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NUMERICAL SIMULATION AND ANALYSIS OF MIGRATION-ACCUMULATION OF OIL RESOURCES

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Abstract. Numerical simulation of migration-accumulation of oil resources in porous media is to describe the history of oil migration and accumulation in basin evolution. It is of great value to the evaluation of oil resources and to the determination of the location and amount of oil deposits. This thesis puts forward a mathematical model, a careful parallel operator splitting-up implicit iterative scheme, parallel arithmetic program, parallel arithmetic information and alternating-direction mesh subdivision. For the actual situation of Tanhai region of Shengli Petroleum Field, our numerical simulation test results and the actual conditions are coincident. For the model problem (nonlinear coupled system) optimal order estimates in l^2 norm are derived to determine the errors. We have successfully solved the difficult problem in the fields of permeation fluid mechanics and petroleum geology.

Key Words. migration-accumulation of oil resources; multilayer parallel arithmetic; careful numerical simulation, l^2 error estimates.

1. Introduction

The oil formation in sediment basins, its displacement, transport and accumulation, and the final formation of oil deposits have been one of the key problems in the exploration of oil-gas resources. How has oil been accumulated in the present loop according to the mechanics of immiscible flow? How is oil distributed in basins? All this is what the numerical simulation of accumulation of oil resources mainly studies[1−5]. With the exploration of the oilfields, efforts have been made to find covered and "potato piece" oil deposits, so basin simulation must be more and more precise become large-scale and develop in parallel direction. In basin simulation, the migration-accumulation of oil resources in particular, the traditional serial computers can hardly solve this problem[4−6].

The fluid dynamics model of migration-accumulation has strong hyperbolic characteristics. Therefore, the numerical method is very difficult in mathematics and mechanics. In this field, Ungerer, P., Walte, D. H., Yukler, M. A. and others have had famous publications[7−9]. They have studied the mathematical model and numerical simulation of the two-dimensional section, which have found their practical application in North Sea Oil Field. In China, Wang Jie, Cha Ming and others have also done important jobs[4,10] centered on petroleum geology. In a word, first fruits in monolayer problems have reaped [4,11−14]. This thesis, from the actual conditions

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and for highly accurate and careful parallel numerical simulation of oil resources migration-accumulation, we put forward a mathematical model and a careful parallel operator splitting-up implicit iterative scheme, parallel arithmetic program, parallel arithmetic information transmission and alternating-direction mesh subdivision. Making use of the present SGI high-performance miniature computer group (8CPU), we have conducted parallel arithmetic of the "careful numerical simulation of migration-accumulation of oil resources". We have made parallel computation and analysis of four schemes, namely, the mesh step lengths are 800m., 400m., 200m., and 100m. Our results are identical with the actual situation. For the model problem (nonlinear coupled system) optimal order estimates in l^2 norm are derived to determine the errors. We have successfully solved the difficult problem in the fields of permeation fluid mechanics and petroleum geology. This thesis discusses the numerical simulation of the migration-accumulation of oil resources, the most difficult part in basin simulation and important in rational evaluation of oil resources and exploration oil deposit locations.

2. The Mathematical Model

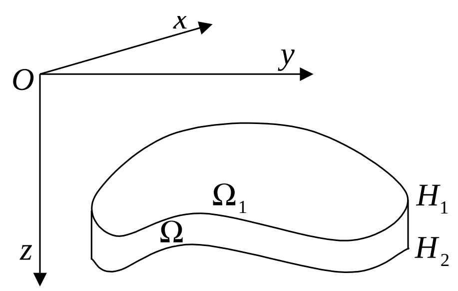

Fig. 1 two-layer sketch map of regions Ω, Ω_1

The mechanism of migration-accumulation of oil resources:

The primary driving force of migration-accumulation is the buoyancy caused by both the density difference between the oil in the carrying bed and that of the water in the porous structure, and the potential gradient formed by all the fluid (water and oil) in the porous structure, while the fluid is trying to migrate to the low-potential area.

The restricting force of migration-accumulation has something to do with the capillary pressure which gets larger while the aperture becomes narrower. If the capillary pressure exceeds the driving force, the migration will be held up. The migration of oil and underground water is mainly a permeation process. Both the oil and water potential fields determine the direction and magnitude of oil and water permeations.

For the numerical simulation of secondary multilayer oil migration in porous media, the flow in the first and third layers is considered as horizontal and in the one between them as vertical. After careful analysis of the model and the scientific numerical test, we propose a creative and rational numerical model. For the mathematical model of multilayer migration-accumulation:

$$\nabla \cdot (K_1 \frac{k_{ro}}{\mu_o} \nabla \psi_o) + B_o q - (K_3 \frac{k_{ro}}{\mu_0} \frac{\partial \psi_0}{\partial z})_{z=H_1} = -\Phi s' (\frac{\partial \psi_0}{\partial t} - \frac{\partial \psi_w}{\partial t}), \quad X = (x,\ y)^T \in \Omega_1,\ t \in J = (0,\ T], \tag{1a}$$

$$\nabla\cdot(K_1\frac{k_{rw}}{\mu_w}\nabla\psi_w)+B_wq-(K_3\frac{k_{rw}}{\mu_w}\frac{\partial\psi_w}{\partial z})_{z=H_1}=\Phi s'(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}),\ X\in\Omega_1,t\in J, \tag{1b}$$

$$\frac{\partial}{\partial z}(K_3\frac{k_{ro}}{\mu_o}\frac{\partial\psi_o}{\partial z})=-\Phi s'(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t})\ ,\ \ X=(x,\ y,\ z)^T\in\Omega,\ t\in J, \tag{2a}$$

$$\frac{\partial}{\partial z}(K_3\frac{k_{rw}}{\mu_w}\frac{\partial\psi_w}{\partial z})=\Phi s'(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}),\quad X\in\Omega,\ \ t\in J, \tag{2b}$$

$$\begin{gathered}\nabla\cdot(K_2\frac{k_{ro}}{\mu_o}\nabla\psi_o)+B_oq+(K_3\frac{k_{ro}}{\mu_0}\frac{\partial\psi_0}{\partial z})_{z=H_2}=-\Phi s'(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}),\\ X=(x,\ y)^T\in\Omega_1,\ t\in J,\end{gathered} \tag{3a}$$

$$\begin{gathered}\nabla\cdot(K_2\frac{k_{rw}}{\mu_w}\nabla\psi_w)+B_wq+(K_3\frac{k_{rw}}{\mu_w}\frac{\partial\psi_w}{\partial z})_{z=H_2}=\Phi s'(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}),\\ X\in\Omega_1,\ \ t\in J,\end{gathered} \tag{3b}$$

where ψ_0 and ψ_w are the potential functions, k_{ro} and k_{rw} are the relative permeabilities for the oil and water phases, respectively. K_1, K_2 and K_3 are the absolute permeabilities in respective layers. μ_0 and μ_w are the viscosities for the oil and water phases. $s'=\frac{ds}{dp_c}$, where s is the water concentration, and p_c is the capillary pressure. B_o and B_w are the flow coefficients, $B_o=\frac{k_{ro}}{\mu_o}(\frac{k_{ro}}{\mu_o}+\frac{k_{rw}}{\mu_w})^{-1}$, $B_w=\frac{k_{rw}}{\mu_w}(\frac{k_{ro}}{\mu_0}+\frac{k_{rw}}{\mu_w})^{-1}$, $q(x,\ t)$ are the source (sink) functions. By Darcy law: $-K_3\frac{k_{ro}}{\mu_0}\frac{\partial\psi_0}{\partial z}=q_{h,\,0}$, $-K_3\frac{k_{rw}}{\mu_w}\frac{\partial\psi_w}{\partial z}=q_{h,\,w}$. The initial conditions and boundary conditions are given.

3. The Numerical Simulation Method

The fluid dynamics model of migration-accumulation has strong hyperbolic characteristics. Therefore, the numerical simulation must be very stable for as long as millions of years. The numerical method is very difficult in mathematics and mechanics. This thesis, starting from the actual conditions and the above characteristics, puts forward a kind of careful parallel operator splitting-up implicit iterative scheme.

