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THE OBLIQUE WATER ENTRY IMPACT OF A TORPEDO AND ITS BALLISTIC TRAJECTORY SIMULATION

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Abstract. To study the water entry ballistic trajectory of a torpedo, the wind tunnel experiment has been done based on the similarity principle. Then the drag coefficient of the torpedo is got when it enters the water, which is amended by the introduction of continuous supercavitation factor and local cavity effect factor. The vertical plane motion equations are established to get the torpedo's trajectory. The large scale nonlinear transient finite element commercial software MSC. dytran is also used to simulate the initial water entry impact of the Disk-Ogive-Head[1] torpedo, including four special high-speed water entry attitude angles. Then the kinematics parameters as the tail of torpedo submerges in water are input into the motion equation as the initial conditions. Finally, two parts of the data are combined to get the whole kinematic and kinetic parameters. During the calculation, the ballistic modeling uses the cavitation number to determine the torpedo's moving status: in the supercavitation stage, in partial cavity stage or in full wet navigation stage. The simulation results will do reference use to the following trajectory design. In addition, the water impact load and over load calculation of high-speed oblique water entry impact will help to design the intensity of torpedo's shell.

Key words. Water entry, MSC. dytran, FE simulation, torpedo, over load, trajectory, impact drag coefficient.

1. Introduction

In modern naval warfare, torpedo and anti-torpedo confrontation is growing more fiercely, improving the concealment of torpedoes is a key issue to be researched, conventional air-dropped torpedo or rocket assisted torpedo usually enters water with a parachute [2]. As the target is so large that it can be easily found by the enemy. In addition, when a torpedo enters water with a buffer cap, if the buffer cap couldn't fully come to pieces, the relic would affect its streamline, and thus affect the torpedo's hydrodynamic characteristic. So high speed torpedo's entering water without a parachute and a buffer cap is a trend.

However, water entry of high-speed naked torpedoes will face enormous fluid impact force. Perhaps the load could cause damage to its structure, so failure and even damage of internal components cannot be ignored. It is necessary to accurately compute the fluid and solid interaction and its effect. Therefore, research on water entry impact of high speed torpedoes and their ballistics trajectory have an important significant background.

As computer-aided engineering technology rapidly develops, finite element analysis software has been widely used in the transient dynamics analysis [3], which could be applied in the process of special transient dynamics simulation analysis, especially in high speed torpedo's water entry impact issues, and it can greatly improve the efficiency and save the spending [4]. In practical work, the experiment research of water entry impact is not only costly but also difficult to operate. Sometimes the results may not be entirely accurate. The transient nonlinear finite element commercial analysis software MSC. dytran can effectively deal with

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multi-material fluid-solid coupling problems, it is very appropriate to simulate the dynamic response when a torpedo hits the water.

In the current investigation, the wind tunnel experiment has been done to get the drag coefficient of the torpedo based on the similarity principle. The continuous supercavitation factor and local cavity effect factor are introduced to amend the drag coefficient, which is combined with the vertical plane motion equations to get the torpedo's tracks. The finite element software MSC. Dytran is used to simulate the initial water entry impact of a Disk-Ogive-Head[1] torpedo, including four special high-speed water entry attitude angles. When the kinematics parameters are got, they are input into the motion equation as the initial conditions. Then two parts of the data are combined to get the whole kinematic and kinetic parameters.

2. Dytran program

2.1. Software description. MSC. Dytran[5] is a large nonlinear transient finite element commercial software, which could be used in aviation, aerospace, marine and automotive fields, as well as a wide range of applications. In MSC. Dytran the solid structure uses Lagrange elements; fluid (including air, water) uses Euler elements. The interface between the two is defined as fluid-solid coupling surface. The coupling algorithms include general coupling algorithm and Arbitrary Lagrangian-Eulerian (ALE) Coupling. By directly grid-coupling Lagrange mesh and Euler mesh the kinematics parameters and motion parameters on the coupling surface can be automatically and accurately calculated and outputted at each time step. In this process, on one hand, Euler pressure caused by material flow through the coupling algorithm automatically loads on the structure grid; on the other hand, the deformation of the structure grid will in turn affects the flow of Euler material and pressure values. So the interaction of structure deformation and fluid makes it possible to get the solution of fluid-solid coupling problem.

