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NEW SHOCK DETECTOR AND IMPROVED CONTROL FUNCTION FOR SHOCK-BOUNDARY LAYER INTERACTION

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Abstract. Standard compact scheme or upwind compact scheme have high order accuracy and high resolution, but cannot capture the shock without oscillations. In this paper, modified compact scheme is developed by using an effective shock detector to block upwinding compact scheme to cross the shock, a control function, and an adaptive scheme which uses some WENO flux near the shock. The new scheme makes the original upwinding compact scheme able to capture the shock sharper than WENO and, more important, keep high order accuracy and high resolution in the smooth area which is particularly important for shock, shock boundary layer interaction and shock acoustic interaction. The scheme is robust and has no case-related coefficients.

Key words. Compact Scheme, WENO, Shock-Boundary Layer Interaction, Shock Detector.

1. Introduction

The flow field is in general governed by the Navier-Stokes system which is a system of time dependent partial differential equations. However, for external flows, the viscosity is important largely only in the boundary layers. The main flow can still be considered as inviscid and the governing system can be dominated by the time dependent Euler equations which are hyperbolic. The difficult problem with numerical solution is the shock capturing which can be considered as a discontinuity or mathematical singularity (no classical unique solution and no bounded derivatives). In the shock area, continuity and differentiability of the governing Euler equations are lost and only the weak solution in an integration form can be obtained. The shock can be developed in some cases because the Euler equation is non-linear and hyperbolic. On the other hand, the governing Navier-Stokes system presents parabolic type behavior and is therefore dominated by viscosity or second order derivatives in the boundary layer. One expects that the equation should be solved by a high order compact scheme to get high order accuracy and high resolution. High order of accuracy is critical in resolving small length scales in flow transition and turbulence processes. However, for hyperbolic systems, the analysis already shows the existence of characteristic lines and Riemann invariants. Apparently, the upwind finite difference scheme coincides with the physics for a hyperbolic system. History has shown the great success of upwind technologies. From the point of view of shocks, it makes no sense to use high order compact schemes for shock capturing. High order compact schemes use all gird points on one grid line to calculate the derivative by solving a tri-diagonal or penta-diagonal linear system. However, the shock does not have finite derivatives and downstream quantities cannot cross shock to affect the upstream points. From the point of view of the above statements, upwind scheme is appropriate for the hyperbolic system. Many upwind or bias upwind schemes have achieved great success in capturing the shocks sharply, such as Godunov [4], Roe [15], MUSCL [19], TVD [5], ENO [6] and WENO [12, 7]. All these shock-capturing schemes are based on upwind or bias

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upwind technology, which is nice for hyperbolic systems, but is not favorable to the N-S system which presents parabolic equation behavior. The small length scale is very important in the flow transition and turbulence process and thus very sensitive to any artificial numerical dissipation. High order compact schemes [10, 20] are more appropriate for simulation of flow transition and turbulence because it is central and non-dissipative with high order accuracy and high resolution.

Unfortunately, the shock-boundary layer interaction, which is important to high speed flow, is a mixed type problem which has shock (discontinuity), boundary layer (viscosity), separation, transition, expansion fans, fully developed turbulence, and reattachment. In the case of shock-boundary layer interaction, there are elliptic (parabolic for time dependent problems) areas (separation, transition, turbulence) and hyperbolic areas (main flow, shocks, expansion fans), which makes the accurate numerical simulation extremely difficult if not impossible. We may divide the computational domain into several parts: the elliptic (parabolic for time dependent problems), hyperbolic, and mixed. The division or detection can be performed by a switch function automatically such as shock detectors which simply sets for the shock area and for the rest. The switch function may give good results for shock-boundary layer interaction, but it will have too many logical statements in the code which may slow down the computation. The switch function could also be case-related and very difficult to adjust. It would also slow down the convergence for steady problems.

A combination of compact and WENO schemes should be desirable. There are some efforts to combine WENO with standard central [9, 1] and WENO with upwinding compact (UCS) schemes [14, 22]. Their mixing function is still some kind complex and has a number of case related adjustable coefficients.

In order to overcome the drawback of the CS scheme, we need to achieve local dependency in shock regions and recover the global dependency in smooth regions. This fundamental idea will naturally lead to a combination of a local dependent scheme, e.g. WENO and global dependent compact schemes which we call "Modified Compact Scheme" (MCS).

Last year, we use WENO to improve 7th order upwinding compact scheme as we called as "modified upwinding compact scheme (MUCS)", which uses a new shock detector to find the shock location and a new control function to mix upwinding compact scheme with WENO. The mixing function is designed in following ways: the new scheme automatically becomes bias when approaching the shock, but rapidly recovers to be upwinding compact, with high order of accuracy and high resolution.

However, the mixing function must be improved for high efficiency. It is required that the mixing function must be smooth (not a switch function), keeps up-winding for shock, keeps enough dissipation before and after shock, and maintain high accuracy in the smooth region.

2. Modified Compact Scheme

Compact scheme is great to resolve small length scales, but cannot be used for the cases when a shock or discontinuity is involved. Our new modified compact scheme is an effort to remove the weakness by introducing WENO flux when the computation is approaching the shock.