3.1. The splitting-up implicit iterative scheme of the three-dimensional problem.

z direction:

$$\begin{gathered}\frac{1}{2}\Delta_{\bar z}(A_{zw}\Delta_z\psi_w^*)+\frac{1}{2}\Delta_{\bar z}(A_{zw}\Delta_z\psi_w^{(l)})+\Delta_{\bar y}(A_{yw}\Delta_y\psi_w^{(l)})+\Delta_{\bar x}(A_{zw}\Delta_x\psi_w^{(l)})\\ -G\psi_w^*+G\psi_o^*=H_{l+1}(\sum A_w)(\psi_w^*-\psi_w^{(l)})-B_w^mq^{m+1}-G\psi_w^m+G\psi_o^m,\end{gathered} \tag{4a}$$

$$\begin{gathered}\frac{1}{2}\ \Delta_{\bar z}(A_{zw}\Delta_z\psi_o^*)+\frac{1}{2}\Delta_{\bar z}(A_{zw}\Delta_z\psi_o^{(l)})+\Delta_{\bar y}(A_{yo}\Delta_y\psi_o^{(l)})+\Delta_{\bar x}(A_{xw}\Delta_x\psi_o^{(l)})\\ +G\psi_w^*-G\psi_o^*=H_{l+1}(\sum\ A_o)\ (\psi_o^*-\psi_o^{(l)})-B_o^mq^{m+1}+G\psi_w^m-G\psi_o^m\ ,\end{gathered} \tag{4b}$$

y direction:

$$\begin{gathered}\frac{1}{2}\Delta_{\bar y}(A_{yw}\Delta_y\psi_w^{**})-\frac{1}{2}\Delta_{\bar y}(A_{yw}\Delta_y\psi_w^{(l)})-G\psi_w^{**}+G\psi_o^{**}\\ =H_{l+1}(\sum A_w)(\psi_w^{**}-\psi_w^*)-G\psi_w^*+G\psi_o^*,\end{gathered} \tag{4c}$$

$$\begin{gathered}\frac{1}{2}\Delta_{\bar y}(A_{yo}\Delta_y\psi_o^{**})-\frac{1}{2}\Delta_{\bar y}(A_{yo}\Delta_y\psi_o^{(l)})+G\psi_w^{**}-G\psi_o^{**}\\ =H_{l+1}(\sum A_o)(\psi_o^{**}-\psi_o^*)+G\psi_w^*-G\psi_o^*,\end{gathered} \tag{4d}$$

x direction:

$$\frac{1}{2}\Delta_{\bar{x}}(A_{xw}\Delta_x\psi_w^{(l+1)}) - \frac{1}{2}\Delta_{\bar{x}}(A_{zw}\Delta_z\psi_w^{(l)}) - G\psi_w^{(l+1)} + G\psi_o^{(l+1)} = H_{l+1}(\sum A_w)(\psi_w^{(l+1)} - \psi_w^{**}) - G\psi_w^{**} + G\psi_o^{**}, \tag{4e}$$

$$\frac{1}{2}\Delta_{\bar{x}}(A_{xo}\Delta_x\psi_o^{(l+1)}) - \frac{1}{2}\Delta_{\bar{x}}(A_{xo}\Delta_x\psi_o^{(l)}) + G\psi_w^{(l+1)} - G\psi_o^{(l+1)} = H_{l+1}(\sum A_o)(\psi_o^{(l+1)} - \psi_o^{**}) + G\psi_w^{**} - G\psi_o^{**}, \tag{4f}$$

where $\Delta_{\bar{x}}(A_x\Delta_x\psi^{m+1})_{ijk} = A_{x,i+1/2,jk}(\psi_{i+1,jk} - \psi_{ijk})^{m+1} - A_{x,i-1/2,jk}(\psi_{ijk} - \psi_{i-1,jk})^{m+1}$, $A_{xw,i+1/2,jk} = \big(\frac{K\Delta y\Delta z}{\Delta x}\frac{k_{rw}}{\mu_w}\big)_{i+1/2,jk}, \cdots$.
Take the value of k_r according to the partial upper reaches principle, and other terms can be defined similarly. $G = -V_p\Phi\dot{s}/\Delta t$, $V_p = \Delta x\Delta y\Delta z$, the $(l+1)$ times iterative computational formula of $\dot{s}$:

$$\dot{s}^{(l+1)} = \omega_1\Big(\frac{s^{(l)} - s^m}{p_c^{(l)} - p_c^m}\Big) + (1-\omega_1)\dot{s}^{(l)}, \tag{5}$$

where l is the iterative time, $0 < \omega_1 < 1$ is the mean factor.

For the purpose of high accuracy, we introduce the residual computational value:

$$P_z = \psi_w^* - \psi_w^{(l)},\ P_y = \psi_w^{**} - \psi_w^*,\ \ P_z = \psi_w^{(l+1)} - \psi_w^{**}, \tag{6a}$$

$$R_z = \psi_o^* - \psi_o^{(l)},\ R_y = \psi_o^{**} - \psi_o^*,\ \ R_z = \psi_w^{(l+1)} - \psi_o^{**}. \tag{6b}$$

Finally, we put forward the careful parallel operator splitting-up implicit iterative scheme.

z direction:

$$\frac{1}{2}\ \Delta_{\bar{z}}(A_{zw}\Delta_z P_z) - (G + H_{l+1}\sum A_w)P_z + GR_z = -[\Delta(A_w\Delta\psi_w^{(l)}) + B_w^m q^{m+1} - G(\psi_w^{(l)} - \psi_w^m) + G(\psi_o^{(l)} - \psi_o^m)], \tag{7a}$$

$$\frac{1}{2}\ \Delta_{\bar{z}}(A_{zo}\Delta_z R_z) - (G + H_{l+1}\ \sum A_o)R_z + GP_z = -[\Delta(A_o\Delta\psi_o^{(1)}) + B_o^m q^{m+1} + G(\psi_w^{(l)} - \psi_w^m) - G(\psi_o^{(l)} - \psi_o^m)], \tag{7b}$$

y direction:

$$\frac{1}{2}\Delta_{\bar{y}}(A_{yw}\Delta_y P_y) - (G + H_{l+1}\sum A_w)P_y + GR_y = -\frac{1}{2}\Delta_{\bar{y}}(A_{yw}\Delta_y P_z), \tag{7c}$$

$$\frac{1}{2}\Delta_{\bar{y}}(A_{yo}\Delta_y P_y) - (G + H_{l+1}\sum A_o)R_y + GP_y = -\frac{1}{2}\Delta_{\bar{y}}(A_{yo}\Delta_y R_z), \tag{7d}$$

x direction:

$$\frac{1}{2}\Delta_{\bar{x}}(A_{xw}\Delta_x P_z) - (G + H_{l+1}\sum A_w)P_x + GR_x = -\frac{1}{2}\Delta_{\bar{x}}(A_{xw}\Delta_x(P_y + P_z)), \tag{7e}$$

$$\frac{1}{2}\Delta_{\bar{x}}(A_{xo}\Delta_x P_z) - (G + H_{l+1}\sum A_o)R_x + GP_x = -\frac{1}{2}\Delta_{\bar{x}}(A_{xo}\Delta_z(R_y + R_z)). \tag{7f}$$

When the iterative error reaches our accuracy index, the iterative values $\psi_o^{(l+1)}$ and $\psi_w^{(l+1)}$ are regarded as ψ_o^{m+1} and ψ_w^{m+1}. Again by

$$s^{m+1} = s^m + \dot{s}(\psi_0^{m+1} - \psi_0^m - \psi_w^{m+1} + \psi_w^m). \tag{8}$$

In practical numerical computation, k_{rw}, k_{ro}, $p_c(s)$ must undergo data processing and filtration so as to get the correct results.