2.2. Numerical model. In this paper, all the output parameters and model use the International Units. The finite element model of the torpedo is shown in figure 1. Euler fluid region is divided into two parts, the upper part is air domain, with the size of 1.2m∗1.2m∗0.8m. It is divided into 200,000 Euler elements, filled with ideal compressible gas. The air domain is described by Gamma state equation [6]:

$$p = (\gamma - 1)\rho e, \tag{1}$$

where p is the air pressure, γ is the ratio of specific heat, taken as 1.4, ρ is the air density, taken as 1.2, e is the specific internal energy of unit mass. The initial pressure for the air region takes a standard atmospheric pressure of $0.1013Pa$. According to Eq. (1) the initial e of air domain could be calculated as $211{,}041 J/g$.

The lower part is the water domain, with the size of 1.2m∗1.2m∗1.5 m, which is divided into 300,000 Euler elements. Non-viscous and compressible fluid medium is used to fill these elements. The pressure of the water region is described by the polynomial equation of state [6], shown as

$$p = \begin{cases} a_1\mu + a_2\mu^2 + a_3\mu^3 + (b_0 + b_1\mu)\rho_0 \\ \mu > 0, incompression \\ a_1\mu + (b_0 + b_1\mu)\rho_0 e \\ \mu < 0, intension \end{cases}, \tag{2}$$

where p is the pressure of water, $\mu = \rho/\rho_0 - 1$, ρ is the density of sea water, ρ_0 is the reference density of water. The true density of sea water takes 1020 m/s^3, the reference water density takes 1000 m/s^3, e is specific internal energy per unit mass.

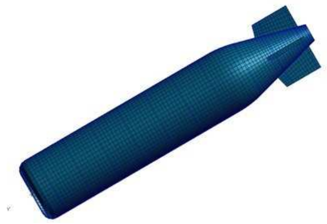

Figure 1. Three-dimensional torpedo FE model.

Normally it is admitted to take the linear part of the state equation. And the a_0 is also known as the bulk modulus of water.

The rigid structure of the torpedo is divided into 5,000 Lagrange shell elements. In addition, in order to take full account of the air cushion when this Disk-Ogive-Head torpedo enters the water, the initial position of torpedo is set leaving the water surface a certain distance.

The torpedo is regarded as a rigid body, the ratio of Lagrange shell element size to Euler hexahedron element size is about 1 to 0.6. The weight of torpedo is 150 kg. The initial conditions are shown in Table 1.

Table 1. Initial motion parameters of the torpedo.

Water entry attack angle	0°	0°	0°	0°
Water entry attack angle	50°	30°	20°	10°
Initial velocity	150m/s	150m/s	150m/s	150m/s

The whole outer surface of torpedo structure is defined as the fluid-solid coupling surface, using the general coupling algorithm. For this algorithm, the grid position in the region is fixed, while air and water material can move freely in the mesh grid. All the free surfaces of air and water are defined as the free flow-in and flow-out boundary by FLOW [6] card. The card MATRIG [6] is used to define the mass center, as well as the shell's polar inertia moment around the mass center and the equatorial inertia moment in local coordinate system. In the global coordinate system, motion parameters of the torpedo outputted include displacement components of mass center in horizontal and vertical directions, velocity, acceleration, as well as the time history of impact force. The output of water and air Euler fluid elements is only the deformation. Global coordinate system and local coordinate system are respectively shown in Figure 3 and Figure 4.

