INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 4, Number 3-4, Pages 392–401

NUMERICAL OPTIMIZATION OF RADIATED ENGINE NOISE WITH UNCERTAIN WAVENUMBERS

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Abstract. In this paper, we investigate an efficient numerical method to identify an optimal impedance factor for mitigating radiated engine noise. The engine tone-noise wavenumber is treated as a random variable. We prove the existence of the sensitivity derivative of the state variable (which is the acoustic pressure) with respect to the random wavenumber. The proposed numerical method is based on the stratified Monte Carlo algorithm whose convergence is accelerated by exploiting the sensitivity derivative information.

Key Words. optimal control, liner impedance factor, random wavenumber, finite element

1. Introduction

The purpose of this paper is to study the optimal design of the acoustic liner to minimize fan noise radiation from commercial aircraft engine nacelles. There have been numerous studies of this problem from an engineering perspective (e.g., [12, 26, 27] and references therein). In [8, 11], the search for the liner material minimizing engine noise radiation was treated as an optimal control problem and liner impedance factors that yielded significant noise reduction were found both theoretically and numerically.

Optimal control of systems governed by partial differential equations is a very active research subject [2, 14, 15, 16, 17, 20]. Most research in this area makes the natural (but rather unrealistic) assumption that the input data and model parameters of the control systems are precisely known. But it is well known that every physical system is subject to uncertainty due to varying operating conditions and imprecise measurements, and that when a mathematical model is formulated, further uncertainties may arise due to modeling and discretization errors [3, 18]. Practical experience suggests that uncertainties in the input data and the parameters of control systems can drastically reduce the reliability and accuracy of the deterministic optimal control approach.

Optimal control of stochastic partial differential equations clearly makes the control models more complex, but more flexible, realistic and hence of practical value. As the effect of parameter uncertainty is built into the model, one would expect that the optimal control will be less sensitive to changes in the model parameters, and hence more robust.

Received by the editors February 22, 2006.

2000 *Mathematics Subject Classification.* 49J20, 58E25, 65K10, 65M60.

Yanzhao Cao's research is supported by Air Force Office of Scientific Research under the grant number FA9550-05-1-0133 .

The key to the numerical solution of the proposed problem is an efficient simulation method to evaluate the cost function. The Monte Carlo simulation is a generally applicable solution method for stochastic problems. However, it is a well-known that as the accuracy requirement increases, the number of realizations (deterministic problems to be solved) grows far too rapidly. Consequently, algorithms that employ the Monte Carlo method may be easy to program, but impossible to employ on problems of practical interest. Alternatives to the Monte Carlo method include moment methods and the polynomial chaos method [24, 23, 25]. However, moment methods are accurate only for small variance problems. The polynomial chaos method may prove to be a viable alternative.

In this paper, we apply a modified Monte Carlo algorithm using the sensitivity derivatives of the state variable with respect to the uncertain model parameter – the acoustic wavenumber in the present case. The method was first developed in [4] and tested extensively in [6, 7, 28]. We focus on the combination of this method and the stratified sampling method. We provide the existence result on sensitivity derivatives as well as the variance reduction analysis on the sensitivity-derivative enhanced stratified Monte Carlo method (SDSMCM). Numerical experiments demonstrate the efficiency of this numerical method.

2. Optimal control model for the optimal impedance factor

2.1. Model formulation. We assume the problem to be axisymmetric [5]. The nacelle geometry has the generic shape represented in Figure 1. The modal composition of the noise source is supposed to be known on the source plane Γ_1. The nacelle boundary is made up of two parts: the first part being the interior boundary Γ_2 to which some acoustic liner material is attached, and the second part being Γ_3 that constitutes the rest of boundary of the nacelle geometry. The boundary Γ_4 is assumed to be sufficiently far from the noise source so that the Sommerfeld radiation boundary condition is an adequate approximation. The nacelle symmetry axis is denoted by Γ_5.