3.2. The mathematical model and numerical method of the quasi-three-dimensional (single layer) problem.

If the actual thickness of the carrying bed is much smaller than the size of the horizontal simulation area, we propose the solution by reducing it to a two-dimensional problem in the following way. So it can also be called a quasi-three-dimensional problem. By integrating z with equations (1a) and (1b), the average results are:

$$\nabla\cdot(\bar{K}\frac{\Delta z k_{ro}}{\mu_o}\nabla\psi_o)+B_o\bar{q}\Delta z=-\bar{\Phi}s'\Delta z(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}), \tag{9a}$$

$$\nabla\cdot(\bar{K}\frac{\Delta z k_{rw}}{\mu_w}\nabla\psi_w)+B_w\bar{q}\Delta z=\bar{\Phi}s'\Delta z(\frac{\partial\psi_o}{\partial t}-\frac{\partial\psi_w}{\partial t}), \tag{9b}$$

where Δz is the thickness of the carrying bed.

$$\bar{K}=\frac{1}{\Delta z}\int_{h_1(x,y)}^{h_2(x,y)}K(x,y,z)dz,$$
$$\bar{\Phi}=\frac{1}{\Delta z}\int_{h_1(x,y)}^{h_2(x,y)}\Phi(x,y,z)dz,\quad \bar{q}=\frac{1}{\Delta z}\int_{h_1(x,y)}^{h_2(x,y)}q(x,y,z)dz,$$

where $h_1(x,\ y)$, $h_2(x,\ y)$ are the depths of the carrying beds for the upper and lower boundaries, respectively.

For the quasi-three-dimensional problem we put forward a kind of careful parallel operator splitting-up implicit iterative scheme.

x direction:

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xw}\Delta_x\psi_w^*)+\Delta_{\bar{y}}(A_{yw}\Delta_y\psi_w^{(l)})-G\psi_w^*+G\psi_o^*\\&=H_{l+1}(\sum A_w)\ (\psi_w^*-\psi_w^{(l)})-B_w^m q^{m+1}-G\psi_w^m+G\psi_o^m\ ,\end{aligned} \tag{10a}$$

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xo}\Delta_x\psi_o^*)+\Delta_{\bar{y}}(A_{yo}\Delta_y\psi_o^{(l)})+G\psi_w^*-G\psi_o^*\\&=H_{l+1}(\sum A_o)\ (\psi_o^*-\psi_o^{(l)})-B_o^m q^{m+1}+G\psi_w^m-G\psi_o^m\ ,\end{aligned} \tag{10b}$$

y direction:

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xw}\Delta_x\psi_w^*)+\Delta_{\bar{y}}(A_{yw}\Delta_y\psi_w^{(l+1)})-G\psi_w^{(l+1)}+G\psi_o^{(l+1)}\\&=H_{l+1}(\sum A_w)\ (\psi_w^{(l+1)}-\psi_w^*)-B_w^m q^{m+1}-G\psi_w^m+G\psi_o^m\ ,\end{aligned} \tag{10c}$$

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xo}\Delta_x\psi_o^*)+\Delta_{\bar{y}}(A_{yo}\Delta_y\psi_o^{(l+1)})+G\psi_w^{(l+1)}-G\psi_o^{(l+1)}\\&=H_{l+1}(\sum A_o)\ (\psi_o^{(l+1)}-\psi_o^*)-B_o^m q^{m+1}+G\psi_w^m-G\psi_o^m\ ,\end{aligned} \tag{10d}$$

where $G=-V_p\Phi s'/\Delta t$, $V_p=\Delta x\Delta y$, H_{l+1} is the iterative factor, $\sum A_w=A_{w,i+1/2,j}$ $+A_{w,i-1/2,j}+\cdots+A_{w,i,j-1/2}$, $\sum A_0=\cdots$.

For high accuracy purpose, we introduce the residual computational value:

$$P_x=\psi_w^*-\psi_w^{(l)},\ P_y=\psi_w^{(l+1)}-\psi_w^*,$$
$$R_x=\psi_o^*-\psi_o^{(l)},\ R_y=\psi_o^{(l+1)}-\psi_o^*.$$

Finally, we put forward the modified method of alternating direction implicit iterative scheme.

x direction:

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xw}\Delta_x P_x)-(G+H_{l+1}\sum A_w)P_x+GR_x\\&=-[\Delta(A_w\Delta\psi_w^{(l)})+B_w q-G(\psi_w^{(l)}-\psi_w^m)+G(\psi_o^{(l)}-\psi_o^m)]=-B_1X^{(l)},\end{aligned} \tag{11a}$$

$$\begin{aligned}&\Delta_{\bar{x}}(A_{xw}\Delta_x R_x) - (G + H_{l+1}\sum A_o)R_x + GP_x \\ &= -[\Delta(A_o\Delta\psi_o^{(l)}) + B_o q + G(\psi_w^{(l)} - \psi_w^m) - G(\psi_o^{(l)} - \psi_o^m)] = -B_2 X^{(l)}.\end{aligned} \tag{11b}$$

As for y direction, the computation is similar. When the iterative error reaches our accuracy index, the iterative values $\psi_w^{(l+1)}$, $\psi_o^{(l+1)}$ are regarded as ψ_w^{m+1}, ψ_o^{m+1}. Again from (8) we find out S^{m+1}.

3.3. The numerical method of the multilayer problem.

The following quasi-three-dimensional numerical schemes can be used to do numerical computation.

The first layer scheme:

$$\nabla\cdot(\bar{K}_1\Delta z_1\frac{k_{ro}}{\mu_o}\nabla\psi_o) + B_o\bar{q}\Delta z_1 + q_{h,o}^1 = -\bar{\Phi}s'(\frac{\partial\psi_o}{\partial t} - \frac{\partial\psi_w}{\partial t}),\ X\in\Omega_1, t\in J, \tag{12a}$$

$$\nabla\cdot(\bar{K}_1\Delta z_1\frac{k_{rw}}{\mu_w}\nabla\psi_w) + B_o\bar{q}\Delta z_1 + q_{h,w}^1 = \bar{\Phi}s'(\frac{\partial\psi_o}{\partial t} - \frac{\partial\psi_w}{\partial t}),\ X\in\Omega_1, t\in J, \tag{12b}$$

where

$$\begin{aligned}&\bar{K}_1 = \frac{1}{\Delta z_1}\int_{h_1^1(x,y)}^{h_2^1(x,y)} K_1(x,y,z)dz,\\ &\bar{\Phi} = \frac{1}{\Delta z_1}\int_{h_1^1(x,y)}^{h_2^1(x,y)} \Phi(x,y,z)dz, \quad \bar{q} = \frac{1}{\Delta z_1}\int_{h_1^1(x,y)}^{h_2^1(x,y)} q(x,y,z)dz.\end{aligned}$$

The second layer scheme:

$$\nabla\cdot(\bar{K}_2\Delta z_2\frac{k_{ro}}{\mu_o}\nabla\psi_o) + B_o\bar{q}\Delta z_2 - q_{h,o}^2 = -\bar{\Phi}s'(\frac{\partial\psi_o}{\partial t} - \frac{\partial\psi_w}{\partial t}),\ \ X\in\Omega_1, t\in J, \tag{13a}$$

$$\nabla\cdot(\bar{K}_2\Delta z_2\frac{k_{rw}}{\mu_w}\nabla\psi_o) + B_w\bar{q}\Delta z_2 - q_{h,w}^2 = \bar{\Phi}s'(\frac{\partial\psi_o}{\partial t} - \frac{\partial\psi_w}{\partial t}),\ \ X\in\Omega_1, t\in J, \tag{13b}$$

where $\bar{K}_2 = \frac{1}{\Delta z_2}\int_{h_1^2(x,\,y)}^{h_2^2(x,\,y)} K_1(x,\,y,\,z)dz,\ \cdots,\ q_{h,\,o}^1 \approx q_{h,\,o}^2,\ q_{h,\,w}^1 \approx q_{h,\,w}^2.$ Numerical Schemes (12) and (13) are combined by applying Darcy's law. Compute equations (12) and (13) respectively by the scheme proposed by the quasi-three-dimensional problem (2.2). The two layers between them are coupled by Darcy's law, that is

$$q_{h,0}^1 = q_{h,0}^2 \approx -\frac{1}{2}\{\bar{K}_1(\frac{k_{ro}}{\mu_o})_1 + \bar{K}_2(\frac{k_{ro}}{\mu_o})_2\}(\psi_{0,2} - \psi_{0,1})/\Delta z, \tag{14a}$$

$$q_{h,w}^1 = q_{h,w}^2 \approx -\frac{1}{2}\{\bar{K}_1(\frac{k_{rw}}{\mu_w})_1 + \bar{K}_2(\frac{k_{rw}}{\mu_w})_2\}(\psi_{w,2} - \psi_{w,1})/\Delta z. \tag{14b}$$

Thus, this important problem can be successfully solved. This method can be used in solving multilayer problems.

