delinquent others, the greater the likelihood of being a delinquent. Youth gang members commit more violent crimes and property crimes than their peers who are not in gangs[11]. Thornberry et al.[21] have found that individual gang members are not fundamentally different from nonmembers, but when they are in the gang, the gang facilitates or enhances their involvement in delinquent behavior. Crane[6] has viewed that juvenile crime is contagious because it spreads through peer influence, which is well supported by the theory on delinquency and differential association. Voss[26] argues that the impact of delinquent peers is strengthened by the frequency, duration, priority, and intensity of association with such peers. Therefore, we take an epidemiological approach to youth gangs and delinquency.

In this study, we focus on contagion of gang membership and juvenile crimes among youths who are particularly susceptible to peer pressure. We treat this sociological problem as an epidemic and take a mathematical modeling approach, so the environmental peer pressure is expressed as the mass action terms used in epidemiological models. We construct a deterministic epidemiological model with a system of four nonlinear differential equations to examine the dynamics of the system through stability analysis. We identify a tipping point which is used to discuss strategies for prevention and control of juvenile crimes. All parameters are approximated to apply sensitivity analysis to the threshold condition, and our results are exploited through deterministic simulations.

2. Model

Individuals of our system are adolescents of ages between 13 and 18 in the low socioeconomic class. Our model consists of four classes: the at-risk susceptible population \(S\), gang members who are not delinquents \(G\), delinquent gang members, but not arrested \(D\), and delinquents who are arrested and law-enforced \(L\). Individuals of \(S\) who join a gang move to \(G\), some members of \(G\) move to class \(D\) by committing crimes. When individuals in class \(D\) are arrested, they transit to class \(L\). We use \(N = S + G + D + L\) to represent the total constant population. The parameter \(\mu\) is the per capita rate at which individuals age into the population, \(S\) and ones age out the system. All transition rates are per capita rate. In our model, a susceptible becomes a gang member due to peer pressure. Therefore, the constant rate \(\alpha\) at which a susceptible individual joins a gang depends on the frequency and intensity of the interaction with gang peers and juvenile delinquents. Youth gang members may learn delinquent behavior from delinquent peers, or their crimes may be motivated by personal issues such as a family problem, girl friend issue or needing money. The former is modeled by the peer pressure rate \(\beta\), which is directly related to the proportion of delinquents already existing among the peer population, and the latter progression to \(D\) is modeled by the rate \(\gamma\), so that the overall per capita delinquency rate is \(\beta D/N + \gamma\). Transition to \(S\) from \(G\) depends
on family, teachers, counselors or fear rather than peer pressure, and is modeled by constant per capita rate \(\phi\). When individuals of \(L\) are released, they may relapse back to \(D\) or \(G\) right away, or may start fresh by transiting to \(S\), which are modeled by per capita rates \(\theta\), \(\psi\) and \(\rho\), respectively. The following figure summarizes the model in schematic form.

![Figure 1](image-url)

**Figure 1.** A schematic diagram of the model

With these assumptions, we obtain a system of four ordinary differential equations, and the governing compartmental model is the following.

\[
\begin{align*}
\frac{dS}{dt} &= \mu N - \alpha \frac{S}{N}(G + D) + \phi G + \rho L - \mu S \\
\frac{dG}{dt} &= \alpha \frac{S}{N}(G + D) - \beta \frac{G}{N} D + \sigma L - (\gamma + \phi + \mu)G \\
\frac{dD}{dt} &= \beta \frac{G}{N} D + \gamma G + \delta L - (\eta + \mu)D \\
\frac{dL}{dt} &= \eta D - (\delta + \sigma + \rho + \mu)L \\
N &= S + G + D + L
\end{align*}
\]