If the meanflow is uniform with Mach number M_0, the governing equation for the acoustic pressure u [21] is

$$(1-M_0^2)\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-2ikM_0\frac{\partial u}{\partial x}+k^2u=0. \tag{2.1}$$

For simplicity, we set the mean Mach number M_0 equal to zero. The acoustic pressure u then satisfies the Helmholtz equation,

$$\Delta u+k^2u=0 \quad \text{on } \Omega, \tag{2.2}$$

subject to the following boundary conditions on the boundary $\partial\Omega$ of Ω:

$$\begin{aligned} u\,\big|_{\Gamma_1} &= g,\\ \left(\tfrac{\partial u}{\partial n}+\tfrac{ik}{\xi}u\right)\Big|_{\Gamma_2} &= 0,\\ \tfrac{\partial u}{\partial n}\Big|_{\Gamma_3} &= 0,\\ \left(\tfrac{\partial u}{\partial n}+iku\right)\Big|_{\Gamma_4} &= 0,\\ \tfrac{\partial u}{\partial n}\Big|_{\Gamma_5} &= 0, \end{aligned} \tag{2.3}$$

where k is the wavenumber and ξ is the complex impedance factor whose real part represents resistance and the imaginary part reactance. Both the dependent and

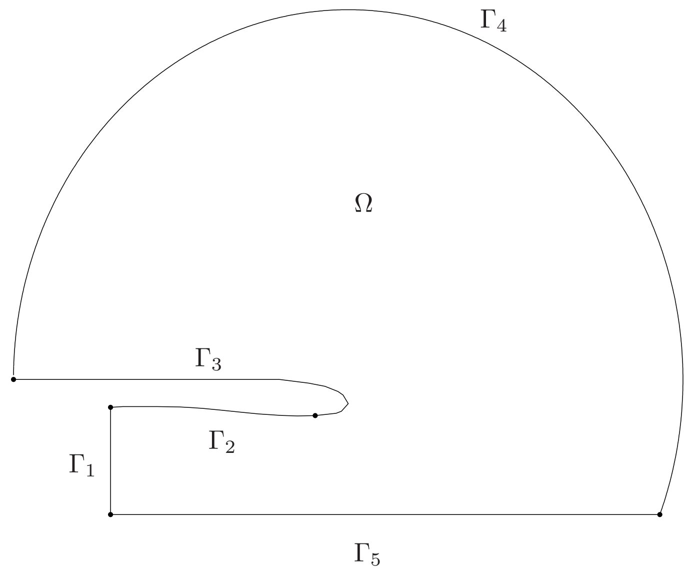


FIGURE 1. The computational domain

the independent variables in the above equations are assumed to be properly non-dimensionalized. The wavenumber k is a random variable. At least two factors contribute to the randomness of k:

- the variability of operating conditions of aircrafts;
- measurement errors in determining the value of k.

We define a functional which measures the level of the noise generated by the engine:

$$J(\xi, u(\xi, k)) = \alpha \int_\Omega u^2 d\Omega + \beta \int_\Omega |\nabla u|^2 d\Omega + \lambda |\xi - \xi_0|^2 \tag{2.4}$$

where $\alpha \geq 0$, $\beta \geq 0$, $\lambda \geq 0$ are constants, and ξ_0 is a given complex number. The optimization problem consists in finding the parameter ξ to minimize the noise propagating to the far field, while respecting some constraints on the boundary shape. More specifically, we want to find ξ that minimizes expected value of the cost functional J. If k is a random variable, the cost functional J is a random function, and we define $\hat{J}(\xi)$ as the expectation of J:

$$\hat{J}(\xi) = \int J(\xi, u(\xi, k))\rho(k)dk$$

where ρ is the probability density function of k. Our goal in this paper is to construct an efficient numerical algorithm to find the minimizer for the cost functional $\hat{J}$.