2.1. Effective New Shock Detector. A very effective shock detector (Oliveira, Lu, Liu and Liu, 2009) [13] has been proposed by C. Liu. The detector has two steps. The first step is to check the ratio of the truncation errors on the coarse and fine grids and the second step is to check the local ratio of the left and right hand slopes. The currently popular shock/discontinuity detectors such as Harten's, Jameson's and WENO can detect shocks, but mistake high frequency waves and critical points as shocks. The schemes then damp the physically important high frequency waves. Preliminary results show that the new shock/discontinuity detector is very delicate. The new detector can detect all shocks including strong, weak and oblique shocks or discontinuities in function and first, second, and third order derivatives without artificial case related constants. However the new detector never mistakes high frequency waves, critical points and expansion waves as shock. This will overcome the bottle neck problem with numerical simulation for the shock-boundary layer interaction, shock-acoustic interaction, image process, porous media flow, multiple phase flow, detonation wave and anywhere the high frequency waves are important, but discontinuity exists and is mixed with high frequency waves. To introduce our new two step shock/discontinuity detector, we need to introduce some popular shock detectors first.

Harten's Switch Function and Jameson's Shock Detector. Harten (1978) defined an automatic switch function that is able to detect large changes in the variation of the function values fi. It generates values between 0 and 1, where 0 is considered smooth and 1 is considered non-smooth.
The switch is defined as:

$$\theta_{j+\frac{1}{2}} = max(\hat{\theta}_j, \hat{\theta}_{j+1}) \tag{2.1}$$

with

$$\hat{\theta}_i = \begin{cases} \left| \frac{|\alpha_{i+1/2}| - |\alpha_{i-1/2}|}{|\alpha_{i+1/2}| + |\alpha_{i-1/2}|} \right|^p, & \text{if } |\alpha_{i+1/2}| + |\alpha_{i-1/2}| > \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha_{i+\frac{1}{2}} = f_{i+1} - f_{i-1}$ and ε is a suitably chosen measure of insignificant variation in f. Jameson's (1981) shock detector is similar, which can be described as:

$$\nu_i = \frac{|p_{i-1} - 2p_i + p_{i+1}|}{|p_{i-1}| + 2|p_i| + |p_{i+1}|} \tag{2.2}$$

which is related to the second order derivative of the pressure.

WENO The WENO weights use smoothness measurements that evaluate the changes in the variation of the function values f_i. Assuming that the three weights have equal contribution, we can determine that a function is smooth if all values are approximately 1/3.

$$\omega_i = \frac{\alpha_i}{\sum\limits_j \alpha_j}; \alpha_i = \frac{1}{(IS_i + \varepsilon)^2}, i = 0,\ 1,\ 2 \tag{2.3}$$

where

$$\begin{aligned} IS_0 &= \tfrac{13}{12}\left(f_{i-2} - 2f_{i-1} + f_i\right)^2 + \tfrac{1}{4}\left(f_{i-2} - 4f_{i-1} + 3f_i\right)^2; \\ IS_1 &= \tfrac{13}{12}\left(f_{i-1} - 2f_i + f_{i+1}\right)^2 + \tfrac{1}{4}\left(f_{i-1} - f_{i+1}\right)^2; \\ IS_2 &= \tfrac{13}{12}\left(f_{i+2} - 2f_{i+1} + f_i\right)^2 + \tfrac{1}{4}\left(f_{i+2} - 4f_{i+1} + 3f_i\right)^2. \end{aligned}$$

New Two Step Shock/Discontinuity Locator(Oliveira, Lu, Liu, Liu, 2009). This new shock detector consists of two main steps: *a multigrid truncation error ratio check and a local slope ratio check.*

Step 1: Determine the multigrid ratio of the approximation of the sum of the 4th, 5th and 6th truncation errors for [F = f + smooth sine wave of small amplitude] and select the points where the ratio is smaller than 4. Theoretically, the ratio of the 4th order truncation error of coarse and fine grids should be 16, but any function that has a ratio of 4 will be considered smooth and passing the test. The points which have a ratio less than 4 will be picked out for the second left and right hand slope ratio check.

The multigrid truncation error ratio check is:

$$MR\left(i,h\right)=\frac{T_{C}\left(i,h\right)}{T_{F}\left(i,h\right)+\varepsilon}, \tag{2.4}$$

where

$$\begin{aligned} T_{C}\left(i,h\right) &= T_{4}\left(i,2h\right)+T_{5}\left(i,2h\right)+T_{6}\left(i,2h\right) \\ &= \frac{\left|f_{i}^{(4)}\right|\left(2h\right)^{4}}{4!}+\frac{\left|f_{i}^{(5)}\right|\left(2h\right)^{5}}{5!}+\frac{\left|f_{i}^{(6)}\right|\left(2h\right)^{6}}{6!} \end{aligned}$$

and

$$\begin{aligned} T_{F}\left(i,h\right) &= T_{4}\left(i,h\right)+T_{5}\left(i,h\right)+T_{6}\left(i,h\right) \\ &= \frac{\left|f_{i}^{(4)}\right|\left(h\right)^{4}}{4!}+\frac{\left|f_{i}^{(5)}\right|\left(h\right)^{5}}{5!}+\frac{\left|f_{i}^{(6)}\right|\left(h\right)^{6}}{6!} \end{aligned}$$

where $T_F(i,h)$ is the truncation error sum (4th, 5th, and 6th) calculated at the fine grid with n points, $T_C(i,h)$ is the truncation error sum calculated at the coarse grid with n/2 points by Taylor expansion. $T_F(i,h)$ and $T_C(i,h)$ have 4th, 5th, and 6th order derivatives which are all calculated by a 6th order compact scheme.