### 3. Analysis

#### 3.1. Re-scaling the system.

The subsequent analysis is facilitated by choosing the dimensionless variables. Let \(S = sN\), \(G = gN\), \(D = dN\), \(L = lN\). Then the last equation of the system (1) provides \(s = 1 - g - d - l\). Denoting the outflow rate from \(D\) by \(\xi\), the rate at which individuals leave \(L\) by \(\psi\), and letting \(\kappa\) express the per capita exit rate of members of \(G\) that depends on reasons other than peer pressure, we have

\[
\xi = \eta + \mu, \quad \psi = \delta + \sigma + \rho + \mu, \quad \kappa = \gamma + \phi + \mu
\]

and obtain the rescaled system:

\[
\begin{align*}
\frac{gs'}{s'} &= \alpha(1 - g - d - l)(g + d) - \beta gd + \sigma l - \kappa g \\
\frac{di'}{i'} &= \beta gd + \gamma g + \delta l - \xi d \\
\frac{li'}{i'} &= \eta d - \psi l
\end{align*}
\]
3.2. Gang and delinquency free equilibrium and $R_0$. The most desirable end state for this model is no gang and no crime equilibrium point $(g^*, d^*, l^*) = (0, 0, 0)$, i.e., $s^* = 1$. We proceed to find the basic reproductive number. A sociological term for this is a tipping point because it provides a point at which a stable system turns to an unstable one or vice versa, which is a threshold condition. In epidemiological models, $R_0$ is interpreted as the average number of secondary cases caused by a typical single infected individual, hence the disease spreads if $R_0 > 1$ and it dies out if $R_0 < 1$. To calculate $R_0$, we use the next generation operator method in Diekerman, Heesterbeek, and Metz[7] since our model requires two stages (generations) for a susceptible to become a delinquent.

**Proposition 1.** The basic reproductive number $R_0$ is

$$R_0 = \frac{1}{2} \left[ \alpha + \delta \eta \xi \psi + \sqrt{\left( \frac{\alpha}{\kappa} - \frac{\delta \eta}{\xi \psi} \right)^2 + 4 \left( \frac{\alpha \gamma}{\kappa \xi} + \frac{\eta \sigma \gamma}{\psi \xi \kappa} \right)} \right].$$

**Proof.** We solve for the equilibrium value $l^* = \eta d^* / \psi$ in Eq. (4) and substitute it in Eqs. (2) and (3). Then the Jacobian $J$ of $g$ and $d$ evaluated at the gang and crime free equilibrium is obtained:

$$J = \begin{bmatrix} \alpha - \kappa & \alpha + \sigma \eta / \psi \\ \gamma & \delta \eta / \psi - \xi \end{bmatrix}$$

By rewriting $J$ in the form $J = F - W$, we have

$$J = \begin{bmatrix} \alpha & \alpha + \sigma \eta / \psi \\ \gamma & \delta \eta / \psi \end{bmatrix} - \begin{bmatrix} \kappa & 0 \\ 0 & \xi \end{bmatrix} \equiv F - W.$$  

Since $R_0$ is the dominant eigenvalue of $FW^{-1}$ and

$$FW^{-1} = \begin{bmatrix} \alpha / \kappa & \alpha / \xi + \sigma \eta / \xi \psi \\ \gamma / \kappa & \delta \eta / \xi \psi \end{bmatrix},$$

$$R_0 = \frac{1}{2} \left[ \frac{\alpha}{\kappa} + \frac{\delta \eta}{\xi \psi} + \sqrt{\left( \frac{\alpha}{\kappa} + \frac{\delta \eta}{\xi \psi} \right)^2 - 4 \alpha \delta \eta / \kappa \xi \psi + 4 \left( \frac{\alpha \gamma}{\kappa \xi} + \frac{\eta \sigma \gamma}{\psi \xi \kappa} \right)} \right].$$

Rewriting the expression in the radical completes the proof.

To interpret $R_0$, we first simplify $R_0$ algebraically by using $\sqrt{a + b} < \sqrt{a} + \sqrt{b}$. Then we obtain

$$R_0 < \frac{\alpha}{\kappa} + \sqrt{\frac{\alpha \gamma}{\kappa \xi} + \frac{\eta \sigma \gamma}{\psi \xi \kappa}}.$$  

The first term of the inequality (5) expresses that individuals enter the first infective class $G$ at a rate of $\alpha$ and leave after an average time of $1/\kappa$. The square root arises from the two stages required for a susceptible to become a delinquent. Note that $R_0$ is independent of $\beta$. Since $R_0$ quantifies reaction to the first appearance of infection, the first peer pressure rate $\alpha$ plays a significant role in $R_0$. We show that $\beta$ is important in determining the size of $D$ in circumstances where $D$ is populated in Proposition 4. The expressions in the radical display two different ways to enter the second infective class $D$: the first is from $S$ to $G$ to $D$ and the second is the relapsed entry to $D$; the sum of these two is a reproductive number for the second infective class $D$. The inequality 5 holds since the entry to $G$ is counted by the term outside the radical and also by the first term inside the radical.