To conclude this subsection, we introduce the notation used throughout the paper. For a given integer m, let $H^m(\Omega)$ be the Sobelev space of complex-valued functions on Ω. Its norm and inner product are denoted by $\|\cdot\|_m$ and $(\cdot,\cdot)_m$, respectively. In particular, $H^0(\Omega)$ is the space of square integrable complex-valued functions on Ω, and its inner product is denoted by $(\cdot,\cdot)$. We also need the space

$H^1_E(\Omega) = \{u \in H^1(\Omega) : u|_{\Gamma_1} = 0\}$, which is a subspace of $H^1(\Omega)$. Similarly, we define $L^2(\Gamma)$, the space of square integrable complex-valued functions on Γ, a part of the boundary of Ω. The inner product and norm on $L^2(\Gamma)$ are denoted by $\langle \cdot, \cdot \rangle_\Gamma$ and $\| \cdot \|_\Gamma$, respectively.

2.2. Existence of sensitivity derivatives. In this subsection, we establish the existence of the sensitivity derivatives that will be used in the Monte Carlo method in the evaluation of the expected cost functional values. To this end, we first formulate the state equation (2.2) together with the boundary conditions (2.3) into the weak form: For given k and ξ, find $u = u(\xi, k) \in H^1(\Omega)$ with $u|_{\Gamma_1} = g$ such that

$$a(\xi, k, u, v) = 0, \quad \forall v \in H^1_E(\Omega), \tag{2.5}$$

where

$$a(\xi, k, u, v) = (\nabla u, \nabla v) - k^2(u, v) + \frac{ik}{\xi}\langle u, v\rangle_{\Gamma_2} + ik\langle u, v\rangle_{\Gamma_4}.$$

Theorem 1. *There is a countable set $\mathcal{K}$ of k such that variational problem (2.5) has a unique solution u for $k \notin \mathcal{K}$ and $\xi \in \Xi = \{$complex numbers with positive real parts$\}$. Furthermore, u is differentiable with respect to k for $k \notin \mathcal{K}$ and $\xi \in \Xi$, and the partial derivative $\frac{\partial u}{\partial k} \in H^1_E(\Omega)$ is the unique solution of the following variational equation:*

$$a(\xi, k, u_k, v) = 2k(u, v) - \frac{i}{\xi}\langle u, v\rangle_{\Gamma_2} - i\langle u, v\rangle_{\Gamma_4}, \quad \forall v \in H^1_E(\Omega). \tag{2.6}$$

Proof. It follows from the trace theorem (see [1]) that there is a function $u_* \in H^1(\Omega)$ such that $u_*|_{\Gamma_1} = g$. The variational problem (2.5) is equivalent to finding $w = u - u_* \in H^1_E(\Omega)$ such that

$$a(\xi, k, w, v) = -a(\xi, k, u_*, v), \quad \forall v \in H^1_E(\Omega). \tag{2.7}$$

Recall that the embeddings from $H^1(\Omega)$ to $L^2(\Omega)$ and $H^{1/2}(\partial\Omega)$ to $L^2(\partial\Omega)$ are compact ([1]). By the Riez Representation Theorem, there are compact bounded linear operators $B_j (j = 0, 2, 4)$ from $H^1_E(\Omega)$ to itself such that

$$(\phi, \psi) = (B_0\phi, \psi)_1, \quad \langle \phi, \psi\rangle_{\Gamma_j} = (B_j\phi, \psi)_1, \quad j = 2, 4, \quad \forall \phi,\ \psi \in H^1_E(\Omega).$$

By the Riez Representation Theorem again, we have for some $f(\xi, k) \in H^1_0(\Omega)$

$$-a(k, \xi, u_*, v) = (f(\xi, k), v)_1, \quad \forall v \in H^1_0(\Omega).$$

Hence, the variational problem (2.7) becomes

$$(I - (1 + k^2)B_0 + ikB_2/\xi + ikB_4)w, v)_1 = (f(\xi, k), v)_1, \quad \forall v \in H^1_0(\Omega),$$

which is the same as the following linear equation in $H^1_0(\Omega)$:

$$L(\xi, k)w = f(\xi, k), \tag{2.8}$$

where

$$L(\xi, k) = I - (1 + k^2)B_0 + \frac{ik}{\xi}B_2 + ikB_4.$$

Since B_0 is compact, there is a countable set $\mathcal{K}$ of k such that $I - (1 + k^2)B_0$ has a bounded inverse for all $k \notin \mathcal{K}$. For $k \notin \mathcal{K}$ and $\xi \in \Xi$, let w be a solution to the associated homogeneous equation of (2.8). Then we have $k \neq 0$ and