Step 2: Calculate the local left and right slope ratio check only at the points which have first ratio less than 4.

The new local left and right slope ratio check is:

$$LR(i)=\frac{\left|\left|\frac{f'_{R}}{f'_{L}}\right|-\left|\frac{f'_{L}}{f'_{R}}\right|\right|}{\left|\frac{f'_{R}}{f'_{L}}\right|+\left|\frac{f'_{L}}{f'_{R}}\right|+\varepsilon}=\left|\frac{\left(f'_{R}\right)^{2}-\left(f'_{L}\right)^{2}}{\left(f'_{R}\right)^{2}+\left(f'_{L}\right)^{2}+\varepsilon}\right|=\left|\frac{\alpha_{R}^{2}-\alpha_{L}^{2}}{\alpha_{R}^{2}+\alpha_{L}^{2}+\varepsilon}\right| \tag{2.5}$$

where $f'_R = 3f_i - 4f_{i+1} + f_{i+2}$, $f'_L = 3f_i - 4f_{i-1} + f_{i-2}$ and ε is a small number to avoid division by zero.

Optional step 3: Use a cutoff value of 0.8 to create a 0/1 switch function on the result of Step 2. If the value is zero, f is considered locally smooth, and if the value is one, f is a shock/discontinuity at that point.

Note that Liu's first step always checks $f + \sigma \sin\left(k\pi x + \varphi\right)$ instead of f, where σ is a small number. Since all derivatives are calculated by a subroutine with a standard compact scheme, the cost of two step checks is relatively inexpensive.

In order to find a universal formula, we need to normalize the data set, $u(i), i = 1, \cdots, n$:

$$u_{diff}=\left|u_{max}-u_{min}\right|, \bar{u}=(u-u_{\min})/u_{diff}$$

Here,u_{max} and u_{min} are the maximum and minimum values of u respectively and $\bar{u}$ is normalized. For simplicity, we disregard the hat of u and use $u(i)$ as the normalized data set. However, this normalization is for finding the shock locator only and not for the function itself which we calculate for derivatives.

2.2. Control function for using WENO to improve CS and UCS. Although the new shock detector can provide accurate location of shock including weak shock, strong shock, oblique shock and discontinuity in function, first, second and third order derivatives, it is a switch function and give one in shock and zero for others.

As we mentioned above, a switch function cannot be directly used to mix CS and WENO and we must develop a rather smooth function to mix CS (2.6) and WENO (2.7):

$$\frac{1}{3}H'_{j-\frac{3}{2}}+H'_{j-\frac{1}{2}}+\frac{1}{3}H'_{j+\frac{1}{2}}=\left[-\frac{1}{36}H_{j-\frac{5}{2}}-\frac{7}{9}H_{j-\frac{3}{2}}+\frac{7}{9}H_{j+\frac{1}{2}}+\frac{1}{36}H_{j+\frac{3}{2}}\right]/h \quad (2.6)$$

$$\begin{aligned}H'_{j-1/2}=&\,\omega_{0,j-1/2}(\tfrac{1}{3}F_{j-3}-\tfrac{7}{6}F_{j-2}+\tfrac{11}{6}F_{j-1})\\&+\omega_{1,j-1/2}(-\tfrac{1}{6}F_{j-2}+\tfrac{5}{6}F_{j-1}+\tfrac{1}{3}F_j)\\&+\omega_{2,j-1/2}(\tfrac{1}{3}F_{j-1}+\tfrac{5}{6}F_j-\tfrac{1}{6}F_{j+1})\end{aligned} \quad (2.7)$$

where F is the original function and H is a primitive function of F and H' is the flux $\hat{F}=H'$. We defined a new control function Ω:

$$(1-\Omega)*CS+\Omega*WENO \quad (2.8)$$

This will lead a tri-diagonal matrix system which is the core of our new scheme:

$$\begin{aligned}&\tfrac{1}{3}(1-\Omega)H'_{j-\frac{3}{2}}+H'_{j-\frac{1}{2}}+\tfrac{1}{3}(1-\Omega)H'_{j+\frac{1}{2}}\\&=(1-\Omega)[\tfrac{1}{36}(H_{j+\frac{3}{2}}-H_{j-\frac{5}{2}})+\tfrac{7}{9}(H_{j+\frac{1}{2}}-H_{j-\frac{3}{2}})]/h\\&+\Omega*[\omega_{0,j-1/2}(\tfrac{1}{3}F_{j-3}-\tfrac{7}{6}F_{j-2}+\tfrac{11}{6}F_{j-1})+\omega_{1,j-1/2}(-\tfrac{1}{6}F_{j-2}+\tfrac{5}{6}F_{j-1}+\tfrac{1}{3}F_j)\\&+\omega_{2,j-1/2}(\tfrac{1}{3}F_{j-1}+\tfrac{5}{6}F_j-\tfrac{1}{6}F_{j+1})]\end{aligned} \quad (2.9)$$

When $\Omega=0.0$, the equation become a standard sixth order compact scheme, but when $\Omega=1.0$ the scheme is a standard WENO scheme.