**Proposition 2.** If $R_0 < 1$, the gang and delinquency free equilibrium is locally asymptotically stable.
Proof. We find the Jacobian of Eq.s (2)-(4) evaluated at the gang and delinquency free equilibrium,

$$J = \begin{bmatrix} \alpha - \kappa & \alpha & \sigma \\ \gamma & -\xi & \delta \\ 0 & \eta & -\psi \end{bmatrix}.$$  

The characteristic equation of $J$ is $\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0$, where

$$a_1 = \xi + \psi + \kappa - \alpha,$$
$$a_2 = \xi \psi - \delta \eta + (\kappa - \alpha)(\xi + \psi) - \gamma \alpha,$$
$$a_3 = (\kappa - \alpha)(\xi \psi - \delta \eta) - \gamma (\alpha \psi + \sigma \eta).$$

To show that the Routh-Hurwitz conditions\([3]\) $(a_3 > 0, a_1 > 0, a_1 a_2 > 0)$ are satisfied, we observe that

$$(6) \quad R_0 < 1 \iff (\xi - \delta \eta/\psi)(\kappa - \alpha) - \gamma (\alpha + \sigma \eta/\psi) > 0.$$  

Therefore, if $R_0 < 1$ then $a_3 > 0$. Since $\xi - \delta \eta/\psi$ is always positive, $\kappa > \alpha$ if $R_0 < 1$, which implies that $a_1 > 0$. Calculations show that the equivalent statements (6) also imply $(\kappa - \alpha)(\xi + \psi) - \gamma \alpha > 0$, which in turn implies $a_1 a_2 > a_3$. The Routh-Hurwitz conditions guarantee that all eigenvalues are negative.

### 3.3. Endemic equilibria.
Possible endemic equilibria are obtained if both $l^*$ and $d^*$ are not zero. Replacing $l^*$ by $\eta d^*/\psi$ in Eq. (3) and solving for $d^*$, we express $d^*$ in terms of $g^*$,

$$(7) \quad d^* = \frac{\gamma g^*}{\xi - \delta \nu - \beta g^*} = \frac{l^*}{\nu}, \quad \text{where} \quad \nu = \frac{\eta}{\psi}.$$  

This implies that

$$(8) \quad \xi - \delta \nu - \beta g^* > 0$$  

in order to have a positive $d^*$.

**Proposition 3.** If $R_0 > 1$, we have either one or three endemic equilibria. If $R_0 < 1$, we have either two endemic equilibria or none. If $\beta = 0$, there is only one endemic equilibrium if $R_0 > 1$, and none otherwise. Multiple endemic equilibria, if any ever exist, do so only for a positive bounded $\beta$.

**Proof.** To solve for $g^*$, we replace $d^*$ and $l^*$ in Eq. (2) by (7), and obtain a cubic polynomial of $g^*$: $f(g^*) = c_1 g^{3*} + c_2 g^{2*} + c_3 g^* + c_4$, where

$$c_1 = \alpha \beta^2,$$
$$c_2 = \beta [\beta (\kappa - \alpha) - \gamma (\beta + \alpha (2 + \nu))] - 2\alpha (\xi - \delta \nu),$$
$$c_3 = (\xi - \delta \nu)[2\beta (\alpha - \kappa) + \gamma (\beta + \alpha (2 + \nu)) + \alpha (\xi - \delta \nu)] + \gamma [\beta (\alpha + \sigma \nu) + \gamma (1 + \nu)],$$
$$c_4 = (\xi - \delta \nu)[(\kappa - \alpha) (\alpha - \kappa) - \gamma (\alpha + \sigma \nu)].$$

c_1 is always positive. Suppose $R_0 > 1$. (6) implies that $f(0) = c_4 < 0$. By continuity of $f$, there is at least one zero of $f$. From (8) we see that $g^* < \frac{\xi - \delta \nu}{\beta}$.