$$0 = \Im((L(\xi, k)w, w)_1) = \Im(a(\xi, k, w, w) = \frac{k\Re(\xi)}{|\xi|^2}\|w\|^2_{\Gamma_2} + k\|w\|^2_{\Gamma_4},$$

which yields that $w = 0$ on Γ_2 and Γ_4. Hence, we have

$$(I - (1 + k^2)B_0)w = 0.$$

Therefore, $w = 0$. It follows from the compactness of B_2 and B_4 that $L(\xi, k)$ has a bounded inverse, i.e., equation (2.8) has a unique solution $u(\xi, k)$ in $H^1_E(\Omega)$, for all $k \notin \mathcal{K}$ and $\xi \in \Xi$.

For $(\xi, k) \in \Xi \times (R \backslash \mathcal{K})$ and $|h|$ sufficiently small, we have

$$\begin{aligned} \frac{u(\xi, k+h) - u(\xi, k)}{h} &= \frac{w(\xi, k+h) - w(\xi, k)}{h} \\ &= L(\xi, k)^{-1} \left((2k + h)B_0 - \frac{i}{\xi} B_2 - iB_4 \right) u(k + h, \xi). \end{aligned}$$

It is apparent that $u(\xi, k)$ defines a continuous mapping from the open set $\Xi \times (R \backslash \mathcal{K})$ to $H^1_E(\Omega)$. Thus, we get

$$\lim_{h \to 0} \frac{u(\xi, k+h) - u(\xi, k)}{h} = L(\xi, k)^{-1} \left(2kB_0 - \frac{i}{\xi} B_2 - iB_4 \right) u(\xi, k) \quad \text{in} \quad H^1_E(\Omega).$$

Hence, u is differentiable with respect to k for $k \notin \mathcal{K}$ and $\xi \in \Xi$, and

$$\left(I - (1 + k^2)B_0 + \frac{ik}{\xi} B_2 + ikB_4 \right) \frac{\partial u}{\partial k}(\xi, k) = \left(2kB_0 - \frac{i}{\xi} B_2 - iB_4 \right) u(\xi, k).$$

It follows that $\frac{\partial u}{\partial k} \in H^1_E(\Omega)$ is the unique solution of (2.6). The proof is complete. □

The assumption that $\Im(\xi) > 0$ is a practical requirement (see [26]). It follows from (2.5) and (2.6) that $u(k)$ and its derivative $\frac{\partial u}{\partial k}$ can be computed by the same finite element procedure.

3. Sensitivity-derivative enhanced stratified Monte Carlo method

In this section, we consider the combination of the stratified sampling Monte Carlo method and the sensitivity-derivative enhanced Monte Carlo method to compute the cost functional $\hat{J}(\xi)$. For notational convenience, we replace $J(\xi, u(k))$ with $J(u(k))$. Let $\Phi(k)$ denote the cumulative distribution function of the random variable k:

$$\Phi(k) = \int_{-\infty}^{k} \rho(\zeta) d\zeta \, . \tag{3.1}$$

The function Φ is non-decreasing with range $[0, 1]$. The interval $[0, 1]$ is divided into S strata, assumed here for simplicity to be of equal length:

$$\left[\eta^s, \eta^{s+1} \right], \quad s = 0, 1, \ldots, S - 1 \, , \tag{3.2}$$

where

$$\eta^s = \frac{s}{S}, \quad s = 0, 1, \ldots, S \, . \tag{3.3}$$

In the standard stratified sampling method, one chooses N_S random samples, $\psi^s_i, i = 1, \ldots, N_S$, uniformly distributed in $\left[\eta^s, \eta^{s+1} \right]$ for each s, and computes the corresponding random samples in the variable ξ by inverting the cumulative distribution function:

$$k^s_i = \Phi^{-1}(\psi^s_i), \quad i = 1, \ldots, N_S \, . \tag{3.4}$$

This procedure ensures that the k_i^s are distributed according to the density function $\rho(k)$. The expected value of J is then approximated by

$$\hat{J} \approx \frac{1}{S\ N_S} \sum_{s=0}^{S-1} \sum_{i=1}^{N_S} J(u(k_i^s)) \ . \tag{3.5}$$