For the modified upwinding compact scheme (MUCS), the final matrix becomes:

$$\begin{aligned}&\tfrac{1}{4}(1-\Omega)H'_{j-\frac{3}{2}}+H'_{j-\frac{1}{2}}+\tfrac{1}{2}(1-\Omega)H'_{j+\frac{1}{2}}\\&=(1-\Omega)*[-\tfrac{1}{60}H_{j-\frac{5}{2}}-\tfrac{31}{48}H_{j-\frac{3}{2}}-\tfrac{1}{3}H_{j-\frac{1}{2}}+\tfrac{11}{12}H_{j+\frac{1}{2}}+\tfrac{1}{12}H_{j+\frac{3}{2}}\\&-\tfrac{1}{240}H_{j+\frac{5}{2}}]/h+\Omega*[\omega_{0,j-1/2}(\tfrac{1}{3}F_{j-3}-\tfrac{7}{6}F_{j-2}+\tfrac{11}{6}F_{j-1})\\&+\omega_{1,j-1/2}(-\tfrac{1}{6}F_{j-2}+\tfrac{5}{6}F_{j-1}+\tfrac{1}{3}F_j)\\&+\omega_{2,j-1/2}(\tfrac{1}{3}F_{j-1}+\tfrac{5}{6}F_j-\tfrac{1}{6}F_{j+1})]\end{aligned} \quad (2.10)$$

Our origin control function is

$$\Omega=\min[1.0,8.0/MR(i,h)]\times LR(i,h) \quad (2.11)$$

in which MR is the multigrid global truncation error ratio and LR is local ratio of left and right side angle ratio. If the shock is met, MR is small, LR is near 1 and , the WENO will be used and the CS is fully blocked. If the area is smooth, MR should be around 16.0 and LR is close to zero (left and right angle are same). Additional requirement is set that any point must compare with left and right neighboring points and we pick the largest among the three neighboring points.

The reason we pick 8.0 is that we treat the fourth order continuous function as smooth function and only need half of LR for Ω. It is easy to find there are no case related adjustable coefficients which is quite different from many other published hybrid schemes. However, as the mixing function, sometimes its value is too small for the shock location, and the scheme smears too much. Our new control function, which is in the following, is better:

Define $A(i,h)=\sqrt{\min[1.0,4.0/MR(i,h)]\times LR(i,h)}$ (2.12)

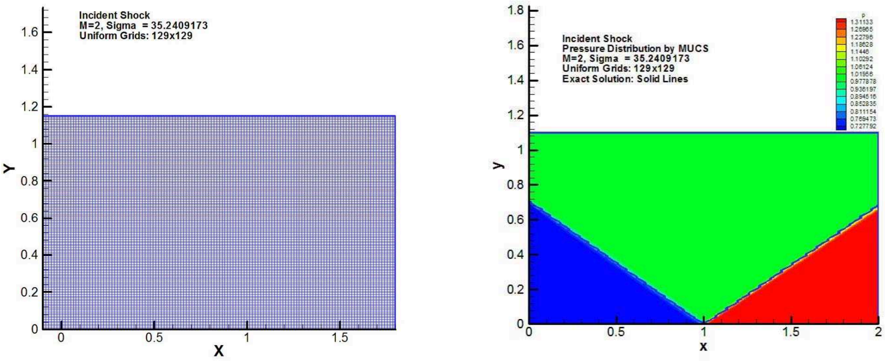
We set

$$\Omega=(A(i-1,h)+A(i,h)+A(i+1,h))/3.0.$$

For $A(i,h)$, we have the square root because min$[1.0\ , 4.0/MR(i,h)] \times LR(i,h)$ is the product of two values and both of them are smaller than one. The consequent value becomes too small for the shock area. Therefore we use the square root to "recover" the value to be near 1.0 as much as we can. We use the average of the three consecutive values as the final weight of WENO because the average can reduce the possibility of misjudgments and makes the control function much smoother. Following is the result of using our new control function, and we made some comparison between the new scheme and the pure WENO.