Evaluating $f$, we obtain

$$f\left(\frac{\xi - \delta \nu}{\beta}\right) = \alpha (1 + \nu) \gamma^2 \left(\frac{\xi - \delta \nu}{\beta}\right)^2 > 0.$$  

Therefore, possible graphs indicate that the cubic polynomial $f$ may have three endemic equilibria if $R_0 > 1$, and we have none or two endemic equilibria if $R_0 < 1$. 
Suppose $\beta = 0$, i.e., no effect of peer pressure from $G$ to $D$. Then $f$ is linear and

$$f(g^*) = c_4 + (\xi - \delta \nu)\gamma(2 + \nu) + (\xi - \delta \nu)^2 + \gamma^2(1 + \nu).$$

Since the slope is positive, one equilibrium exists if $c_4 < 0$, i.e., $R_0 > 1$, and none otherwise.

Suppose that $\beta$ is arbitrarily large, then we see that $g$ is arbitrarily small from (8). In this case, $d^*$ and $l^*$ become zero as seen from the Eqs (2)-(4). Therefore, no endemic equilibria exist if $\beta$ is arbitrarily large.

From Proposition 3, we see that a bifurcation occurs at $R_0 = 1$. Since we are interested in the effects of peer pressure, $\alpha$ and $\beta$, and $R_0$ is independent of $\beta$, we study bifurcation types.

**Proposition 4.** The bifurcation at $R_0 = 1$ is forward in $R_0$ only if

$$\gamma \beta \alpha > \gamma \beta \mu(1 + \nu) + (\xi - \delta \nu)\gamma(2 + \nu) + (\xi - \delta \nu)^2 + \gamma^2(1 + \nu).$$

**Proof.** Since $f(0) = 0$ when $R_0 = 1$, to prove a forward bifurcation, it is enough to show that $f'(0) > 0$ in a sufficiently small neighborhood of $R_0 = 1$, in the sense that there is a small endemic equilibrium $g^*$ for $R_1 > 1$. From (6), we have

$$R_0 = 1 \iff (\xi - \delta \nu)(\kappa - \alpha) = \gamma(\alpha + \sigma \nu).$$

Then $f'(0) = c_3$ becomes

$$f'(0) = (\xi - \delta \nu)[\beta(\alpha - \kappa) + \gamma(\beta + \nu)] + (\xi - \delta \nu)^2 + \gamma^2(1 + \nu)$$

$$= \gamma \beta(\mu + \nu - \alpha) + (\xi - \delta \nu)\gamma(2 + \nu) + (\xi - \delta \nu)^2 + \gamma^2(1 + \nu),$$

and $f'(0) > 0$ if (9) is satisfied.

### 4. Parameter Estimation

The entrance and removal rate, $\mu$, is $1/5$ yr$^{-1}$ since the adolescents considered are to be ages between 13 and 18, making $1/\mu = 5$ years.

Between 1997 and 2001, the National Longitudinal Survey of Youth (NLSY97) annually interviewed a nationally representative sample of nearly 9,000 youth whose ages were between 12 and 16 on December 31, 1996. The survey found that 8 percent had belonged to a gang by age 17 in 2001. However, in the first survey year, 1997, only 3 percent indicated that they were gang members[4][17]. This gives a per capita rate of growth in incidence of $(0.08-0.03)/4$ yr = 0.0125 and $\alpha = 0.0125/0.03 \approx 0.42$ yr$^{-1}$. The data is ideal because the survey is longitudinal although it is a bit old. If we consider a sample of at-risk youth, the value of $\alpha$ is much higher: Rochester Youth Development Project (RYDP) found that 30% joined gangs between the ages of 14 and 18[17]. In this case, $\alpha$ is 0.6 yr$^{-1}$.