For the model problem, theory of differential equation prior estimates and techniques are made use of. We can obtain the convergence theorem of this numerical method.

4. Validity Analysis of Careful Parallel Arithmetic

We adopt the geology parameters of Tanhai region. Simulation region: Taihai region, earth-coordinate (m) (20611700.00, 4169000.00) and (2071700.00, 4253000.00), horizontal scale=8845.2km^2. The simulation includes two layers, that is Sand third middle section and Sand third upper section. According to the structure of Tanhai region, Chengzikou-Qingyun ridge, Yihezhuang-Wudiningjin ridge, Chenjiazhuang-Binxian ridge and Qingtuozi- Kendong ridge are located from northwest to southeast. In between horizontally located are Chengbei hollow, Huanghekou hollow, Bonan hollow, Gunan hollow and other oil-bearing hollows.

Simulation computation of the following four schemes:

Scheme 1: In x direction the mesh step length is 810m, and there are 130 meshes; in y direction the mesh step length is 840m, and there are 100 meshes. So on the plane of each layer there are 13000 meshes.

Scheme 2: Each mesh in Scheme 1 is further divided into four. Thus in x direction the number of meshes is 260, and the step length is 405m. In y direction we have 200 meshes, and the step length of each is 420m. One layer has 52000 meshes, Two layers has 104000.

Scheme 3: Each mesh in Scheme 2 is further divided into four. Thus in x direction the mesh step length is 202.5m, and there are 520 meshes; In y direction the mesh step length is 220m, and there are 400 meshes. One layer has 208000 meshes, Two layers has 416000 meshes.

Scheme 4: Consider only numerical simulation of monolayer—Sand third upper section. In x direction the mesh step length is 101.25m, and there are 1040 meshes; in y direction the mesh step length is 100m, and there are 800meshes. So on the plane of a simple layer there are 832000 meshes.

Simulation begins with the computation of Dongying Group, continues through sediment interruption of the upper and lower third systems, Guantao group, Minghuazhen group and finally to the present fourth system, covering thirty million geological years. Thus careful precise numerical parallel simulation computation has been completed.

Table 1 illustrates the general situation of schemes 1∼4, the computation time of each geological year and the overall computation time of 30 million years. From Table 1 we can see that when the mesh step length reduces from 800m to 400m, the computation time increases 3.84 times. When the mesh step length reduces from 400m to 200m, the computation time increases 6.14 times.

Simulation results: Figures 2a and 2b show the oil concentration distribution, in two layers (Sand third upper region and Sand third middle region) during 1.8×10^7 years. Figures 3a and 3b show the present oil concentration isograms in these two layers during 3.0×10^7 years. The results of numerical simulation indicate that the oil in Sand third middle region migrates along the fault towards Sand third upper region and accumulates on the uplifted zone around the low-lying area and on the slope, that is Chengdao area, Laohekao, Stake No.5 and Gudong area. The present situation of oil exploration of Shengli Oilfield is basically the same.

The above computation and analysis indicate that our large-scale careful parallel numerical simulation system (when mesh step length is 200m) can perform precise numerical simulation by using three-dimensional seismic interpretation results without losing a single small stratigraphic trap and, therefore, can be used to evaluate present oil resources and explore new oilfields.

Table 1 computation time

Scheme	Mesh number	Mesh step lenght	Layer number	Dongying lower group 2 (10^6 years) 8	Dongying upper group 2 (10^6 years) 8	Hiatus 8 (10^6 years) 10	Guantao group 6 (10^6 years) 11	Minghua-zhen group 6 (10^6 years) 12	Fourth system 2 (10^6 years) 13	Total computation time 30 (10^6 years)
1	130×100	800(m)	2 (s.t.u, s.t.m)	Compute time(s) 532.406	Compute time(s) 614.770	Compute time(s) 2017.250	Compute time(s) 1898.020	Compute time(s) 3211.400	Compute time(s)	Compute time(s)
2	260×200	400	2	1228.8739	1141.8978	3070.6578	7626.4464	14680.8095	6643.7076	34395.4111
3	520×400	200	2	13513.8603	11993.3600	19487.1931	50540.4141	88354.4157	2760.2591	211049.5023
4	1040×800	100	1 (s.t.u)	22349.8655	20432.7689	27606.0855	96192.4874	199378.8870	48215.1896	414075.2836

