ROBUST DISSIPATIVE CONTROL FOR TAKAGI-SUGENO FUZZY DESCRIPTOR SYSTEM

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Abstract. This paper is concerned with the problem of robust dissipative control for Takagi-Sugeno (T-S) fuzzy descriptor system. A new theory and new method of dissipative analysis and control for nonlinear descriptor systems are proposed by using T-S fuzzy descriptor systems. The sufficient conditions for the existence of dissipative controller is given by using properties of partitioned matrices and Lagrange interpolation polynomial theory and so on. With the aid of the Linear Matrix Inequality techniques and related computer language, simulation programs of the controller are given. By selecting different parameters in the controller design, different control processes can be achieved for target systems in the same design process. So it could potentially save cost and time in the controller design for the actual physical system.

Key Words. T-S fuzzy descriptor system, dissipative control, uncertain, robust

1. Introduction

In 1972, Willems introduced the notion of dissipative systems which unified passivity and the small gain concept in one framework[1]. Hill and Moylan applied dissipativeness in the stability analysis of nonlinear systems [2]. After that, the concept of dissipativeness played an important role in the study of electric circuit, network, and control theory [1-4]. The core method of dissipative system theory is to construct a nonnegative energy function such that the consumption energy of a system is less than the supply rate of the energy. The main advantage of the dissipative method is that a complex system can be conveniently described from an energy view. The dissipative system theory is mainly applied in stability analysis of nonlinear systems. Many stability theories based on the Lyapunov function can be explained in the view of the dissipativeness.

$H_{\infty}$ Robust control and the passive theories are two important branches in the dissipative system theory. Designed controllers with the small gain technology and the passivity have been widely studied and applied in many areas [5-8]. The dissipativeness provides an appropriate framework between gain and phase for a less conservative robust controller design, especially in practice problems where both gain and phase performances are considered [9,10].

Compared to the normal system, descriptor systems could maintain better physical properties, such as mechanical systems, electric circuits, network systems, neural
networks, limited robots, biological systems and so on (see [11, 12] and the references therein). The dissipative concept has been extended to the area of descriptor system and has played a very important role in the analysis and the synthesis of control systems [13-15].

It is very difficult to find the storage function for a nonlinear dissipative system in general [1]. Some scholars devote themselves to looking for simple solutions of the controllers [16]. So far, it is still a challenging problem in dissipative research. In 2000, Taniguchi, Tanaka, et al. first presented T-S fuzzy descriptor system [17]. The proposed systems opened up a new way to solve control problems for a kind of nonlinear descriptor system. So far, the problems of passive analysis and $H_\infty$ control for the T-S fuzzy descriptor system have been studied [18,19] and the dissipative problems for T-S fuzzy system also have been stepped in [20,21]. However the dissipative for T-S fuzzy descriptor system little attention on are rarely seen.

This paper is concerned with the problem of robust dissipative control for T-S fuzzy descriptor system. A new theory and new method of dissipative analysis and control for nonlinear descriptor systems are proposed by using T-S fuzzy descriptor systems. The sufficient conditions for the existence of dissipative controller is given by using properties of partitioned matrices and Lagrange interpolation polynomial theory. With the aid of the Linear Matrix Inequality technique and related computer language, simulation programs of the controller are given. By selecting different parameters in the controller design, different control processes can be achieved for target systems in the same design process. So it could potentially save cost and time in the controller design for the actual physical system.

This paper is organized as follows. In section 2, problem formulation and preliminaries is presented. In section 3, the dissipative analysis is given for T-S fuzzy descriptor systems. In section 4, the problem of dissipative control is considered for a kind of T-S fuzzy descriptor system. In section 5, the problem of robust dissipative control is studied for a kind of uncertain T-S fuzzy descriptor system. Two numerical examples are given respectively to demonstrate the validity and feasibility of the control method proposed in section 4 and 5. Finally, concluding remarks are made in section 6.

2. Problem Formulation and Preliminaries

In this section, we consider T-S fuzzy descriptor system, its $i$th fuzzy rule is of the following form

\begin{equation}
R_i: \text{ if } \xi_1(t) \text{ is } M_{1i} \text{ and } \xi_2(t) \text{ is } M_{2i} \ldots \text{ and } \xi_p(t) \text{ is } M_{pi},
\end{equation}

then $E\dot{x}(t) = A_ix(t) + B_{1i}w(t) + B_{2i}u(t)$,

\begin{equation}
z(t) = C_i x(t) + D_{1i}w(t) + D_{2i}u(t), \quad i = 1, 2, \ldots, r,
\end{equation}

where $M_{ji}(j = 1, 2, \ldots, p)$ is the fuzzy set, $r$ is the number of if-then rules, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^l$ is the exogenous input, and $z(t) \in \mathbb{R}^l$ is the controlled output, $E, A_i, B_{1i}, B_{2i}, C_i, D_{1i}, D_{2i}$ are known constant matrices with appropriate dimensions. $\text{rank} E \leq n$, $\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_p(t)]^T$ is the premise variables. The overall fuzzy model can be inferred as follows:

