INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 2, Supp , Pages 131–137

OPTIMIZATION FOR AUTOMATIC HISTORY MATCHING

SHUGUANG WANG, GUOZHONG ZHAO, LUOBIN XU,
DEZHI GUO AND SHUYAN SUN

Abstract. History matching is an inverse problem of partial differential equation on mathematics. We adopt the constrained non-linear optimization to handle this problem, defining the objective function as the weighted square sum of differences between the wells simulation values and the corresponding observation values. We develop an optimization computing program that include Zoutendijk feasible direction methodQuasi-Newton method (BFGS) and improved Nelder-Mead simplex method, combined with a black-oil simulator, and discuss the convergence characters of algorithms in case studies about determining average porosity and directional permeability, determining low permeability strip between two wells and determining oil-water relative permeability curves.

Key Words. reservoirs numerical simulation, automatic history matching, inverse problem, optimization.

1. Problem

History matching is absolutely necessary for a real reservoir simulation, which is to find a suitable set of values for the simulator's input parameters such that the simulator correctly predicts the fluid outputs and the pressures of the wells on the reservoir. It is an inverse problem of partial differential equation on mathematics, and is not a well-posed problem [1-20]. Yet there must exist a solution reflecting real formation condition for a real reservoir problem. So we would focus attention on the stability of the history matching problem model and the algorithm feasibility, not to be concerned with the existence and singleness of the solution.

2. Mathematic Model

We adopt the constrained non-linear optimization most in use for inverse problem of partial differential equation to handle history matching problem, define the objective function as the weighted square sum of differences between the wells simulation values and the corresponding observation values:

$$f(X) = \sum_{i=1}^{n_w} \sum_{j=1}^{n_t} \sum_{k=1}^{n_k} \omega(i,j,k)[y^{obj}(i,j,k) - y^{cal}(i,j,k)]^2 \tag{1}$$

where y^{obj}, y^{cal} denote the observation values and simulator computing values respectively, ω denotes parameter scale coefficient *i, j, k* denote well number, time segment and data kind respectively, n_w, n_t, n_k are the maximum of *i, j, k* respectively , X denotes optimal vector.

For a general history matching problemthe objective function is an implicit function of the optimal vectorit needs to carrying out a simulation run to gain a objective function value, it is the uppermost computing cost. Therefore dealing equality constrained history matching problem, should adopt elimination method to reduce variable number, so as to optimization computing converge rapidly. So a general history problem can be posted as an inequality constrained nonlinear optimization problem

$$\begin{array}{lll} min & f(X) & X \in E^n \\ s.t & g_i(X) \geq 0 & i = 1, \cdots, m \end{array} \tag{2}$$

The optimal vector X, the objective function $f(X)$ and the inequality constrained function vector $G(X)$ are different for different history matching problem.

3. Algorithms

We develop an optimization computing program that include *Zoutendijk feasible direction methodQuasi-Newton method (BFGS) and improved Nelder-Mead simplex method* [21], combined with a black-oil simulator, and discuss the convergence characters of algorithms in some case studies.

Zoutendijk feasible direction method is a constrained nonlinear optimization method, it is in different ways to deal linear constraints and nonlinear constraints.

For linear inequality constraints optimization problem

$$\begin{array}{ll} min & f(X) \\ s.t & AX \geq b \end{array} \tag{3}$$

where, $f(\mathbf{X})$ is differential function, $\mathbf{A}$ is $m \times n$ matrix. $X \in E^n$, $\mathbf{b}$ is $\mathbf{m}$ dimension column vector. *Zoutendijk feasible direction method* transform determinating descent feasible direction $\mathbf{d}$ to solving following linear programming problem, according necessary conditions $\bigtriangledown f(X)^T d0, \quad A_1 d \geq 0$,

$$\begin{array}{ll} min & \bigtriangledown f(X)^T d \\ s.t & A_1 d \geq 0 \\ |d_j| \leq 1 & j = 1, \cdots n \end{array} \tag{4}$$

Linear search step restriction:

$$\lambda_{max} = \begin{cases} min\{\frac{B_j}{D_j} | D_j < 0\}, & D < 0 \\ \infty & D > 0 \end{cases} \tag{5}$$

where, $A_1\mathbf{X} = \mathbf{b}_1$, $A_2\mathbf{X} > \mathbf{b}_2$, $A = \left[\begin{array}{c} A_1 \\ A_2 \end{array} \right]$, $b = \left[\begin{array}{c} b_1 \\ b_2 \end{array} \right]$, $B = \mathbf{b}_2 - A_2\mathbf{X}_i, D = A_2\mathbf{d}_i$