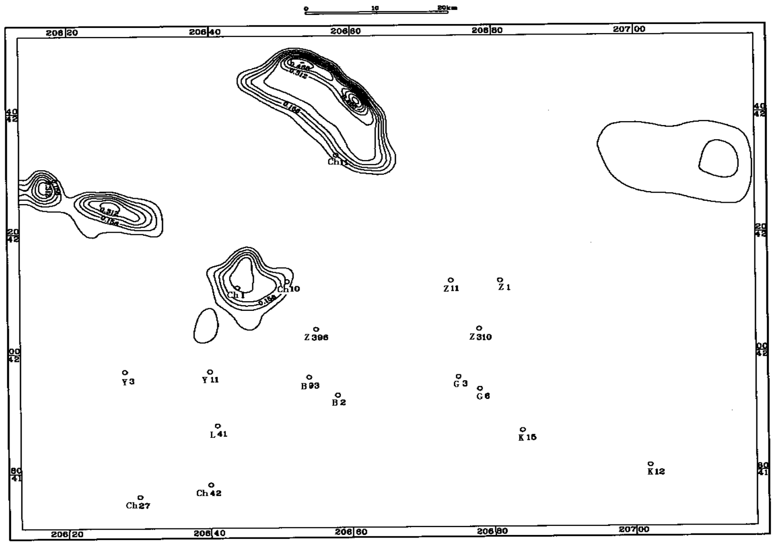

Fig.2a 1.8×10^7 year's Sand Third Upper oil concentration isogram

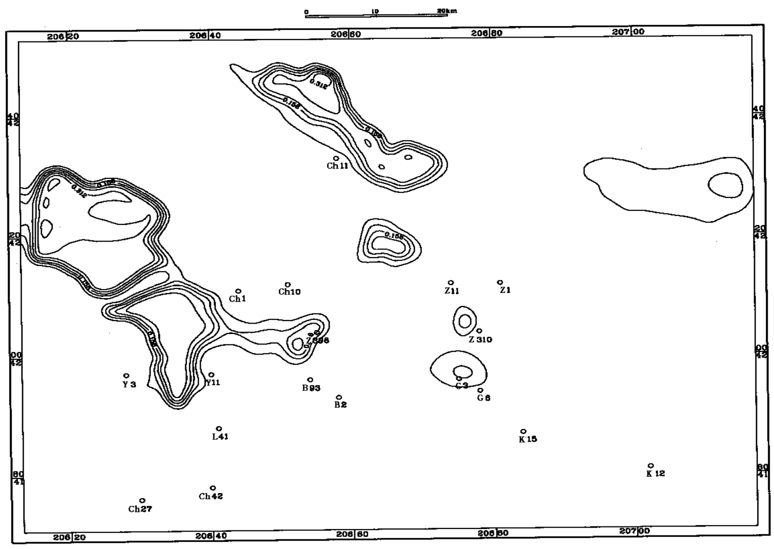

Fig.2b 1.8×10^7 year's Sand Third Middle oil concentration isogram

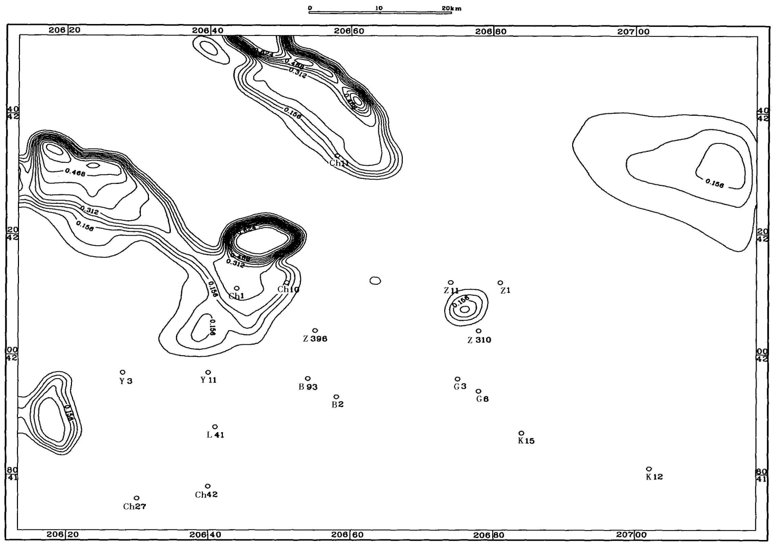

Fig.3a 3.0×10^7 year's Sand Third Upper oil concentration isogram

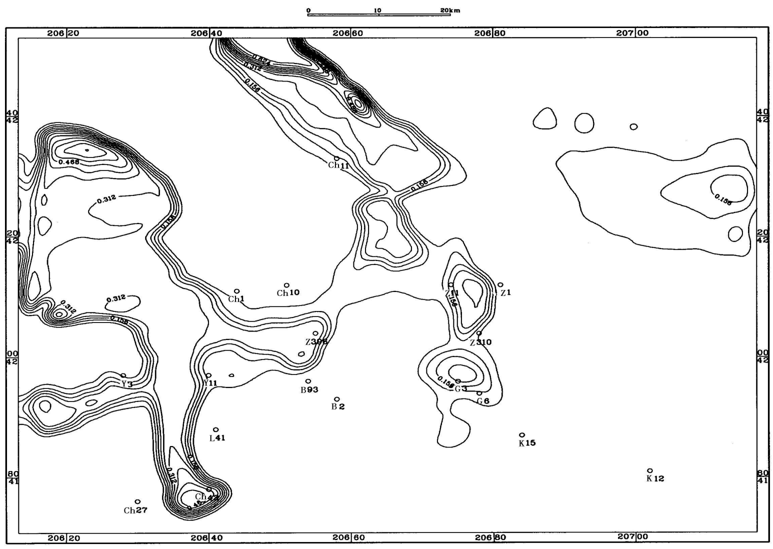

Fig.3b 3.0×10^7 year's Sand Third Middle oil concentration isogram

5. Numerical Analysis of the Model Problem

As for the numerical method of oil migration-accumulation of the multilayer in porous media, for the sake of brevity we consider one model problem, the nonstationary flow computation of mutilayer fluid dynamics in porous media. We have to find out the following nonlinear convection-dominated diffusion coupling systems with initial-boundary value problem[11−14]:

$$\Phi_1(x,\ y)\frac{\partial u}{\partial t} + \vec{a}(x,\ y,\ t)\cdot\nabla u - \nabla\cdot(K_1(x,\ y,u)\nabla u) - K_2(x,\ y,\ z)\frac{\partial w}{\partial z}|_{z=1}$$
$$= Q_1(x,\ y,\ t,\ u),\quad (x,\ y)^T \in \Omega_1,\ \ t \in J = (0,\ T], \tag{15a}$$

$$\Phi_2(x,\ y,\ z)\frac{\partial w}{\partial t}=\frac{\partial}{\partial z}(K_2(x,\ y,\ z)\frac{\partial w}{\partial z}),\ \ (x,\ y,\ z)^T\in\Omega,\ \ t\in J, \tag{15b}$$

$$\begin{aligned}&\Phi_3(x,y)\frac{\partial v}{\partial t}+\vec{b}(x,y,t)\cdot\nabla v-\nabla\cdot(K_3(x,y,v)\nabla v)+K_2(x,y,z)\frac{\partial w}{\partial z}|_{z=0}\\&\qquad=Q_3(x,\ y,\ t,\ v),\quad (x,\ y)^T\in\Omega_1,\quad t\in J,\end{aligned} \tag{15c}$$

where

$$\Omega=\{(x,y,z)|0<x<1,0<y<1,0<z<1\},\Omega_1=\{(x,y)|0<x<1,0<y<1\}.$$

We assume the boundary condition:

$$u(x,\ y,\ t)|_{\partial\Omega_1}=0,\quad v(x,\ y,\ t)|_{\partial\Omega_1}=0,\quad w(x,\ y,\ z,\ t)|_{\partial\Omega}=0,\quad t\in J, \tag{16a}$$

$$w(x,y,z,t)|_{z=1}=u(x,y,t),\ w(x,y,z,t)|_{z=0}=v(x,y,t),\ (x,y)^T\in\Omega_1,t\in J. \tag{16b}$$

The initial conditions:

$$\begin{aligned}&u(x,\ y,\ 0)=u_0(x,\ y),\ (x,\ y)^T\in\Omega_1,\\&w(x,\ y,\ z,\ 0)=w_0(x,\ y,\ z),\ (x,\ y,\ z)^T\in\Omega,\\&v(x,\ y,\ 0)=v_0(x,\ y),\ (x,\ y)^T\in\Omega_1.\end{aligned} \tag{17}$$

The unknown functions u, w and v are the potential functions, ∇u, ∇v and $\frac{\partial w}{\partial z}$ are Darcy's velocity, $\Phi_\alpha(\alpha=1,\ 2,\ 3)$ is the porosity, $K_\alpha(\alpha=1,\ 2,\ 3)$ is the stratigraphical permeability, $\vec{a}(x,\ y,\ t)=(a_1(x,\ y,\ t),\ a_2(x,\ y,\ t))^T$, $\vec{b}(x,\ y,\ t)=(b_1(x,\ y,\ t),\ b_2(x,\ y,\ t))^T$ are the convection coefficients. $Q_1(x,\ y,\ u)$, $Q_2(x,\ y,\ v)$ are the external volumetric flow rates.

Let $h=\frac{1}{N},t^n=n\Delta t$, $U(x_i,y_j,t^n)=U^n_{ij},V(x_i,y_j,t^n)=V^n_{ij},W(x_i,y_j,z_k,t^n)=W^n_{ijk}$. Let $\delta_x,\delta_y,\delta_z$ and $\delta_{\bar{x}},\delta_{\bar{y}},\delta_{\bar{z}}$ be the forward and backward difference quotients, respectively. $d_tu^n_{ij}$ is the forward quotient of net function u^n_{ij}.

For equation (15a), the upwind finite difference fractional steps scheme is given by

$$\begin{aligned}&(\hat{\Phi}_1-\Delta t(1+\frac{h_1}{2}\frac{|a^n_1|}{K_1(U^n)})^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}})+\Delta t\delta_{a^n_1,U^n,x})U^{n+1/2}_{ij}\\&=\hat{\Phi}_{1,ij}U^n_{ij}+\Delta t\{K^n_{2,ij,N-1/2}\delta_{\bar{z}}W^{n+1}_{ij,N}+Q(x_i,y_j,t^n,U^{n+1}_{ij})\},\quad 1<i<N,\end{aligned} \tag{18a}$$

$$U^{n+1/2}_{ij}=0,\quad (x_i,y_j)\in\partial\Omega_{1,h}, \tag{18b}$$

$$\begin{aligned}&(\hat{\Phi}_1-\Delta t(1+\frac{h_1}{2}\frac{|a^n_2|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar{y}})+\Delta t\delta_{a^n_2,U^n,y})U^{n+1}_{ij}\\&=\hat{\Phi}_{1,ij}U^{n+1/2}_{ij},\quad 1<j<N,\end{aligned} \tag{18c}$$