2.3. Mesh size determination. In the present work, the size ratios of Euler to Lagrange grid SR=0.5, 0.8, 1.0, 1.2, 1.4, 1.8, 2.0, 3.0 are tested separately. The only difference is the Euler elements number, other parameters such as sampling frequency, motion parameters outputted, Euler domain boundary condition and rigid body grid elements number make no difference.

It can be seen from figure 5 that MSC. dytran calculation has close relationship with finite element mesh. We must choose a reasonable mesh scale, so that the results are reliable. It is shown in the chart that the curves changes with the elements number, as the fluid domain grid is meshed with more finer grid, the

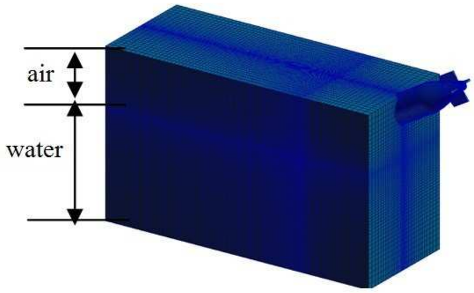

Figure 2. Three-dimensional water FE model.

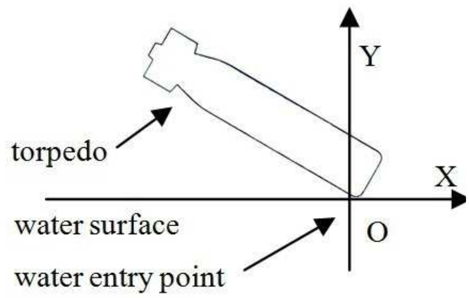

Figure 3. The global coordinate system.

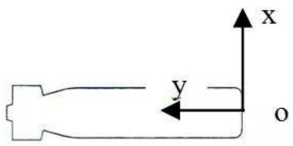

Figure 4. The local coordinate system.

curves becomes more smooth and straight, with an upward trend moving closer between each other. In this paper, for taking full account of time-consuming and computer reliability, the grid ratio is chosen as 0.6 to 1.0.

3. Theoretical modeling

3.1. Experiment and drag coefficient testing. The experiment was performed in $NF3$ wind tunnel in Northwestern Polytechnical University in China. Mechanics parameters of a small model which is similar to the torpedo researched in this paper were tested. The main parameter is the relationship between attitude angle and fluid drag coefficient. In the experiment, the model has the same cross sectional area with the torpedo, which is the characteristics area for drag coefficient. After testing, the drag coefficient is amended with the whole area of the torpedo. The

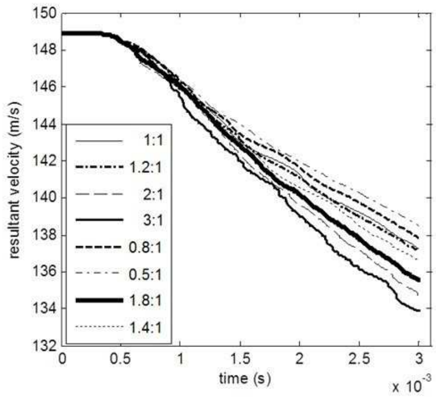

FIGURE 5. The time history of velocity for different SR.

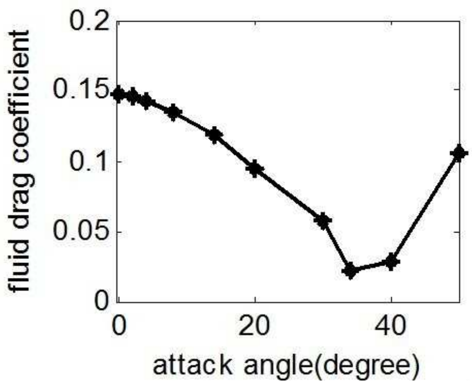

FIGURE 6. Relationship of sideslip angle and drag characteristic of the model.