OJJDP[12] lists three sets of data on duration of gang membership: The Seattle Social Development Project (SSDP) found 69 percent belonged for 1 year or less and 31 percent belonged for longer; Denver Youth Survey researchers and Rochester Youth Development Study reported 67 percent and 53 percent, respectively, for less than 1 year, and 33 percent and 47 percent, respectively, for more than 1 year; Approximately 7 percent of the Rochester subjects remained in a gang for 4 years or longer. Averaging them, we estimate that it takes less than 2 years in average leaving a gang, making $\phi$ greater than 0.5 yr$^{-1}$. Studies show that at-risk youth who join gangs are more than twice as likely to remain in a gang for more than 1 year[12], which substantially decreases the value of $\phi$.

According to the prevalence of various youth crimes among gang and nongang youth ages 13 to 18[12], about 43 percent of gang members were involved in crimes. Therefore, we take for the proportion of gangs who engage in criminal activity before
leaving the system \( \frac{\beta d + \gamma}{\mu + \beta d + \gamma} = 0.43 \). With our value \( \mu \), we obtain \( \beta d + \gamma = 0.15 \).

There were about 774,000 gang members in the U.S. in 2008[9]. A 2004 survey indicated that youth gang membership was estimated to consist of 41% juveniles and 59% 18 or older[17]. The 2009 report of National Youth Gang Center[25] reads that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group.

We find the sensitivity indices of leaving the system \( \beta d \). These are involved in crimes, which makes \( d \) among 20 million youths of our age group are in gangs, and about 50 percent of juveniles. Hence, we assume that about 400,000 gang members of our age group.

There were about 774,000 gang members in the U.S, in 2008[9]. A 2004 survey indicated that youth gang membership was estimated to consist of 41% juveniles and 59% 18 or older[17]. The 2009 report of National Youth Gang Center[25] reads that more than 65% of youth gang members in smaller cities and rural counties are juveniles. Hence, we assume that about 400,000 gang members of our age group.

With the lack of data that measures how many crimes committed by gang members are peer-driven and how many are not, we assume that \( \beta \) is one order of magnitude greater than \( \gamma \), taking \( \beta = 1 \text{ yr}^{-1} \) and \( \gamma = 0.14 \text{ yr}^{-1} \).

Thornberry et al.[21] have found that although gang members were only 30 percent of the studied population, they were 54 percent of all arrests. Even with this conservative value, \( \eta \approx 0.235 \text{ yr}^{-1} \) from \( \eta/(\mu + \eta) = 0.54 \text{ yr}^{-1} \).

Researchers agree that arrest and sanctioning have no impact on subsequent delinquent behavior[14][21], which means that juvenile offenders who were arrested go back to \( D \) eventually. Among committed juveniles, 80% had been in the facility for at least 30 days, 68% for at least 60 days, and 57% at least 90 days. After a full year, 13% of committed offenders remained in placement according to the 2006 National Report on juvenile offenders and victims[17]. The report also lists 113 median days as offenders’ average time in various facilities. Therefore, we take \( 1/(\delta + \sigma + \rho) = 1/3 \) year. With lack of available data, we assume that \( \delta = \sigma = \rho = 1.0 \text{ yr}^{-1} \). To discuss preventative strategies, we vary the values of these parameters using the equation \( \delta + \sigma + \rho = 3 \).

5. Simulation and discussion

5.1. Equilibria and stability. Using the parameter values estimated in the previous section, we denote the set \{\( \mu = 0.2, \beta = 1, \eta = 0.235, \gamma = 0.14, \delta = \sigma = \rho = 1 \}\) by \( A \). With \( \alpha = 0.42, \phi = 0.5 \) and \( A \), we obtain \( R_0 \approx 0.6 < 1 \). Note that the longitudinal study that provided the value of \( \alpha \) was conducted in 1997-2001. Youth gang activity declined significantly in 2000-2002, which agrees with our simulation. Figure 2a shows the solution curves. To simulate the recent situation of at-risk youth in big cities such as the result of RYDP, we use \( \alpha = 0.6, \phi = 0.3 \) and \( A \). Then \( R_0 \approx 1.01 > 1 \) and there is one endemic equilibrium as seen in Figure 2b with \((g^*, d^*, t^*) \approx (0.18, 0.13, 0.01)\). It shows about 30 percent \((g^* + d^*)\) of youth involved in gang activities as shown in RYDP[17]. Phase portraits \((g(t), d(t))\) in Figure 2c shows local asymptotic stability although we could not confirm it analytically.