$$U^{n+1}_{ij}=0,\quad (x_i,y_j)\in\partial\Omega_{1,h}, \tag{18d}$$

where
$\delta_{a^n_1,U^n,x}u_{ij}=a^n_{1,ij}[H(a^n_{1,ij})K_1(U^n)^{-1}_{ij}\cdot K_1(U^n)_{i-1/2,j}\delta_{\bar{x}}+(1-H(a^n_{1,ij}))K_1(U^n)^{-1}_{ij}\cdot K_1(U^n)_{i+1/2,j}\delta_x]u_{ij}$, $\delta_{a^n_2,U^n,y}u_{ij}=a^n_{2,ij}[H(a^n_{2,ij})K_1(U^n)^{-1}_{ij}\cdot K_1(U^n)_{i,j-1/2}\delta_{\bar{y}}+(1-H(a^n_{2,ij}))K_1(U^n)^{-1}_{ij}\cdot K_1(U^n)_{i,j+1/2}\delta_y]u_{ij}$, $K_1(U^n)^{-1}_{ij}=(K_1(U^n)_{ij})^{-1}$, $H(z)=\left\{\begin{array}{l}1,z\geq 0,\\0,z<0.\end{array}\right.$ In practical computation, $\delta_{\bar{z}}W^{n+1}_{ij,N}$ in (18a) is approximately taken as $\delta_{\bar{z}}W^n_{ij,N}$, while U^{n+1}_{ij} is taken as U^n_{ij}.

For equation (15b), the finite difference scheme is expressed as

$$\Phi_{2,ijk}\frac{W^{n+1}_{ijk}-W^n_{ijk}}{\Delta t}=\delta_z(K^n_2\delta_{\bar{z}}W^{n+1})_{ijk},\quad 0<k<N,\quad (i,j)\in\Omega_{1,h}, \tag{19}$$

For equation (15c), the upwind finite difference fractional steps scheme is given by

$$\begin{aligned}(\hat{\Phi}_3 - \Delta t(1+\frac{h_1}{2}\frac{|b_1^n|}{K_3(V^n)})^{-1}\delta_x(K_3(V^n)\delta_{\bar{x}}) + \Delta t\delta_{b_1^n,V^n,x})V_{ij}^{n+1/2} \\ = \hat{\Phi}_{3,ij}V_{ij}^n + \Delta t\{-K_{2,ij,1/2}^n\delta_z W_{ij,0}^{n+1} + Q(x_i,y_j,t^n,V_{ij}^{n+1})\}, \\ i_1(j) < i < i_2(j),\end{aligned} \tag{20a}$$

$$V_{ij}^{n+1/2} = 0, \quad (x_i,y_j) \in \partial\Omega_{1,h}, \tag{20b}$$

$$\begin{aligned}(\hat{\Phi}_3 - \Delta t(1+\frac{h_1}{2}\frac{|b_2^n|}{K_3(V^n)})^{-1}\delta_y(K_3(V^n)\delta_{\bar{y}}) + \Delta t\delta_{b_2^n,V^n,y})V_{ij}^{n+1} \\ = \hat{\Phi}_{3,ij}V_{ij}^{n+1/2}, \quad j_1(i) < j < j_2(i),\end{aligned} \tag{20c}$$

$$V_{ij}^{n+1} = 0, \quad (x_i,y_j) \in \Omega_{1,h}, \tag{20d}$$

where
$\delta_{b_1^n,V^n,x}v_{ij} = b_{1,ij}^n[H(b_{1,ij}^n)K_3(V^n)_{ij}^{-1}\cdot K_3(V^n)_{i-1/2,j}\delta_{\bar{x}} + (1-H(b_{1,ij}^n))\cdot K_3(V^n)_{ij}^{-1}$ $K_3(V^n)_{i+1/2,j}\delta_x]u_{ij}$, $\delta_{b_2^n,V^n,y}v_{ij} = b_{2,ij}^n[H(b_{2,ij}^n)K_3(V^n)_{ij}^{-1}\cdot K_3(V^n)_{i,j-1/2}\delta_{\bar{y}} + (1-$ $H(b_{2,ij}^n))K_3(V^n)_{ij}^{-1}\cdot K_3(V^n)_{i,j+1/2}\delta_y]v_{ij}$. In practical computation, $\delta_z W_{ij,0}^{n+1}$ in (20a) is approximately taken as $\delta_z W_{ij,0}^n$, and V_{ij}^{n+1} as V_{ij}^n.

The algorithm for a time step is as follows. Assuming that the approximate solution $\{U_{ij}^n, W_{ijk}^n, V_{ij}^n\}$ at time $t=t^n$ is known, one needs to find out the approximate solution $\{U_{ij}^{n+1}, W_{ijk}^{n+1}, V_{ij}^{n+1}\}$ at time t^{n+1}. First, from schemes (18a) and (18b), method of speedup is used to get the solution of transition sheaf $\{U_{ij}^{n+1/2}\}$ along x direction. Second, from schemes (18c) and (18d) we obtain solution $\{U_{ij}^{n+1}\}$. Next, from (20a) and (20b), by using method of speedup, we get the solution of transition sheaf $\{V_{ij}^{n+1/2}\}$ along x direction; from (20c) and (20d) we obtain the solution$\{V_{ij}^{n+1}\}$. Finally, from scheme (19) we obtain $\{W_{ijk}^{n+1}\}$. Only in this way, can we proceed continuously so that a complete time step can be taken. Finally, because of the positive definite condition, this finite difference solution exists and is the sole one.

Theorem Suppose that the exact solution of problems (15)$\sim$(17) satisfies smooth condition: $\frac{\partial^2 u}{\partial t^2}$, $\frac{\partial^2 v}{\partial t^2} \in L^\infty(L^\infty(\Omega_1))$, u, $v \in L^\infty(W^{4,\infty}(\Omega_1))\bigcap W^{1,\infty}$ $(W^{1,\infty}(\Omega_1))$, $\frac{\partial^2 w}{\partial t^2} \in L^\infty(L^\infty(\Omega)), w \in L^\infty(W^{4,\infty}(\Omega))$. Adopt the second order upwind finite difference fractional steps schemes (18), (19) and (20). Then the following error estimates hold:

$$\begin{aligned}\|u-U\|_{\bar{L}^\infty(J;l^2)} + \|v-V\|_{\bar{L}^\infty(J;l^2)} + \|w-W\|_{\bar{L}^\infty(J;l^2)} + \|u-U\|_{\bar{L}^2(J;h^1)} \\ +\|v-V\|_{\bar{L}^2(J;h^1)} + \|w-W\|_{\bar{L}^2(J;h^1)} \le M\{\Delta t + h^2\},\end{aligned} \tag{21}$$

where $\|g\|_{\bar{L}^\infty(J;X)} = \underset{n\Delta t\le T}{\mathrm{Sup}}\|f^n\|_X, \|g\|_{\bar{L}^2(J;X)} = \underset{L\Delta t\le T}{\mathrm{Sup}}\{\sum_{n=0}^{L}\|g^n\|_X^2\Delta t\}^{1/2}$.

Proof Let $\xi = u-U$, $\zeta = v-V$, $\omega = w-W$, where u, v, w are exact solutions of problems (15)$\sim$(17), and U, V, W are the difference solutions of schemes (18), (19) and (20).