FIGURE 7. Model shape and installation in wind tunnel.

model is fixed as shown in figure 7, and the drag coefficient of corresponding attitude angle of the model in the experiment is shown in figure 6. The drag coefficient is amended as

$$C_t = \frac{S_t}{S_m} A_t, \tag{3}$$

where C_t is the torpedo's drag coefficient, S_t is the torpedo's surface area, S_m is model's surface area, A_t is the largest cross-sectional area of torpedo.

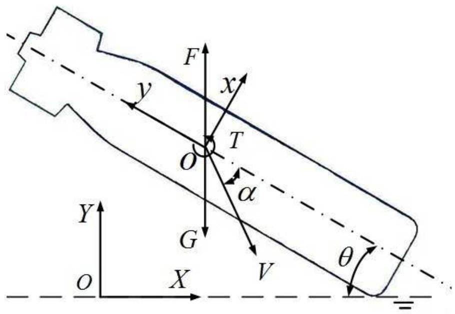

FIGURE 8. Diagram of torpedo showing attack, pitch and trajectory angles.

3.2. Theoretical modeling. In this paper, the theoretical model describes the motion of torpedo after its tail fully submerging, until the stable navigation. There is not yet an exact trajectory model. Cavitation is a stage that the torpedo moves in mixed lower density medium. The study shows that the drag coefficient is smaller than that of the fully wet stage. In addition, the drag coefficient changes nonlinearly with the cavitation number. Therefore, here we have adopted a method of amendment to the drag coefficient by introducing the supercavitation continuous factor and the partial cavitation impact factor, which change with the cavity number. Based on the cavity number, we determine the stage of the torpedo in supercavitation stage, in partial cavity stage or in full wet stage navigation.

In early stage of water entry, the most part of torpedo's surface contacts with air. The torque from air pressure exists, while it is far smaller than the fluid impact force and its torque, so the establishing of the six freedom degrees space motion equations only considers the hydrodynamic force on the torpedo.

Motion equations of the mass center are created in semi-speed coordinate system, the origin is located in mass center of torpedo. Rigid body rotational equations are created in torpedo coordinate system, with the origin locating in the floating center. All the hydrodynamic forces are converted to the corresponding coordinate system by the momentum of rigid body movement and momentum theorem. We assume that [7]:

(1) Mass center is close to the buoyancy center, the velocity v_B of buoyancy center is equal to the velocity v of mass center, that is $v_B = v$.

(2) All the axes of the torpedo coordinate system are its principal axes of inertia, so all the moment of inertia are zero, that is $J_{xy} = J_{xz} = J_{yz} = 0$.

(3) The sideslip angle of the torpedo always equals zero, the initial attack angle equals zero too. The attack angle is always a small quantity as it moves.

The two-dimensional kinematic equations in the vertical plane are listed as

$$m\frac{dv}{dt} = F_x + mg\cos\theta, \tag{4}$$

$$mv\frac{d\theta}{dt} = F_y + mg\cos\theta, \tag{5}$$

$$J_z\frac{d^2\phi}{dt^2} = T_z, \tag{6}$$

$$\omega_z = \frac{d\theta}{dt}, \tag{7}$$

$$\frac{dx}{dt} = v\cos\Theta, \tag{8}$$

$$\frac{dy}{dt} = v\sin\Theta, \tag{9}$$

and

$$\Theta = \theta - \alpha, \tag{10}$$

where, m is the mass of torpedo, J_z is the moment of inertia of torpedo, α is the attack angle, T_z is the torque of fluid force on torpedo around z axis in the global coordinate system, θ is the attitude angle, v is torpedo's speed and Θ is the trajectory angle. F_x and F_y[8] are the fluid force components on the torpedo in the horizontal and vertical directions. Before the cavity collapses, they are calculated by a semi-empirical formula on the basis of amendment to the experiment data. For in the initial impact stage, dynamics parameters have been calculated with software, so now they are calculated by the following formula:

$$F_x = \big[0.25R + 0.22L_s(L - L_f)\big]\rho L_w(v_x + \omega_z L_w)^2, \tag{11}$$