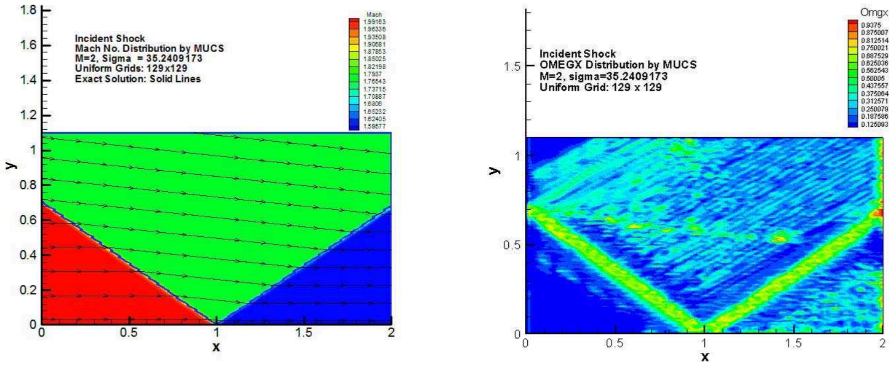
3. Computational Results by New MUCS

3.1. New MUCS for 2-D Euler Equations. An incident shock case with an inflow Mach number of 2 and attach angle of $\vartheta = 35.241^0$ was chosen as a sample problem to compare the WENO and MUCS results with the exact solution. Since the incident shock has exact solution, it is a good prototype problem for scheme validation and comparison. It is also a difficult problem to get sharp shock without visible oscillation for any high order scheme since it has oblique shocks involved. The computational domain is x=2.0 and y=1.1 and a uniform grids was used. We find that modified compact scheme (MCS) worked well on coarse and middle size grids, but has oscillations on the fine grids (129×129). While modified upwinding compact scheme or MUCS does not have serious oscillation, even better than WENO after the second shock. On the other hand MUCS captured shock sharper than WENO for all grids. The control (mixing) function did use WENO (red and yellow: WENO dominated) to block the UCS and used UCS for smooth area (blue area: UCS dominated). All of the comparisons are made by using same code and same boundary treatment but different subroutines (WENO or MUCS) for derivatives only. The fine grids (129×129) results are depicted on Figures 1-3 From these figures we can find the 7th order MUCS results are very comparable with exact solution and are better than that obtained by 5th order WENO scheme. Figure 2 (b) shows that our shock detector works pretty well and captures the shock accurately. Our new control function (mixing function) also gives good weights for WENO and UCS, which indicates that when the shock is met, WENO becomes dominant gradually, but in smooth area, the scheme is dominated by UCS. Figures 3 (a) and (b) give us the comparison of pressure on the wall between our numerical solution and exact solution, which shows that our result is very near the exact solution although it is a little overshooting after the second shock. In Figures 3 (c), (d), (e), (f) and (g), the comparisons of pressure on the wall and K=30 between our new scheme and pure WENO are given. Figures 3 (e), (f), (g) are locally enlarged for comparison. As seen, for pressure on the wall, although both of our new scheme and WENO have a little overshooting and oscillation immediately after the second shock (Figure 3.3 (f)), the pure WENO smears flow a lot. This smearing also occurs in all other level, e.g. K=30 (Figures 3 (d) and (g)). We know the smearing should be avoided for small length scales, especially for turbulence. Therefore, our new scheme is much more favorable for small length scales.



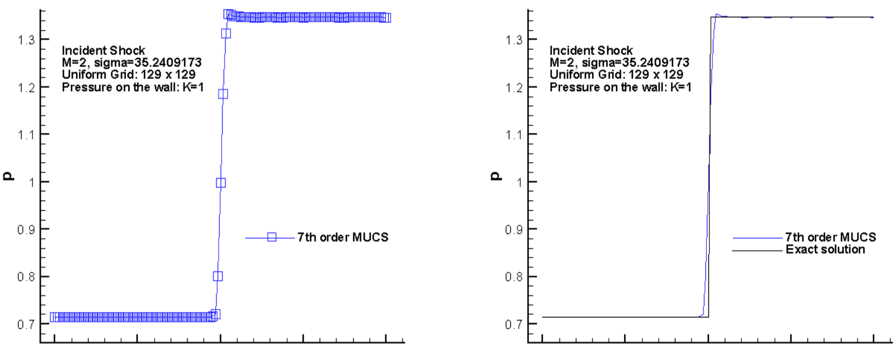
(a) (b)

FIGURE 1. **Numerical test for 2D incident Shock on fine grids** (a) Grids (129×129) (b) Pressure contour



(a) (b)

FIGURE 2. **Numerical test for 2D incident Shock on fine grids** (a) Mach number (b) Control function (red and yellow: WENO dominated; blue: UCS dominated)



(a) (b)