For nonlinearinequality constraints optimization problem,

$$\begin{array}{lll} min & f(X) & \\ s.t & g_i(X) \geq 0 & i = 1, \cdots, m \end{array} \tag{6}$$

where$\mathbf{X} \in E^n, f(\mathbf{X}), g_i(\mathbf{X})$ are differentiable functions. *Zoutendijk feasible direction method* transform determinating descent feasible direction $\mathbf{d}$ to solving following *linear programming problem*, according necessary conditions $\bigtriangledown f(\mathbf{X})^T d < 0$, $\bigtriangledown g_i(\mathbf{X})^T d > 0$, $i \in I$, $I = \{i | g_i(X) = 0\}$

$$\begin{array}{lll} min & Z & \\ s.t & \bigtriangledown f(\mathbf{x})^T d - Z \leq 0 & \\ & \bigtriangledown g_i(\mathbf{x})^T d - Z \geq -g_i(\mathbf{x}), & i = 1, \cdots, m \\ & |d_j| \leq 1 & i = 1, \cdots, m \end{array} \tag{7}$$

Linear search step restriction:

$$\lambda_{max} = sup\{\lambda | g_i(X_k + \lambda d_k) \geq 0, i = 1, 2, \cdots, m\}$$

Zoutendijk feasible direction method obtain: steepest descent direction when search point in the linear inequality constraints feasible region or steepest descent direction pointing to inside feasible region, projection direction of the steepest descent on the active constraint surfaces when search point on the linear inequality constraint surfaces and steepest descent direction pointing to outside feasible region; angle bisector direction between the steepest descent direction and the gradient vector of active nonlinear inequality constraint surfaces when search point on the nonlinear inequality constraint surfaces, the more far from nonlinear inequality constraint surfaces, the more closed with steepest descent direction when search point in the nonlinear inequality constraint region.

Quasi-Newton method (BFGS) is an unconstrained nonlinear optimization method, it approximates the inverse matrix of the *Hession* matrix in *Newton's method* in iteration method with the gradient vector. If we known the approximate matrix H_i of the A_i^{-1}let the approximate matrix H_{i+1} of the A_{i+1}^{-1} be $H_{i+1} = H_i + E_i, E_i$ is ith updated matrix. *BFGS formula* make choice $H_1 = I$, and define the ith updated matrix

$$E_i = \left(1 + \frac{\mathbf{q}_i^T \mathbf{H}_i \mathbf{q}_i}{\mathbf{p}_i^T \mathbf{q}_i}\right) \frac{\mathbf{p}_i \mathbf{p}_i^T}{\mathbf{p}_i^T \mathbf{q}_i} - \frac{\mathbf{p}_i \mathbf{q}_i^T \mathbf{H}_i + \mathbf{H}_i \mathbf{q}_i \mathbf{p}_i^T}{\mathbf{p}_i^T \mathbf{q}_i} \tag{8}$$

where$\mathbf{p}_i = \mathbf{X}_{i+1} - \mathbf{X}_i$, $\mathbf{q}_i = \bigtriangledown f(\mathbf{X}_{i+1}) - \bigtriangledown f(\mathbf{X}_i)$, when iteration steps reach the variable number, the initial value of approximate matrix will be reset, iteration will be restarted.

If $\bigtriangledown f(\mathbf{X}_i) \neq 0, i1, \cdots, n$, the constructed approximate matrix $H_i(i1, \cdots, n)$ is positive definite matrix; If objective function is positive definite quadratic function, the conjugated search direction is obtained and the minimum point must be reached by this formula in finite step iterations.

In computing, we force *Quasi-Newton method (BFGS)* turn into *Zoutendijk feasible direction method* on the next iteration when search stop on the inequality constraint surfaces*improved Nelder-Mead simplex method* can be used to handle inequality constraints optimization problemWhen descent feasible direction has been obtained, a linear investigation with increasing step length will be carried out to find high-low-high three points in the direction (or minimum point on inequality constraint surface), then a three points quadratic interpolation will be performed.

4. Case Studies

Three case studies are carried out with algorithms above in matching well pressures and water cut. The reservoir model is 1 layer and 11×11 blocks, one injection well and three production wells (figure 1), distance between wells is 200m. Simulation carries out on a three phase black oil simulator with automatic history matching function, with all implicit method equation solvers.