5.2. Sensitivity analysis and control strategies. The basic reproduction number \( R_0 \) in epidemiology is often described as a point at which a stable system turns to an unstable one or vise versa. In sociology, it refers to a tipping point at which a rare phenomenon becomes a common one. Hence, we use \( R_0 \) as a threshold condition. To see how a small perturbation made to a parameter \( q \) affects a threshold condition \( R \), we define the sensitivity index of \( R \) for \( q \) as

\[
S_q = \frac{\partial R}{\partial q} R.
\]

We find the sensitivity indices of \( R_0 \) for all parameters in order to determine parameters to which our system is most sensitive.

Applying \( A \) to sensitivity indices, we find that the system is most sensitive to \( \alpha \) over all. Burns et al.[2] states, "Any program that targets children and child
delinquents should include a strong prevention component with a focus on discouraging gang involvement." Although education and family help, it is hard to control $\alpha$, peer pressure. Among the parameters that can be influenced by intervention strategies, the system is most sensitive to $\phi$. Gatti et al.[11] find that not only entry into the gang, but also prolonged membership is associated with a greater risk of deviant behavior, and suggests the importance not only of preventing gang formation, and youths from joining gangs, but also of acting to reduce the duration of their membership. Early intervention by teachers, family and friends increases $\phi$. Some effective prevention programs target parents to improve parenting skills and involve community policing [2].

The third parameter to which our system is most sensitive is $\gamma$. Since both $\gamma$ and $\beta$ model the transition from $G$ to $D$, and $R_0$ is independent of $\beta$, we simulate the system using $(\beta,\gamma) = (0,0.15), (1,0.14)$ and $(2,0.13)$. As the values of $\beta$ change from 0 to 2, $d^*$ increases by almost 200 percent, approximately from 0.08 to 0.23, and $g^*$ decreases about 25 percent, approximately from 0.19 to 0.14 as seen in phase portraits of Figure 3a. This strengthens the fact that adolescents are more likely to engage in delinquent behaviors while they are in a group rather than by themselves.

Many studies show that arrest and sanctioning have no impact on subsequent behavior, and most likely increase future delinquent behavior al.[14][21], which indicates that the value of $\rho$ in our model is almost zero practically, and the values of $\sigma$ and $\delta$ are higher than our approximations. The absolute values of sensitivity indices of these are $|S_\rho| \approx 0.024$, $|S_\sigma| \approx 0.01$ and $|S_\delta| = 0.01$. In order to study the efficacy of law enforcement, we vary the values of $(\rho,\delta,\sigma)$, but fix all other parameter values. In Figure 3b, the dotted curve shows the phase portrait $(g(t),l(t))$ with $(\rho,\delta,\sigma) = (0,1.5,1.5)$, and the solid curve depicts the ideal situation with $(\rho,\delta,\sigma) = (3,0,0)$. Note that $d^*$ is reduced by almost 70 percent, approximately from 0.19 to 0.06. The ideal outcome is to develop appropriate punishment and effective intervention strategies. Above all, chronic delinquency for young offenders should be avoided. According to labeling theory, once juvenile offenders are labeled as criminal and deviant they are more likely to offend. They accept the role of deviancy and are more likely to associate with others who have been similarly labeled[8], which makes rehabilitation so hard. Well-trained law officers, secure facilities, training schools and followup care are necessary to increase $\rho$. Howell[13]
lists graduated sanctions: immediate intervention, intermediate sanctions, community confinement, and secure corrections. He also emphasizes that accountability and sanctions with increasingly intensive treatment and rehabilitation services should be combined to achieve an effective juvenile system.

References

report submitted to the U.S. Department of Justice, Office of Justice Programs, National Institute of Justice.


Department of Mathematics and Computer Science, Manchester College, N. Manchester, IN 46962, USA

E-mail: yslee@manchester.edu

Department of Mathematics Education, Kwandong University, Kanglung, 210-701, South Korea

E-mail: taesdo@kwandong.ac.kr