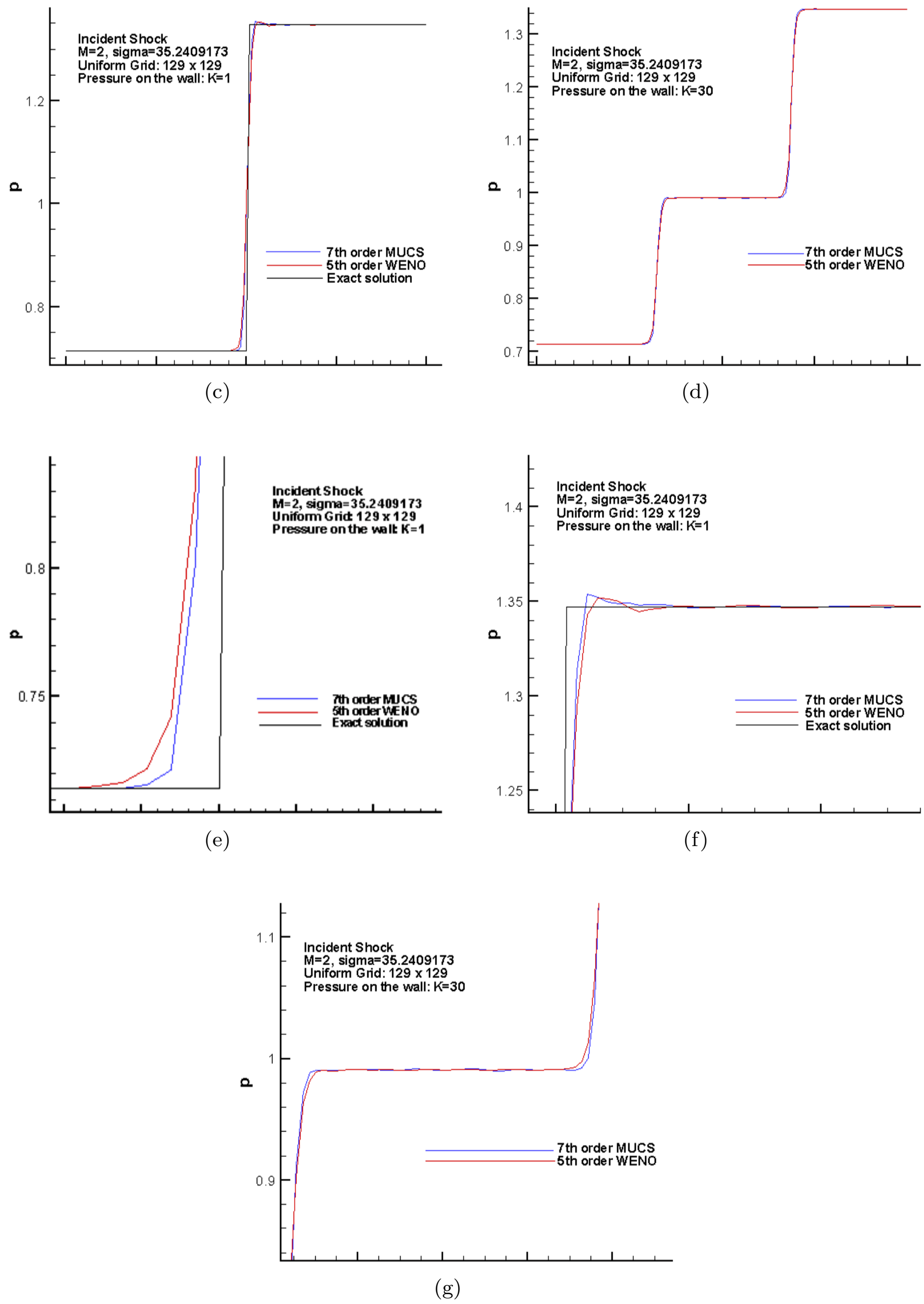
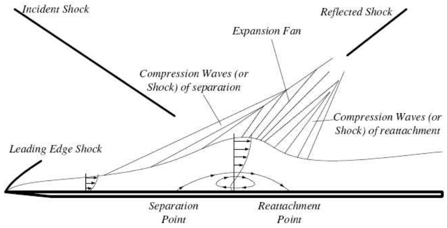


FIGURE 3. **Numerical test for 2D incident Shock on fine grids** (a) 7th order MUCS (b) MUCS and Exact (c) 7th order MUCS , 5th order WENO and exact solution (d) Pressure on the level K=30 (e) (f) (g) Locally enlarged comparisons

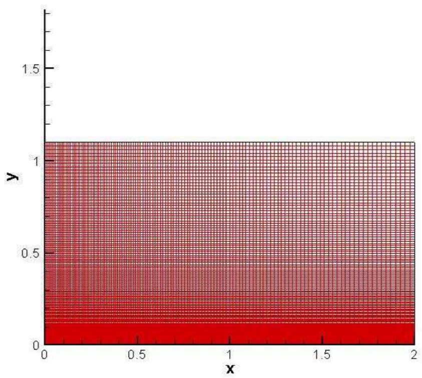
3.2. New MUCS for 2-D incident shock-boundary layer interaction. In order to compare the performance of our MUCS scheme with standard WENO, we select an incident shock case with Mach number 3 and Reynolds number$Re = 3\times10^4$ and the attack angle$\vartheta = 32.24^0$ (Figure 4). The grids 129×129 and 257×257 are selected.

Figure 5 (b), Figure 6 (b) give distribution contours for pressure with stream track for grid 129×129 and 257×257, respectively. The incident shock, leading shock, separation shock, expansion waves and reflecting shock are all clearly captured. A structure of separation bubble with 5 vertexes is captured. Figure 5 (d) (e) and Figure 6 (d) (e) depict the distribution of control function of OMGX and OMGZ in x- and z- directions, respectively. We can find that the new scheme capture the shock sharply. The blue area is the area where compact scheme is dominated and the yellow area represents the area where the WENO scheme is dominated. We also did some comparison between our new scheme and pure WENO which is depicted in Figure 5 (f) (g) and Figure 6 (f) (g) (h) (i). From Figure 5 (f) (g) and Figure 6 (f) (g) (h) (i), we found the pure WENO has a result with a lot of oscillations and smearing, while our new MUCS is much better.

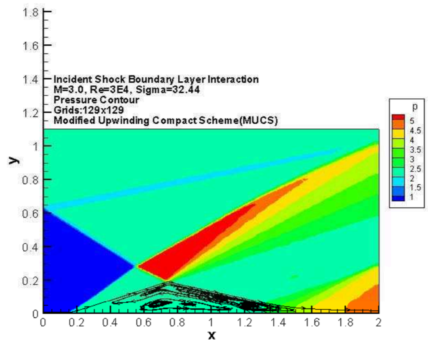


(a)

FIGURE 4. **Sketch of incident shock-boundary layer interaction**



(a)



(b)