For the the sensitive-derivative enhanced stratified sampling, one first computes the contribution to (3.5) from each stratum by applying the sensitivity derivative enhanced Monte Carlo method (see [10] for details of the method). In particular,

$$\hat{J} \approx \frac{1}{S} \sum_{s=1}^{S} \tilde{J}^s \ , \tag{3.6}$$

where

$$\tilde{J}^s = J(u(\bar{k}^s)) + \frac{1}{N_S} \sum_{i=1}^{N_S} \left(J(u(k_i^s)) - J_1^s(k_i^s) \right) \ , \tag{3.7}$$

with

$$J_1^s(k) = J\left(u\left(\bar{k}^s\right)\right) + J_u\left(u\left(\bar{k}^s\right)\right) u_k\left(\bar{k}^s\right)\left(k - \bar{k}^s\right) \ , \tag{3.8}$$

where J_u denotes the gradient of J, u_k denotes the sensitivity derivative of u with respect to k and $\bar{k}^s$ is the mean value of k in the s-th stratum, given by

$$\bar{k}^s = \frac{\int_{k^s}^{k^{s+1}} k\rho(k)dk}{\int_{k^s}^{k^{s+1}} \rho(k)dk}, \tag{3.9}$$

where k^s is computed from the η^s from (3.4).

Next we examine the variance reduction effects of the sensitivity derivative enhanced stratified sampling method. Assume that k is uniformly distributed on $[a, b]$, i.e., $k \sim U(a, b)$. Let $k^i \sim U(\mu_i, \mu_{i+1})$. Then the standard stratified sampling method (3.5) for the evaluation of the expected value of $J(u(k))$ is equivalent to the evaluation of the expectation of the random variable $\frac{1}{S}\sum_{i=1}^{S} J(u(k^i))$ by the standard Monte Carlo method, while the sensitivity derivative enhanced stratified sampling method is equivalent to the evaluation of the expectation of the random variable $\frac{1}{S}\sum_{i=1}^{S}\left(J\left(u\left(k^i\right)\right) - J_1\left(k^i\right)\right)$ by the standard Monte Carlo method. Let $E(\chi)$ and $V(\chi)$ denote the expectation and variance of the random variable χ, respectively. We have the following estimates for the variances of these two random variables.

Theorem 2. *Let* $m = \max\left|\frac{d}{dk}J(u(k))\right|$ *and* $M = \max\left|\frac{d^2}{dk^2}J(u(k))\right|$. *Then*

$$V\left(\frac{1}{S}\sum_{i=1}^{S} J\big(u(k^i)\big)\right) \le \frac{m^2}{6S^3}(b-a)^2 \tag{3.10}$$

and

$$V\left(\frac{1}{S}\sum_{i=1}^{S}\Big(J\big(u(k^i)\big) - J_1\big(k^i\big)\Big)\right) \le \frac{7M^2}{720S^5}(b-a)^4. \tag{3.11}$$

Proof. By Equation (7) of [9], we have that $V(J(u(k^i))) \le \frac{m^2}{2}V(k^i)$. Thus

$$V\left(\frac{1}{S}\sum_{i=1}^{S} J\big(u(k^i)\big)\right) \le \frac{1}{S^2}\sum_{i=1}^{S}\frac{m^2}{2}V(k^i) = \frac{m^2}{6S^3}(b-a)^2.$$

From equation (8) of [9], we have that

$$V\big(J(u(k^i)) - J_1(k^i)\big) \le \frac{M^2}{2}\left[V^2(k^i) + E(k-k_i))^4\right].$$