Determining average porosity and directional permeabilitys is carried out on a model with 0.27 Porosity, 300md **x** directional permeability and 75md **y** directional permeability. Constrained conditions are $1md \leq K_x, K_y \leq 3000md$ and $0.005 \leq Por \leq 0.5$. Initial values are $K_x = K_y = 180md, Por = 0.35$. The result is:

The result indicates that the computing is convergent and optimal variables are determinable.

Determining low permeability strip between two wells is carry out on a model with 0.27 Porosity, 300md **x** and **y** directional permeability, with 10md x directional

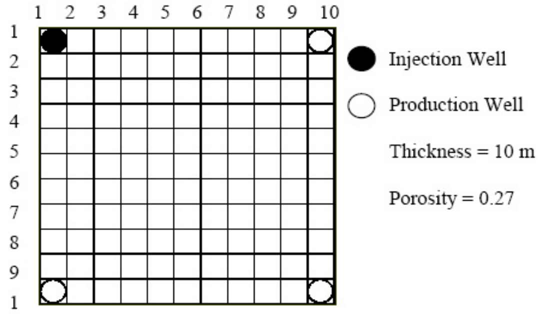
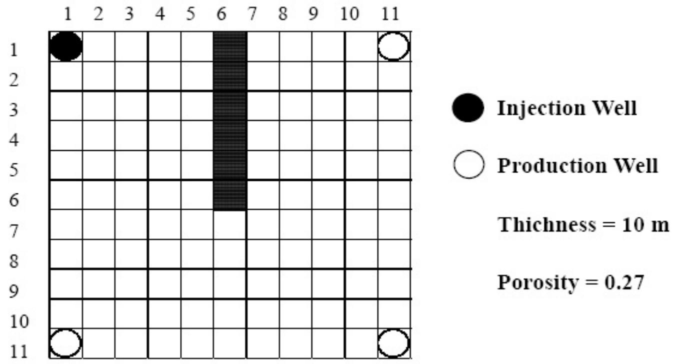


Figure 1

ALGORITHM	ITN	SIMN	OBJFUN	DIFFERENCE		VARIABLE		
				P (KPa)	Wcut(%)	K_x (md)	K_y (md)	Por(f)
Steepest Descent	26	191	2.17835	6.7371	8.6966×10^{-4}	299.99	74.994	0.26999
BFGS	9	73	2.88884	0.1118	5.9589×10^{-4}	299.98	74.990	0.26999
DFP	9	74	2.87725	0.1127	5.7228×10^{-4}	299.98	74.990	0.26999
Neld-Mead	80	189	1.64387	9.2108	2.5470×10^{-4}	300.00	75.004	0.27000
Simplex	105	240	0.28179	3.9036	8.1264×10^{-5}	299.99	75.000	0.27000

permeability including six blocks low permeability strip (figure 2). Constrained conditions are $1md \leq K_{xv} \leq 3000md$. The result is:



Vinit	ALGORITHM	ITN	SIMN	OBJFUN	DIFFERENCE	
					P (KPa)	Wcut(%)
300.00	Steepest Descent	26	250	85061.866	21.7386	
30.000	Steepest Descent	40	380	371.86478	1.43733	
	BFGS	41	399	582.15780	1.79839	
3.0000	Steepest Descent	30	306	8.0894455	0.21199	
	BFGS	13	141	2.1172348	0.10845	
	Steepest Descent	70	696	140.48230	0.32429	8.2178×10^{-3}
	BFGS	30	321	5.2567500	0.07746	1.5264×10^{-3}

Variables	Vinit	Steepestdescent V_f (Matching P)	BFGS V_f (Matching P)	Steepest descent V_f (Matching P, Wcut)	BFGS V_f (Matching P, Wcut)
X1	300.000	1.307396			
X2	300.000	1.320158			
X3	300.000	1.428910			
X4	300.000	1.556308			
X5	300.000	52.11080			
X6	300.000	329.1404			
X1	30.0000	5.978256	4.193198		
X2	30.0000	5.179856	5.533061		
X3	30.0000	21.19928	23.86267		
X4	30.0000	13.72538	12.37608		
X5	30.0000	8.870324	10.06122		
X6	30.0000	6.336989	6.685886		
X1	3.00000	9.251298	9.540971	9.845584	9.905584
X2	3.00000	10.13492	10.06274	9.880600	9.876773
X3	3.00000	10.77611	10.84602	10.21839	10.61945
X4	3.00000	10.77133	10.70558	10.50321	9.631231
X5	3.00000	10.01385	7.363978	10.13577	9.703287
X6	3.00000	8.314169	11.98132	8.575823	10.29446

The result indicates that the computing is convergent and the determinability of the optimal variables is relative to initial values.