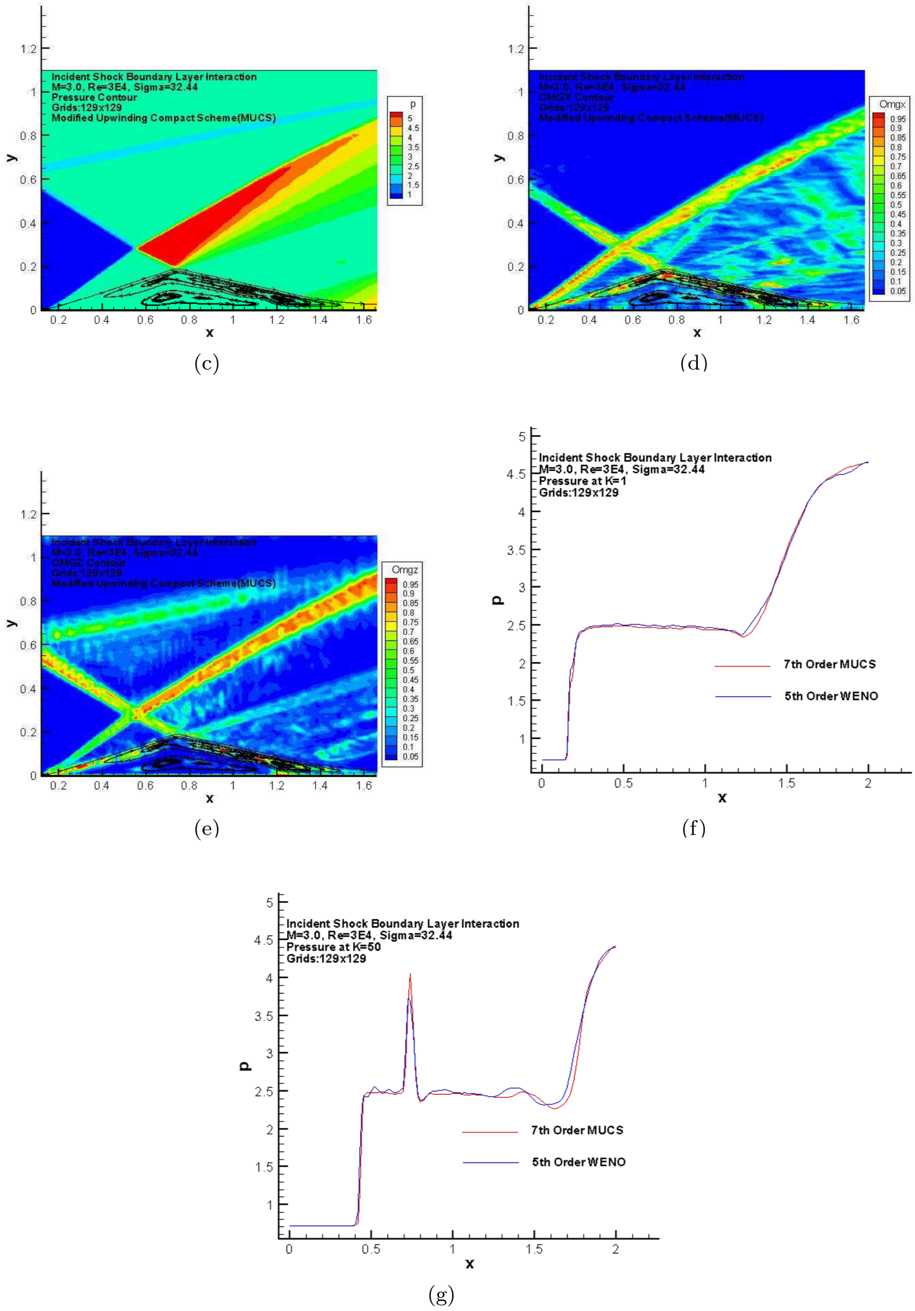
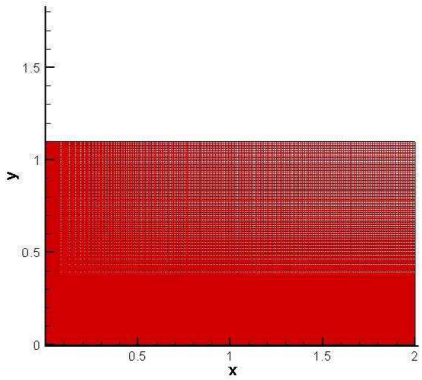
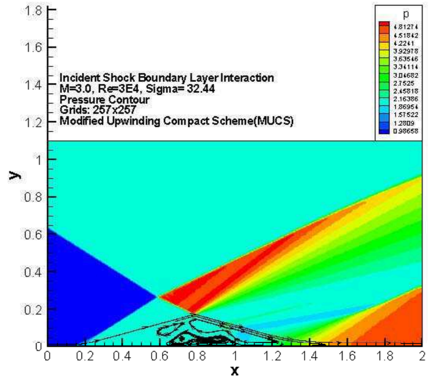


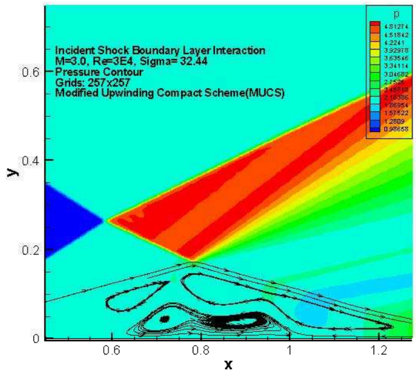
FIGURE 5. **Numerical results for grids** 129 × 129 (a) grids of 129 × 129 (b) pressure distribution (c) Locally enlarged pressure distribution (d) Omega in x direction (e) Omega in z direction (f) comparison of pressure on the wall between MUCS and WENO (g) comparison of pressure at K=50 between MUCS and WENO