Therefore

$$V\left(\frac{1}{S}\sum_{i=1}^{S}(J(u(k^i)) - J_1(k^i))\right) \le \frac{1}{S^2}\sum_{i=1}^{S}\frac{M^2}{2}\left[V^2(k^i) + E(k-k_i))^4\right]$$

$$= \frac{M^2}{2S^2}\sum_{i=1}^{S}\left(\frac{(b-a)^4}{144S^4} + \frac{(b-a)^4}{80S^4}\right) = \frac{7M^2}{720S^5}(b-a)^4.$$

□

Based on (3.10) and (3.11), we observe that

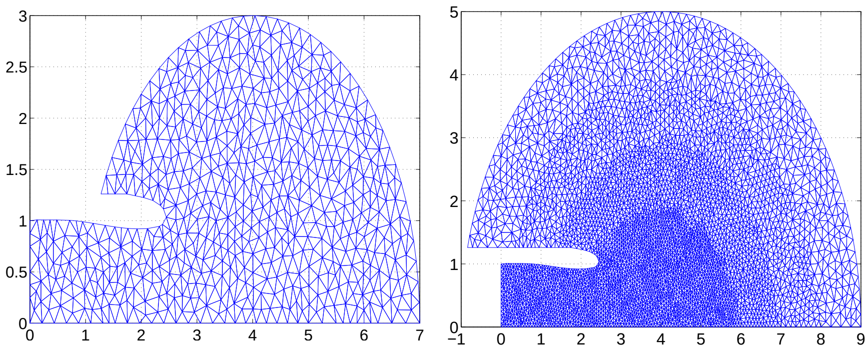
- the smaller the variance of k, the more effective is the stratified sampling method, and
- for sufficiently large S, the standard stratified sampling method accelerates the convergence of the standard Monte Carlo method by variance reduction, and the sensitivity-derivative enhanced stratified sampling method further accelerates convergence of sampling.

4. Numerical experiments

In this section, we present numerical results to show that the choice of optimal impedance parameter results in reducing the noise level significantly. The variational problem (2.5) is solved by a linear finite-element method. The computational domains are triangulated by *Triangle* (by J.R. Shewchuk at http://www.cs.cmu.edu/~quake/triangle.html). The resulting linear system is solved by the BiCG method. In the following, we take $\gamma = 0$ and the noise source $g(y) = 20.0 + \exp(3y)\sin(10\pi y)$. We also assume that k is uniformly distributed on $(2\pi - 0.2, 2\pi + 0.2)$, which is simulated by the random number generator *gsl_ran_uniform* of the GNU Scientific Library (http://www.gnu.org/software/gsl/).

First, we compare the sensitivity-derivative enhanced stratified Monte Carlo method (SDSMCM) with the traditional stratified Monte Carlo (SMCM). To this end, we consider computing $\hat{J}(\xi)$ for $\xi = 5 - i$ with the computational domain and its triangulation specified in Figure 2 (a), for which the mesh size is approximately 0.2919. Using the Simpson rule, we obtain $\hat{J}(5-i) = 1355.1192$ which is taken as the "exact value". For $S = 10$, we display the errors (with respect to this "exact value") of SMCM and SDSMCM as well as the ratios between the them in Table 1, which show that SDSMCM converges much faster than SMCM. We also observe that SDSMCM is quite accurate even when the total sample size is only $32 \times 10 = 320$.

Next, we compare the value of $\hat{J}$ when the hard-wall condition is imposed on Γ_2 and its value at a minimum ξ^* obtained by using APPSPACK 4.0 (see [13, 22]). It should be pointed out that the hard-wall condition corresponds to $|\xi| = \infty$. In this case, we use the computational domain and its triangulation specified in Figure 2



(a) (b)

FIGURE 2. The computational domains and their triangulation

TABLE 1. SMCM and SDSMCM

N_S	SMCM	SDSMCM	SMCM/SDSMCM
4	0.83359988	0.18739076	4.4485
8	0.55246987	0.13558235	4.0748
16	0.19973568	0.11324156	1.7638
32	0.17677950	0.06135548	2.8812
64	0.15025451	0.04273177	3.5162
128	0.10705324	0.02690782	3.9785
256	0.06522164	0.02046095	3.1876
512	0.04597298	0.01471749	3.1237

(b), for which the mesh size is approximately 0.0415 near the noise source and 0.2917 in the far field. In Table 2, we display the results for $S = 10$ and $N_S = 32$. Table 2 shows that the noise level is reduced significantly when the optimal impedance factor is used.