$$F_y = \big[0.25R + 0.22L_s(L - L_f)\big]\rho L_w(v_y + \omega_z L_w)^2 * sign(v_x), \tag{12}$$

and

$$\begin{aligned} T_z = 0.125\rho R\big[L_w^2(v_y + \omega_z L_w)^2 - (L_w - L_c)^2 v_y + \omega_z L_w)^2\big]\rho L_w(v_y + \\ + \omega_z L_w)^2 * sign(v_x) + 0.22\rho L_s(L - L_f)L_w^2(v_y - \omega_z d)^2, \end{aligned} \tag{13}$$

where R is the radius of the torpedo, with unit of meter, L is torpedo's length, with the unit of meter too, L_s is the length of fin span, $sign$ is the symbol function.

The drag coefficient can be calculated as

$$C_x = C'_x - 8\alpha^2, \tag{14}$$

where α is attack angle, C'_x is the drag coefficient, which is related to cavity status.

Here, an approach of amendment to the drag coefficient is adopted. During the water entry stage, the drag coefficient is related to fin, rudder, steady flow loop, as well as head shape and cavitation status. The supercavitation continuance factor and the local cavity impact factor are introduced, which changes with the cavitation number. Based on the cavitation number, the status of the torpedo is determined, whether in supercavitation stage, in local cavitation stage or in totally wet navigation stage. First of all, quotient of critical sea water saturation vapor pressure to supercavitation continuance factor is calculated and contrasted to the cavitation number, making the following judgments:

If the cavitation number is less than or equal to the quotient, thus the torpedo is in supercavitation navigation stage.

If the cavitation number is bigger than quotient and less than critical seawater saturation vapor pressure, thus the torpedo is in partial cavitation navigation stage.

If the cavitation number is greater than or equal to critical seawater saturation vapor pressure, thus the torpedo moves in totally wet stage.

Then in different stage, the drag coefficient is amended correspondingly.

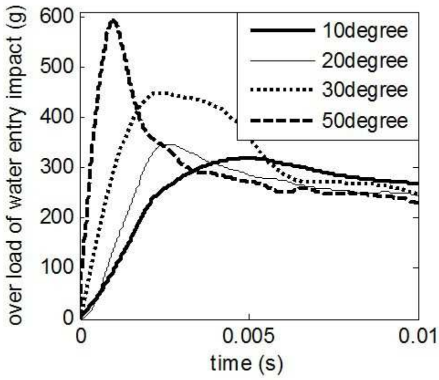

FIGURE 9. Over load of water entry impact.

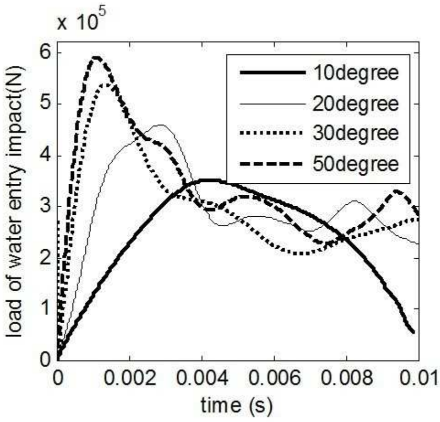

FIGURE 10. Load of water entry impact.

4. Result and analysis

4.1. Analysis of impact loading and over load of oblique water entry. The impact over load and load curve of water entry at the early stage are got with the MSC software, which are shown in figure 9 and figure10. It can be seen that as the water entry attitude angle increases, over load and load peak value also increase correspondingly, and the peak position for the bigger attitude angle appears correspondingly a little earlier. It is obviously concluded that the vertical water entry situation has the biggest peak value, which is understandable. In the program, the lowest points of the torpedo for the four situations have the same height away from the water surface, so the 50° water entry torpedo touches the water surface first. In addition, it is shown from the datas of the two groups that after the impact peak value, the two group curves nearly level off. In general, the water entry impact has a very short duration, with the level of a millisecond[9].