First, consider (18). For (18a)$\sim$(18d), by eliminating $U^{n+1/2}$, we get the following equivalent form:

$$
\begin{aligned}
&\hat{\Phi}_{1,ij}\frac{U_{ij}^{n+1}-U_{ij}^{n}}{\Delta t}-\{(1+\frac{h}{2}\frac{|a_{1,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}})\\
&+(1+\frac{h}{2}\frac{|a_{2,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_y(K_1(U^n)\delta_{\bar{y}})\}U_{ij}^{n+1}+\delta_{a_1^n,U^n,x}U_{ij}^{n+1}\\
&+\delta_{a_2^n,U^n,y}U_{ij}^{n+1}+\Delta t(1+\frac{h}{2}\frac{|a_{1,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_x(K_1(U^n)\\
&\quad\cdot\delta_{\bar{x}}(\hat{\Phi}_1^{-1}(1+\frac{h}{2}\frac{|a_2^n|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar{y}}U^{n+1})\cdot)_{ij}\\
&-\Delta t\{(1+\frac{h}{2}\frac{|a_{1,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}}(\hat{\Phi}_1^{-1}\delta_{a_2^n,U^n,y}U^{n+1}))_{ij}\\
&+\delta_{a_1^n,U^n,x}(\hat{\Phi}_1^{-1}(1+\frac{h}{2}\frac{|a_2^n|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar{y}}U^{n+1}))_{ij}\\
&-\delta_{a_1^n,U^n,x}(\hat{\Phi}_1^{-1}\delta_{a_2^n,U^n,y}U^{n+1})_{ij}\}\\
&=K_{2,ij,N-1/2}^{n}\delta_{\bar{z}}W_{ij,N}^{n+1}+Q(U_{ij}^{n+1}),\quad 1\le i,j\le N-1,
\end{aligned}\tag{22a}
$$

$$
U_{ij}^{n+1}=0,\quad (x_i,y_j)\in\partial\Omega_1.\tag{22b}
$$

Next, for (20a)~(20d), by eliminating $V^{n+1/2}$, we get the following equivalent form:

$$
\begin{aligned}
&\hat{\Phi}_{3,ij}\frac{V_{ij}^{n+1}-V_{ij}^{n}}{\Delta t}-\{(1+\frac{h}{2}\frac{|b_{1,ij}^{n}|}{K_3(V^n)_{ij}})^{-1}\delta_x(K_3(V^n)\delta_{\bar{x}})\\
&+(1+\frac{h}{2}\frac{|b_{2,ij}^{n}|}{K_3(V^n)_{ij}})^{-1}\delta_y(K_3(V^n)\delta_{\bar{y}})\}V_{ij}^{n+1}+\delta_{b_1^n,V^n,x}V_{ij}^{n+1}\\
&+\delta_{b_2^n,V^n,y}V_{ij}^{n+1}+\Delta t(1+\frac{h}{2}\frac{|b_{1,ij}^{n}|}{K_3(V^n)_{ij}})^{-1}\delta_x(K_3(V^n)\\
&\quad\cdot\delta_{\bar{x}}(\hat{\Phi}_3^{-1}(1+\frac{h}{2}\frac{|b_2^n|}{K_3(V^n)})^{-1}\delta_y(K_3(V^n)\delta_{\bar{y}}V^{n+1})\cdot)_{ij}\\
&-\Delta t\{(1+\frac{h}{2}\frac{|b_{1,ij}^{n}|}{K_3(V^n)_{ij}})^{-1}\delta_x(K_3(V^n)\delta_{\bar{x}}(\hat{\Phi}_3^{-1}(\delta_{b_2^n,V^n,y}V^{n+1})\cdot)_{ij}\\
&+\delta_{b_1^n,V^n,x}(\hat{\Phi}_3^{-1}(1+\frac{h}{2}\frac{|b_2^n|}{K_3(V^n)})^{-1}\delta_y(K_3(V^n)\delta_{\bar{y}}V^{n+1}))_{ij}\\
&-\delta_{b^n,V^n,x}(\hat{\Phi}_3^{-1}\delta_{b_2^n,V^n,y}V^{n+1})_{ij}\}\\
&=-K_{2,ij,1/2}^{n}\delta_{z}W_{ij,0}^{n+1}+Q(V_{ij}^{n+1}),\quad 1\le i,j\le N-1,
\end{aligned}\tag{23a}
$$

$$
V_{ij}^{n+1}=0,\quad (x_i,y_j)\in\partial\Omega_1.\tag{23b}
$$

For problems (15)~(17), we have the following error equations:

$$
\begin{aligned}
&\hat{\Phi}_{1,ij}\frac{\xi_{ij}^{n+1}-\xi_{ij}^{n}}{\Delta t}-\{(1+\frac{h}{2}\frac{|a_{1,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}}\xi^{n+1})_{ij}\\
&+[(1+\frac{h}{2}\frac{|a_{1,ij}^{n+1}|}{K_1(u^{n+1})_{ij}})^{-1}\delta_x(K_1(u^{n+1})\delta_{\bar{x}}u^{n+1})_{ij}\\
&-(1+\frac{h}{2}\frac{|a_{1,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar{x}}u^{n+1})_{ij}]\}\\
&-\{(1+\frac{h}{2}\frac{|a_{2,ij}^{n}|}{K_1(U^n)_{ij}})^{-1}\delta_y(K_1(U^n)\delta_{\bar{y}}\xi^{n+1})_{ij}
\end{aligned}
$$

$$
\begin{aligned}
&+[(1+\frac{h}{2}\frac{|a^{n+1}_{2,ij}|}{K_1(u^{n+1})_{ij}})^{-1}\delta_y(K_1(u^{n+1})\delta_{\bar y}u^{n+1})_{ij}\\
&-(1+\frac{h}{2}\frac{|a^{n}_{2,ij}|}{K_1(U^{n})_{ij}})^{-1}\delta_y(K_1(U^{n})\delta_{\bar y}u^{n+1})_{ij}]\}\\
&+\{\delta_{a^n_1,U^n,x}\xi^{n+1}_{ij}+\delta_{a^{n+1}_1,u^{n+1},x}u^{n+1}_{ij}-\delta_{a^n_1,U^n,x}u^{n+1}_{ij}\}\\
&+\{\delta_{a^n_2,U^n,y}\xi^{n+1}_{ij}+\delta_{a^{n+1}_2,u^{n+1},y}u^{n+1}_{ij}-\delta_{a^n_2,U^n,y}u^{n+1}_{ij}\}\\
&+\Delta t\{(1+\frac{h}{2}\frac{|a^{n}_{1,ij}|}{K_1(U^{n})_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar x}(\hat\Phi_1^{-1}(1+\frac{h}{2}\frac{|a^n_2|}{K_1(U^n)})^{-1}\\
&\quad\cdot\delta_y(K_1(U^n)\delta_{\bar y}\xi^{n+1})\cdot)_{ij}+[(1+\frac{h}{2}\frac{|a^{n+1}_{1,ij}|}{K_1(u^{n+1})_{ij}})^{-1}\delta_x(K_1(u^{n+1})\\
&\quad\cdot\delta_{\bar x}(\hat\Phi_1^{-1}(1+\frac{h}{2}\frac{|a^{n+1}_2|}{K_1(u^{n+1})})^{-1}\delta_y(K_1(u^{n+1})\delta_{\bar y}u^{n+1})\cdot)_{ij}\\
&-(1+\frac{h}{2}\frac{|a^{n}_{1,ij}|}{K_1(U^{n})_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar x}(\hat\Phi_1^{-1}(1\\
&\quad+\frac{h}{2}\frac{|a^n_2|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar y}u^{n+1})\cdot)_{ij}]\}\\
&-\Delta t\{(1+\frac{h}{2}\frac{|a^{n}_{1,ij}|}{K_1(U^{n})_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar x}(\hat\Phi_1^{-1}\delta_{a^n_2,U^n,y}\xi^{n+1}))_{ij}\\
&+[(1+\frac{h}{2}\frac{|a^{n+1}_{1,ij}|}{K_1(u^{n+1})_{ij}})^{-1}\delta_x(K_1(u^{n+1})\delta_{\bar x}(\hat\Phi_1^{-1}\delta_{a^{n+1}_2,u^{n+1},y}u^{n+1}))_{ij}\\
&-(1+\frac{h}{2}\frac{|a^{n}_{1,ij}|}{K_1(U^{n})_{ij}})^{-1}\delta_x(K_1(U^n)\delta_{\bar x}(\hat\Phi_1^{-1}\delta_{a^n_2,U^n,y}u^{n+1}))_{ij}]\}\\
&-\Delta t\{\delta_{a^n_1,U^n,x}(\hat\Phi_1^{-1}(1+\frac{h}{2}\frac{|a^n_2|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar y}\xi^{n+1}))_{ij}\\
&+[\delta_{a^{n+1}_1,u^{n+1},x}(\hat\Phi_1^{-1}(1+\frac{h}{2}\frac{|a^{n+1}_2|}{K_1(u^{n+1})})^{-1}\delta_y(K_1(u^{n+1})\delta_{\bar y}u^{n+1}))_{ij}\\
&-\delta_{a^n_1,U^n,x}(\hat\Phi_1^{-1}(1+\frac{h}{2}\frac{|a^n_2|}{K_1(U^n)})^{-1}\delta_y(K_1(U^n)\delta_{\bar y}u^{n+1}))_{ij}]\}\\
&+\Delta t\{\delta_{a^n_1,U^n,x}(\hat\Phi_1^{-1}\delta_{a^n_2,U^n,y}\xi^{n+1})_{ij}\\
&+[\delta_{a^{n+1}_1,u^{n+1},x}(\hat\Phi_1^{-1}\delta_{a^{n+1}_2,u^{n+1},y}u^{n+1})_{ij}-\delta_{a^n_1,U^n,x}(\hat\Phi_1^{-1}\delta_{a^n_2,U^n,y}u^{n+1})_{ij}]\}\\
&=K^n_{2,ij,N-1/2}\delta_{\bar z}\omega^{n+1}_{ij,N}+Q(u^{n+1}_{ij})-Q(U^{n+1}_{ij})+\varepsilon^{n+1}_{1,ij},\quad 1\le i,j\le N-1,
\end{aligned}\tag{24a}
$$