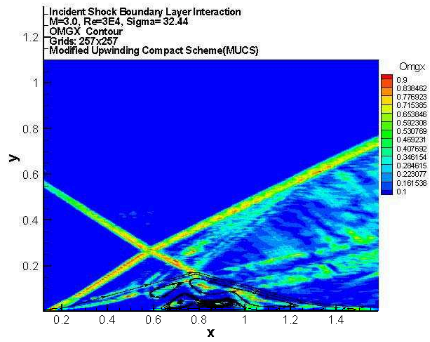
(a)


Incident Shock Boundary Layer Interaction
M=3.0, Re=3E4, Sigma= 32.44
Pressure Contour
Grids: 257x257
Modified Upwinding Compact Scheme(MUCS)

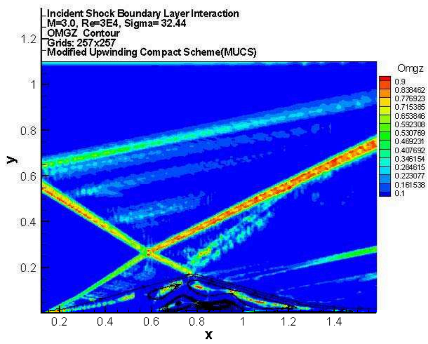
(b)


Incident Shock Boundary Layer Interaction
M=3.0, Re=3E4, Sigma= 32.44
Pressure Contour
Grids: 257x257
Modified Upwinding Compact Scheme(MUCS)

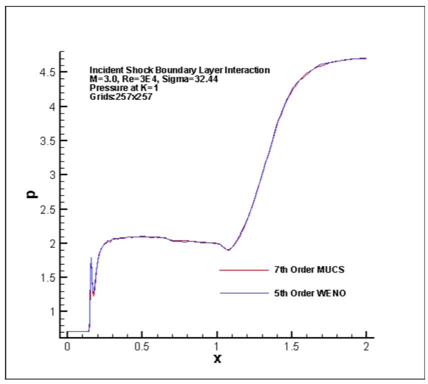
(c)


Incident Shock Boundary Layer Interaction
M=3.0, Re=3E4, Sigma= 32.44
OMGX Contour
Grids: 257x257
Modified Upwinding Compact Scheme(MUCS)

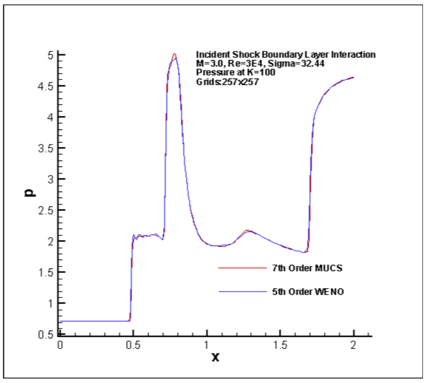
(d)


Incident Shock Boundary Layer Interaction
M=3.0, Re=3E4, Sigma= 32.44
OMGZ Contour
Grids: 257x257
Modified Upwinding Compact Scheme(MUCS)

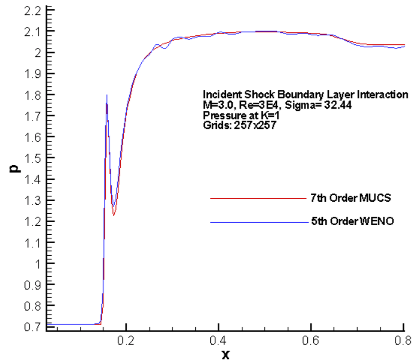
(e)


Incident Shock Boundary Layer Interaction
M=3.0, Re=3E4, Sigma=32.44
Pressure at K=1
Grids:257x257
7th Order MUCS
5th Order WENO

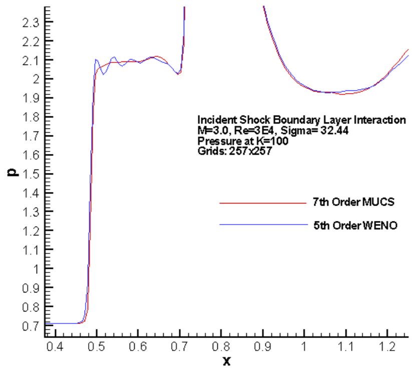
(f)



(g)



(h)



(i)

FIGURE 6. **Numerical results for grids** 257×257 (a) grids of 257×257 (b) pressure distribution (c) Locally enlarged pressure distribution (d) Omega in x direction (e) Omega in z direction (f) comparison of pressure on the wall between MUCS and WENO (g) comparison of pressure at K=100 between MUCS and WENO (h) locally enlarged pressure at K=1 (i)locally enlarged pressure at K=100

4. Conclusion

1. Modified up-winding compact scheme (MUCS) with a new shock detector and new mixing function, which uses WENO to improve upwind compact scheme, can be used for Euler and Navier-Stokes equations for sharp shock capturing and high resolution for small length scales.
2. The new scheme, MUCS, does not have case- related parameters.

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