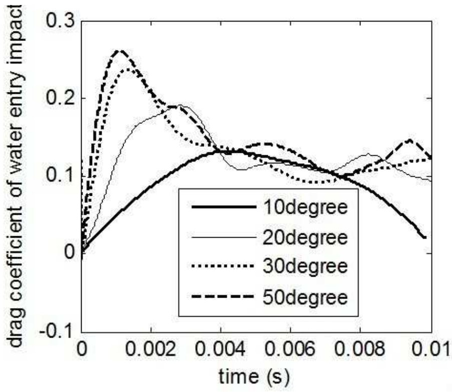

FIGURE 11. Time history of impact drag coefficient.

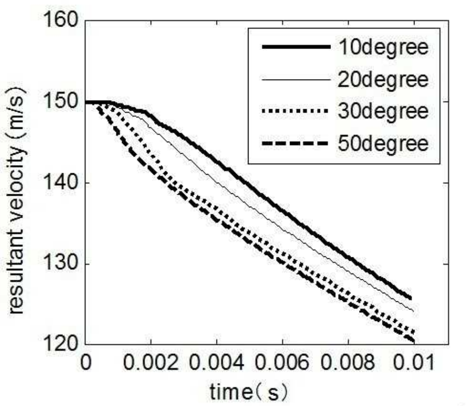

FIGURE 12. Time history of water entry velocity.

4.2. Impact coefficient of water entry. The impact coefficients of the four angles are compared, which are calculated as

$$C_D = \frac{F_{imp}}{\frac{1}{2}\rho_w \pi V^2 R_0^2}, \tag{15}$$

where F_{imp} is water entry impact load, ρ_w is fluid density, V is the velocity of torpedo, R_0 is the torpedo's diameter.

The time history of impact coefficients has been shown in Figure 11, it can be seen from the figure that the impact coefficients appear the same trend with the over load and load curves.

4.3. Water entry velocity at the early impact stage. The four time history curves of water entry velocity are shown in figure 12. At the early stage of impact, the resultant velocity drop dramatically for the huge over load. We can see from table 2, as the water entry load and over load increase with the attitude angle, the negative velocity increments also increase. The 10° angle water entry has a velocity drop of 24.42 m/s, while for that of 50° it is 29.57 m/s.

4.4. Whip analysis and the water entry attitude angle. When the torpedo enters water in a high speed, the vertical and bend torque caused by fluid force

Table 2. Parameters of water entry impact comparison.

Initial trajectory angle (°)	10	20	30	50
Negative velocity increment (m/s)	24.42	25.99	28.41	29.57
Attitude angle increment (°)	1.19	0.82	0.76	0.49
Peak value of impact overload (g)	309.2	345.9	449.0	591.8
Peak value of impact loading (N)	351837	459350	536982	591275

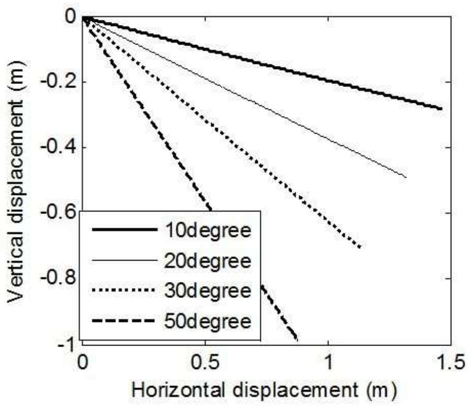

Figure 13. Tracks of oblique water entry.