$$
\xi^{n+1}_{ij}=0,\quad (x_i,y_j)\in\partial\Omega_1,\tag{24b}
$$

where $|\varepsilon^{n+1}_{1,ij}|\le M\{\left\|\frac{\partial^2u}{\partial t^2}\right\|_{L^\infty(L^\infty)},\|u\|_{L^\infty(W^{4,\infty})}\}\{\Delta t+h^2\}$.

$$
\Phi_{2,ijk}\frac{\omega^{n+1}_{ijk}-\omega^n_{ijk}}{\Delta t}=\delta_z(K^n_2\delta_{\bar z}\omega^{n+1})_{ijk}+\varepsilon^{n+1}_{2,ijk},\quad 1\le i,j,k\le N-1,\tag{25}
$$

where $\left|\varepsilon^{n+1}_{2,ijk}\right|\le M\{\left\|\frac{\partial^2w}{\partial t^2}\right\|_{L^\infty(L^\infty)},\|w\|_{L^\infty(W^{4,\infty})}\}\{\Delta t+h^2\}$.

Testing (24a) and (25) against $2\Delta t\xi^{n+1}_{ij}$ and $2\Delta t\omega^{n+1}_{ijk}$, summing them up by parts, and using (24b) we can obtain

$$\begin{aligned}&\left\|\hat{\Phi}_2^{1/2}\xi^{n+1}\right\|^2-\left\|\hat{\Phi}_1^{1/2}\xi^n\right\|^2+(\Delta t)^2\left\|\Phi_2^{1/2}d_t\xi^n\right\|^2+\Delta t\{\left\|K_1^{n,1/2}\delta_{\bar{x}}\xi^{n+1}\right\|^2\\&+\left\|K_1^{n,1/2}\delta_{\bar{y}}\xi^{n+1}\right\|^2\}\leq M\{(\Delta t)^2+h^4+\left\|\xi^{n+1}\right\|^2+\|\xi^n\|^2\}\Delta t.\end{aligned}\tag{26}$$

Similarly, for equation (23) we have

$$\begin{aligned}&\left\|\hat{\Phi}_2^{1/2}\zeta^{n+1}\right\|^2-\left\|\hat{\Phi}_3^{1/2}\zeta^n\right\|^2+(\Delta t)^2\left\|\Phi_2^{1/2}d_t\zeta^n\right\|^2+\Delta t\left\|K_3^{n,1/2}\delta_{\bar{x}}\zeta^{n+1}\right\|^2\\&+\left\|K_3^{n,1/2}\delta_{\bar{y}}\zeta^{n+1}\right\|^2\}\leq M\{(\Delta t)^2+h^4+\left\|\zeta^{n+1}\right\|^2+\|\zeta^n\|^2\}\Delta t.\end{aligned}\tag{27}$$

For error equation (25) we have

$$\begin{aligned}&\left\|\Phi_2^{1/2}\omega^{n+1}\right\|^2-\left\|\Phi_2^{1/2}\omega^n\right\|^2+(\Delta t)^2\left\|\Phi_2^{1/2}d_t\omega^n\right\|^2+2\Delta t\left\|K_2^{1/2}\delta_{\bar{z}}\omega^{n+1}\right\|^2\\&\leq 2\Delta t\sum_{i,j=1}^{N-1}\{K^n_{2,ij,N-1/2}\delta_{\bar{z}}\omega^{n+1}_{ij,N}\cdot\xi^{n+1}_{ij}-K^n_{2,ij,1/2}\delta_z\omega^{n+1}_{ij,O}\cdot\zeta^{n+1}_{ij}\}h^2\\&+M\Delta t\{(\Delta t)^2+h^4+\left\|\omega^{n+1}\right\|^2\}.\end{aligned}\tag{28}$$

Combining (26)$\sim$(28), summing up $0\leq n\leq L$, and noting that $\xi^0=\zeta^0=\omega^0=0$, we have

$$\begin{aligned}&\{\left\|\hat{\Phi}_1^{1/2}\xi^{L+1}\right\|^2+\left\|\hat{\Phi}_3^{1/2}\zeta^{L+1}\right\|^2+\left\|\Phi_2^{1/2}\omega^{L+1}\right\|^2\}\\&+\Delta t\sum_{n=0}^{L}\{\left\|\hat{\Phi}_1^{1/2}d_t\xi^n\right\|^2+\left\|\hat{\Phi}_3^{1/2}d_t\zeta^n\right\|^2+\left\|\Phi_2^{1/2}d_t\omega^n\right\|^2\}\Delta t\\&+\sum_{n=0}^{L}\{\left\|K_1^{n,1/2}\delta_{\bar{x}}\xi^{n+1}\right\|^2+\left\|K_1^{n,1/2}\delta_{\bar{y}}\xi^{n+1}\right\|^2\\&+\left\|K_3^{n,1/2}\delta_{\bar{x}}\zeta^{n+1}\right\|^2+\left\|K_3^{n,1/2}\delta_{\bar{y}}\zeta^{n+1}\right\|+\left\|\Phi_2^{1/2}\delta_{\bar{z}}\omega^{n+1}\right\|^2\}\Delta t\\&\leq M\{\sum_{n=0}^{L}[\left\|\xi^{n+1}\right\|^2+\left\|\zeta^{n+1}\right\|^2+\left\|\omega^{n+1}\right\|^2]\Delta t+(\Delta t)^2+h^4\}.\end{aligned}\tag{29}$$

Applying the discrete Gronwall inequality, we have

$$\begin{aligned}&\left\|\hat{\Phi}_1^{1/2}\xi^{L+1}\right\|^2+\left\|\hat{\Phi}_3^{1/2}\zeta^{L+1}\right\|^2+\left\|\Phi_2^{1/2}\omega^{L+1}\right\|^2\\&+\Delta t\sum_{n=0}^{L}\{\left\|\hat{\Phi}_1^{1/2}d_t\xi^n\right\|^2+\left\|\hat{\Phi}_3^{1/3}d_t\zeta^n\right\|^2+\left\|\Phi_2^{1/2}d_t\omega^n\right\|^2\}\Delta t\\&+\sum_{n=0}^{L}\{\left\|K_1^{n,1/2}\delta_{\bar{x}}\xi^{n+1}\right\|^2+\left\|K_1^{n,1/2}\delta_{\bar{y}}\xi^{n+1}\right\|^2\\&+\left\|K_3^{n,1/2}\delta_{\bar{x}}\zeta^{n+1}\right\|^2+\left\|K_3^{n,1/2}\delta_{\bar{y}}\zeta^{n+1}\right\|+\left\|K_2^{n,1/2}\delta_{\bar{z}}\omega^{n+1}\right\|\}\Delta t\\&\leq M\{(\Delta t)^2+h^4\}.\end{aligned}\tag{30}$$

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