\begin{equation}
E\dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[A_i x(t) + B_{1i}w(t) + B_{2i}u(t)],
\end{equation}

where $h_i(\xi(t))$ is the membership function.
\[
z(t) = \sum_{i=1}^{r} h_i(\xi(t))[C_i x(t) + D_{1i} w(t) + D_{2i} u(t)], \quad i = 1, 2, \ldots, r,
\]

\[
\beta_i(\xi(t)) = \prod_{j=1}^{p} M_{ji}(\xi_j(t)) \geq 0, \quad h_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{i=1}^{r} \beta_i(\xi(t))} \geq 0, \quad \sum_{i=1}^{r} h_i(\xi(t)) = 1,
\]

where \(M_{ji}(\cdot)\) is the grade of membership of \(\xi_j(t)\) in \(M_{ji}\) and \(h_i(\xi(t))\) can be regarded as the normalized weight of each if-then rule.

The following lemmas are used later.

**Lemma 1** (Xie et al. [22]). Given matrices \(D, E\) and \(Y\) with appropriate dimensions, and \(Y\) is symmetric, then

\[
Y + DFE + E^T F^T D^T < 0
\]

for all \(F\) satisfying \(F^T F \leq I\), if and only if there exists some scalar \(\varepsilon > 0\) such that

\[
Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.
\]

**Lemma 2** (Ben-Istael et al. [23]). The square root of the semi-positive definite matrix is existent and unique.

**Lemma 3** (Zhang et al. [24]). The square root \(S = \sqrt{Q}\) of the semi-positive definite matrix \(Q\) is a polynomial of degree \(k - 1\) on \(Q\), where \(k\) is the number of different characteristic root of the matrix. Namely, there exists unique polynomial \(\varphi(\lambda)\) of degree \(k - 1\) such that \(S = \varphi(Q)\).

In this paper, we let the notation \(A^T\) denote the transpose of the matrix \(A\). The asterisk denotes the transpose of symmetric position element in the matrix.

### 3. Dissipative Analysis

Consider T-S fuzzy descriptor system of the following form

\[
E \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))A_i x(t).
\]

**Definition 1.** The system (3) is said to be regular if

\[
\det(sE - \sum_{i=1}^{r} h_i(\xi(t))A_i) \neq 0(\forall t \geq 0).
\]

The system (3) is said to be impulse-free if it is regular and satisfies the following equation

\[
\text{deg}_s \det(sE - \sum_{i=1}^{r} h_i(\xi(t))A_i) = \text{rank} E \quad (\forall t \geq 0).
\]

The system (3) is said to be stable if it is regular and satisfies \(\Re(s) < 0(\forall t \geq 0)\) for \(s \in \sigma(E, \sum_{i=1}^{r} h_i(\xi(t))A_i)\) where \(\sigma(E, \sum_{i=1}^{r} h_i(\xi(t))A_i) = \{s \mid \det(sE - \sum_{i=1}^{r} h_i(\xi(t))A_i) = 0\}\). The system (3) is said to be admissible if it is regular, impulse-free and stable.

We always assume that the system to be considered is regular and impulse-free in the rest of this paper.

**Theorem 1.** The system (3) is admissible if there exists a common nonsingular matrix \(P\) such that the following inequalities hold

\[
E^T P = P^T E \geq 0,
\]
Definition 2. The system of the form (2) with zero initial condition is said to be strictly dissipative if there exists a positive constant $\delta$ such that the following matrix inequality holds

$$V(x(\tau)) \leq \int_0^\tau [z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t)]dt - \delta \int_0^\tau w^T(t)w(t)dt$$

for all positive constant $\tau$ and $w(t) \in L_2[0, \tau]$, where $V$ is a nonnegative continuous real function satisfying $V(0) = 0$, and $Q, S$ and $R$ are matrices with suitable dimension. $Q$ and $R$ are symmetrical.

We always assume matrix $Q \leq 0$ and $Q^1$ stands for the square root of the semi-positive definite matrix $-Q$ in the following part.

Theorem 2. For system (2) with the zero initial condition, when $u(t) = 0$, if there exist a common nonsingular matrix $P \in \mathbb{R}^{n \times n}$ and a positive constant $\delta$ such that the following matrix inequalities hold

$$(4a) \quad E^TP = P^TE \geq 0,$$

$$(4b) \quad -S^TD_{ii} - D_{ii}^TS + \delta I - R < 0,$$

$$(4c) \quad \begin{pmatrix} A_i^TP + P^TA_i & P^TB_{ii} - C_i^TS & C_i^TQ_1 \\ * & -S^TD_{ii} - D_{ii}^TS + \delta I - R & D_{ii}^TQ_1 \\ * & * & -I \end{pmatrix} < 0,$$

$i = 1, 2, \ldots, r$

then the system (2) is admissible and strictly dissipative. $Q, S$ and $R$ are matrices with appropriate dimension. $Q$ and $R$ are symmetrical.