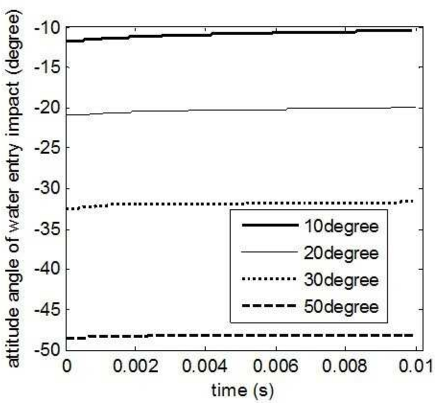

Figure 14. Time history of oblique water entry attitude angles.

will give birth to angular velocity mutation around the horizontal axis, the low pressure phenomenon appearing on the wet surface would be exacerbated. When this phenomenon become serious, whip and ricochet behavior will happen, this should be avoided. The curves of oblique water entry tracks and pitch angles are shown in figure 13 and figure 14. As a result of its special Disk-Ogive-Head, which is similar to the disk head, the four curves in figure 13 are almost a straight line, the whip could be avoided. In addition, in figure 14 four attitude angles only decrease a little, we can conclude that the torpedo does not confront whip.

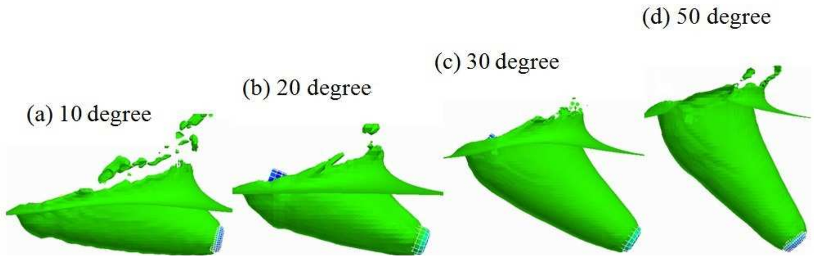

FIGURE 15. Torpedo oblique water entry supercavitation.

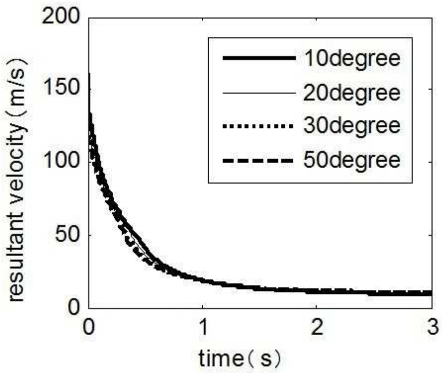

FIGURE 16. Time history curve of resultant velocity.

4.5. Change of water level and analysis of oblique water entry supercavitation. The water entry of high-speed torpedo could give rise to strong fluid-solid coupling interaction. For this strong interaction, water medium in supercavitation region could get a high velocity [10]. In addition, the water medium couldn't bear tension; therefore this part of water is separated from the whole and splashing forms. Splashing and supercavitation phenomenon don't belong to the field of continuum medium mechanics, it is very difficult to be solved by theory analytical method. However, it can be simulated by computer aided technology. The splashing and supercavitation effect of the oblique water entry of torpedoes are shown in figure 15. When the ends of the torpedo completely submerge in water, supercavitation size decreases as the angle increases.

The water entry ballistic trajectory is the most complex section of the whole, which has a significant impact on torpedo's normal working. For if this part of ballistic trajectory is in trouble, the torpedo will not open the engine to work normally. Here, the work stage is not considered, the torpedo moves under initial conditions and gravity.

4.6. The whole trajectory resultant velocity and cavity status. The time history curve of the resultant velocity in the first 3 seconds is shown in figure 16. It can be seen that during the period between the torpedo hits water surface and the cavity collapses the velocity drops dramatically. At the end of 3rd second the velocities level off.

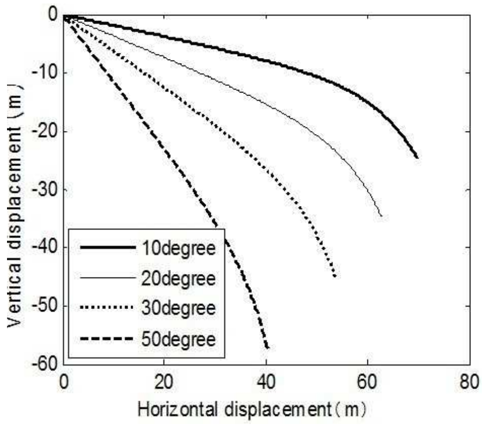

FIGURE 17. Water entry track curve.