TABLE 2. The optimal impedance factors

α	β	$\hat{J}(\infty)$	ξ^*	$\hat{J}(\chi^*)$	$(\hat{J}(\infty) - \hat{J}(\chi^*))/\hat{J}(\infty)$
1	0	3375	$0.390625 - 1.09375i$	2012	40%
1	1	137613	$0.46875 - 1.09375i$	85158	38%

Finally, we display the contour maps of the amplitude of $E(u)$ in Figures 3 and 4. We observe that the distribution of noise is practically confined to the proximity of the inlet when the liner impedance factor ξ is optimal as in the case of the deterministic wavenumber (see [8]).

5. Conclusions

A sampling technique is proposed for the numerical solution of optimal control of stochastic partial differential equations, and its performance is demonstrated in the case of engine noise mitigation problem. Its performance is found to be significantly better than the traditional Monte Carlo sampling. As the sensitivity derivatives can be computed at a fraction of the cost of an analysis computation, and as they are usually available as part of many analysis codes, this sampling

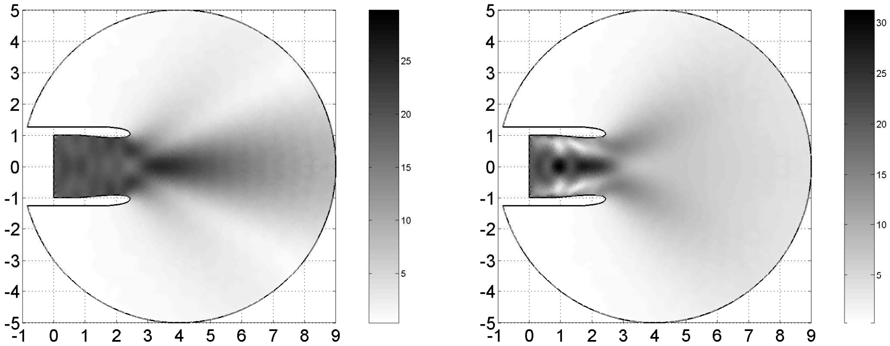


FIGURE 3. The contour maps of $|E(u(\xi))|$ for $\alpha = 1,\ \beta = 0:$
(left) $\xi = \infty$ and (right) $\xi = \xi^*$

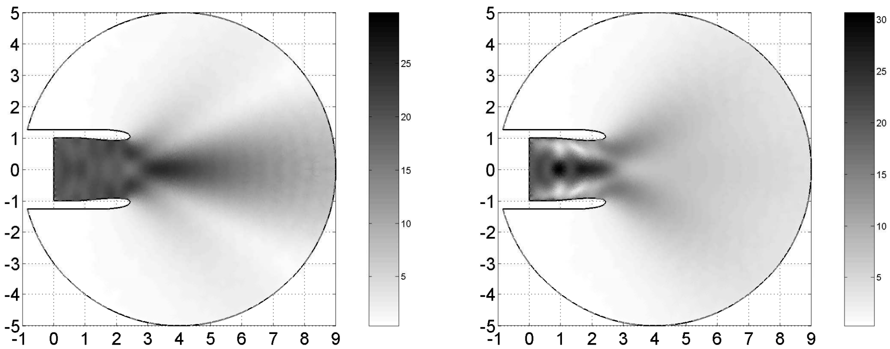


FIGURE 4. The contour maps of $|E(u(\xi))|$ for $\alpha = 1,\ \beta = 1:$
(left) $\xi = \infty$ and (right) $\xi = \xi^*$

technique has the potential for application to practical problems. Future research will focus on multiple random parameters, random control variables and enhancing the performance of quasi Monte Carlo and Latin Hypercube methods with the sensitivity derivative information.

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