Determining oil-water relative permeability curves

Assuming connate water saturation and residual oil saturation are fixed, and five points on both oil relative permeability curve and water relative permeability curve to be optimized. The initial values are on two straight lines. Optimal method use *BFGS*. The constrained conditions are:

$$\begin{array}{ll} K_{r0}(Swc) - K_{r1} > 0, & K_{r1} - K_{r2} > 0, \\ K_{r2} - K_{r3} > 0, & K_{r3} - K_{r4} > 0, \\ K_{r4} - K_{r5} > 0, & K_{r5} > 0, \\ K_{r6} > 0, & K_{r7} - K_{r6} > 0, \\ K_{r8} - K_{r7} > 0, & K_{r9} - K_{r8} > 0, \\ K_{r10} - K_{r9} > 0, & K_{rw}(1 - Sor) - K_{r10} > 0, \end{array}$$

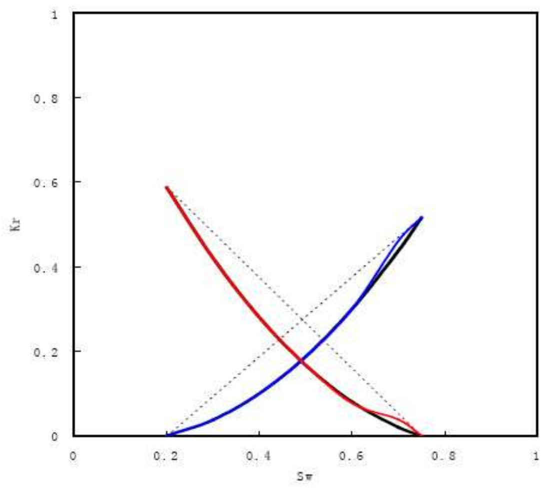
The result indicates that the computing is convergent and most optimal variables are determinable except the last two points.

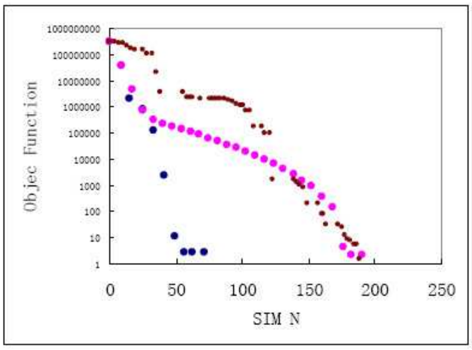
5. Convergence

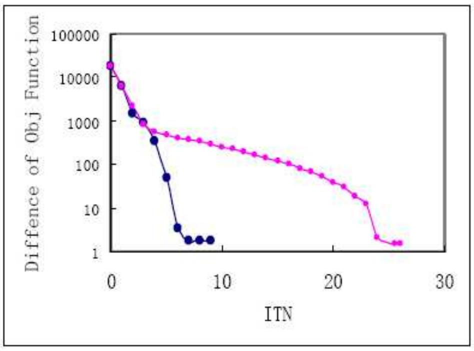
The following figures indicate the different convergence rate of *improved Nelder-Mead simplex methodsteepest descent method and Quasi-Newton method (BFGS)* . *BFGS* is the most rapid, *steepest descent* is the second, and the *improved Nelder-Mead simplex method* is the slowest.

6. Experiences and Conclusions

(1) Case studies indicate: All three algorithms are stabile and feasible; in the first four iterations, there are no evident difference on the results obtained from *Quasi-Newton method (BFGS)* and *steepest descent method; Quasi-Newton method (BFGS)* converges far more rapidly than steepest descent method in the latter iterations; *Nelder-Mead simplex method's* convergence rate is the slowest. But the







evident difference between *Quasi-Newton method (BFGS)* and *steepest descent method* occurs after objective function descend near three orders, it is difficult to say the significance of the difference in engineering here.

(2) Some experiences: Finding the relations about variables, performing variable elimination, descending optimization model freedom and variable relativity as far as possible; attaching importance to line search. When there are a great deal variables to optimize, suggesting to optimize the averages of the interrelated variables first or to introduce constraints temporarily, for example, the relative permeability curves may be appointed in a definite function form.

(3) The fluctuation of the well water cut could occur when *IMPES* formula is used in reservoir simulator, and it often makes optimizing process failed for determining variable accurately.