Cavitation situation of torpedo in its initial trajectory is shown in table 3. The cavity of torpedo is tightly related to its head type, in some cases the head can be compare as a cavitation device. In addition, the initial velocity and angle also has great influence on cavitation. From the table, as the attitude angle becomes smaller, the torpedo's supercavitation navigation and partially cavitation navigation last longer. In addition, the bigger attitude angle torpedo relatively also has a bigger stable velocity, as the smaller attitude angle one could get bigger drag in stable navigation.

TABLE 3. Cavitation status of torpedo in different water entry angle (second, s)

Angles	Supercavitation navigation	Local cavity navigation	Full wet navigation	Steady velocity
50°	0-0.24s	0.24-0.72s	0.72-3s	10.55m/s
30°	0-0.29s	0.29-0.81s	0.81-3s	10.23m/s
20°	0-0.38s	0.38-0.93s	0.93-3s	9.94m/s
10°	0-0.43s	0.43-1.10s	1.10-3s	9.65m/s

4.7. Whole water entry ballistic trajectory angle and curve. For the oblique water entry of many Conical-Nosed-head and Ogive-head torpedoes, the ultimate goal is to avoid whip and ricochet behaviors. The best way to judge the whip of torpedoes is to research the trend of track curves and the time course of trajectory angle, which are shown in figure 17 and figure 18.

As it can be seen from the charts, water entry of this Disk-Ogive-head torpedo is similar to that of a cylinder head torpedo, which will get an immersed torque. So the whip phenomenon couldn't happen. During initial stage of water entry, because of its special head, the torpedo almost moves along a straight line. When the velocity drops to a certain extent, as well as the role of gravity, the trajectory curve has a downward trend in the latter stage. In addition, during the navigating, due to the role of the tail fin, the rudder, and steady flow loop, if the torpedo does not start the engine to work in very long time, the trajectory angle will tend to 90°.

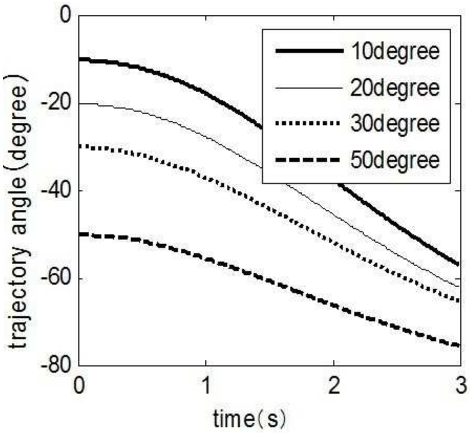

FIGURE 18. Time history curve of trajectory angle.

5. Conclusion

In this paper, the time history regulation of some important kinematics and dynamics parameters of torpedo are deeply studied, so that we can have a detailed understanding of the process of torpedo's entering the water. The torpedo's trajectory is divided into two parts, the initial water entry impact stage and the stage after the tail fully submerging. During the first stage, MSC. dytran software is applied to get the load, over load, velocity and the displacement. Water entry impact duration is very short, about a millisecond, while the hydrodynamic force may cause damage to its head and has a great influence on the torpedo trajectory behavior. The vertical movement equations are got by the introduction of supercavitation continuous factor and local cavitation impact factor to amend the torpedo drag coefficient. The drag coefficient needed to be amended is tested in the wind tunnel in Northwestern Polytechnical University. Then two parts of data are combined together to get the whole trajectory. From the analysis, the initial velocity and angle has a great influence on the ballistic behavior, while for this type of torpedo whip couldn't happen when it hits the water surface as angles of $10°$, $20°$, $30°$ and $50°$ for its Disk-Ogive-Head. So the torpedo could start the engine to work when the cavity completely collapses.

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