Preface

The phase field concept was originally grounded in physics. In the 1880's, Josiah Willard Gibbs developed a phenomenological framework for equilibrium via the free energy functional within multiphase systems. This was further developed by Johannes Diderik van der Waals to include a density function, which allowed for phase equilibria to be characterized by a single free energy with a smooth order parameter. This was later developed by such notables as Sam Allen, John Cahn, and John Hilliard to model the coarsening and separation of phases.

In the 1980's, the phase-field, or diffuse-interface, method was used by several researchers to model solidification. Here, the essential idea was to relax the sharp interface to a diffuse layer for the sake of computational and modeling simplicity. This made for simpler tracking of the interface and easily enabled topological changes. Later, applied mathematicians were able to use asymptotic analyses to justify this approximation as the diffuse interface width shrunk to zero and recover the sharp interface solution (at least away from topological changes). Moreover, computational methods were developed to leverage the phase-field technique to tackle complex, multi-physics problems, which were built on thermodynamically-consistent models based on the theory of non-equilibrium thermodynamics, entropy production, and Onsager's principles.

Contemporary research in phase-field methods continues to grow into many new areas, such as multi-phase fluid dynamics with more than two phases, and multi-physics problems involving fluids, electro-statics, and transport of chemical species. In addition, phase-field methods are being used to solve problems in topology optimization, in geometry (e.g. the planar Steiner minimal tree problem), and in modeling biological cell migration. The phase-field method is a powerful tool of which model builders and computational scientists should be aware. This special issue is dedicated to advances in modeling, analysis, and numerical methods within the phase-field methodology.

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