References

[1] Jacquard, P., Jain, C., Permeability Distribution from Field Pressure Data, SPEJ (Dec. 1965) 281-294.

[2] Jahns, Hans O.: A Rapid Method for Obtaining a Two-Dimensional Reservoir Description from Well Pressure Response Data, SPEJ (Dec. 1966) 315-327.

[3] Coast, K. H., Dempsey, J. R. and Handerson, J. H.: A New Technique for Determining Reservoir Description from field Performance Data, SPEj (March 1970).

[4] Thomas, L. K., Hellums,L. J. and Reheis, G. M.: A nonlinear Automatic History Matching Technique for Reservoir Simulation Models, SPEJ (Dec. 1972).

[5] Chen, W. H., Gavalas, G. R. and Wasserman, M. L.: A New Algorithm for Automatic History Matching, SPEJ (Dec. 1974).
[6] Chavent, G., Dupuy, M. and Lemonnier, P.: History Matching by Use of Optimal Theory, SPEJ (Feb. 1975).
[7] Gavalas, G. R., Shah, P. C. and Seinfeld, J. H.: Reservoir History Matching by Bayesian Estimation, SPEJ (Dec. 1976).
[8] Van den Bosch and Seinfeld, J. H.: History Matching in Two-phase Petroleum Reservoir Incompressible Flow, SPEJ (Dec. 1977)
[9] Watson, A.T., Seinfeld, J. H, Gavalas, G. R. and.Woo, P. T.: History Matching in Two-phase Petroleum Reservoir, SPEJ (Dec. 1980).
[10] Dogru, A. H. and Seinfeld, J. H.: Comparison of Sensitivity Coefficien Calculation Methods in Automatic History Matching, SPEJ (Oct. 1981).
[11] Carter, R.D., Kemp,L.F. and Pierce, A.C.: Discussion of Comparison of Sensitivity Coefficien Calculation Methods in Automatic History Matching, SPEJ (April 1982).
[12] Watson, A.T., Gavalas, G. R. and Seinfeld, J. H.: Identifiability of Estimates of Two-Phase Reservoir Properties in History Matching, SPEJ (Dec. 1984).
[13] Yang, P. H. and Watson, A.T.: Automatic History Matching with Variable-Metric Methods, SPE Reservoir Engineering, August 1988.
[14] Yang, P. H. and Watson, A.T.: A Bayesian Methodology for Estimating Relative Permeability Curves, SPE Reservoir Engineering, May 1991.
[15] Tan, T. B. and Kalogerakis, N.: A Fully Implicit Three-Dimensional Three-Phase Simulator with Automatic History Matching Capability, paper SPE 21205 presented at the Eleventh SPE symposium on Reservoir Simulation, Anaheim, Feb. 17-20, 1991.
[16] Tan, T. B. and Kalogerakis, N.: A three-Dimensional Three Phase Automatic History Matching Model: Reliability of Parameter Estimates, Journal of Canadian Petroleum Technology, Vol 31, No. 3, pp, 34-41, 1992.
[17] Tan, T. B. and Kalogerakis, N.: Improved Reservoir Characterization Using Automatic History Matching Procedures, Journal of Canadian Petroleum Technology, Vol 32, No. 6, pp, 26-33, 1993
[18] Smith, R. A. W. and Tan, T. B.: Reservoir Characterization of a Fractured Reservoir Using automatic History Matching, paper SPE 25251, presented at the 12th SPE Symposium on Reservoir Simulation held in New Orleans, LA, U. S. A., February 28-March 3, 1993.
[19] Sultan, A. J., Ouenes, A. and Weiss, W.; A Automatic History matching for an Integrated Reservoir Description and Improving Oil Recovery, paper SPE 27712 presented at the 1994 SPE Permian Basin Oil and Gas Recovery Conference held in Midland, Texas, 16-18 March 1994.
[20] Ouenes, A., Weiss, W., and Sultan, A. J. et al.: Parallel Reservoir Automatic History Matching Using a Network of Workstations and PVM, paper SPE 29107 presented at the 13th SPE Symposium on Reservoir Simulation held in San Antonio, TX, U. S. A., 12-15 February 1995.
[21] Wang Shuguang. And Guo Dezhi, Popularization of Nelder-Mead Simplex Algorithem and Its Application in Automatic History Matching, Petroleum Geology & Oilfield Development in Daqing, Vol.17, No.4, Aug., 1998.

Exploration & Development Research Institute of Daqing Oilfield Co. Ltd., Heilongjiang, China