Proof. When $w(t) = 0$, the system (2) is admissible according to Theorem 1 and the inequalities (4). Consider the nonnegative functional $V(x(t))$ in form $V(x(t)) = x^T(t)E^TPx(t)$. Then from Definition 2, we can obtain that the system (2) is strictly dissipative.

Remark 1. When normalized membership functions $h_i(\xi(t)) = 1, h_j(\xi(t)) = 0, \forall j \neq i$, and $\text{rank}E = n$, the conclusion in Theorem 2 is a sufficient condition to check the strictly dissipativeness for the linear system. When normal membership functions $h_i(\xi(t)) = 1, h_j(\xi(t)) = 0, \forall j \neq i$, and $\text{rank}E < n$, the conclusion is a sufficient condition to test admissibility and strictly dissipativeness for the descriptor system.

4. Dissipative Control

4.1. Dissipative Control. For the fuzzy model (2), we construct the following fuzzy controller via the PDC [25]

$$u(t) = \sum_{i=1}^r h_i(\xi(t))K_i x(t),$$

where $K_i$ is the local feedback gain. By substituting (5) into the system (2), we obtain the following closed-loop system
When Proof. Then the system (6) is admissible and strictly dissipative.

\[
\begin{aligned}
E\dot{z}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_j(\xi(t))h_j(\xi(t))[(A_i + B_2iK_j)x(t) + B_4i w(t)], \\
z(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_j(\xi(t))h_j(\xi(t))[(C_i + D_2iK_j)x(t) + D_4i w(t)].
\end{aligned}
\]

**Theorem 3.** For system (6) with zero initial condition, if there exist a common nonsingular matrix \(P \in \mathbb{R}^{n \times n}\), matrices \(K_i\) and a positive constant \(\delta\) such that the following matrix inequalities hold

\[
\begin{equation}
(7a)
E^T P = P^T E \geq 0,
\end{equation}
\]

\[
(7b)
-S^T D_{11} - D_{11}^T S + \delta I - R < 0,
\]

\[
(7c)
\begin{pmatrix}
\Delta_i
\end{pmatrix}
\begin{pmatrix}
\Phi_i
\end{pmatrix}
\begin{pmatrix}
\Theta_i
\end{pmatrix}
< 0, \quad i = 1, 2, \ldots, r,
\]

\[
(7d)
\begin{pmatrix}
\Delta_{ij}
\end{pmatrix}
\begin{pmatrix}
\Omega_{ij}
\end{pmatrix}
\begin{pmatrix}
\Phi_{ij}
\end{pmatrix}
\begin{pmatrix}
\Theta_{ij}
\end{pmatrix}
< 0, \quad i < j \leq r,
\]

where

\[
\begin{array}{ll}
\Delta_1 &= (A_i + B_2iK_j)^T P + P^T (A_i + B_2iK_j), \\
\Omega_1 &= P^T B_{11} - (C_i + D_2iK_j)^T S, \\
\Phi_1 &= (C_i + D_2iK_j)^T Q_1, \\
\Theta_1 &= D_{11}^T Q_1, \\
\Delta_2 &= (A_i + B_2iK_j)^T P + P^T (A_i + B_2iK_j) + \\
&\quad (A_j + B_2jK_i)^T P + P^T (A_j + B_2jK_i), \\
\Omega_2 &= P^T B_{11} - (C_i + D_2iK_j)^T S + P^T B_{11} - (C_j + D_2jK_i)^T S, \\
\Phi_2 &= (C_i + D_2iK_j)^T Q_1 + (C_j + D_2jK_i)^T Q_1, \\
\Theta_2 &= D_{11}^T Q_1 + D_{11}^T Q_1, \\
\end{array}
\]

then the system (6) is admissible and strictly dissipative. \(Q, S\) and \(R\) are matrices with appropriate dimension. \(Q\) and \(R\) are symmetrical.

**Proof.** When \(w(t) = 0\), the system (6) is admissible by Theorem 1 and the inequalities (7). Consider the nonnegative functional \(V(x(t))\) in form \(V(x(t)) = x^T(t)E^T P x(t)\). Along the trajectory of the closed-loop system (6), we have

\[
\begin{aligned}
\dot{V}(x(t)) &= z^T(t)Qz(t) - 2z^T(t)Sw(t) - w^T(t)Rw(t) + \delta w^T(t)w(t) \\
&= [x^T(t), w^T(t)] \Xi \begin{pmatrix} x(t) \\ w(t) \end{pmatrix}
\end{aligned}
\]
where

\[ \Xi = \begin{bmatrix} 
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) & (A_i + B_{2i}K_j)^T P_+ + \sum_{i=1}^{r} h_i(\xi(t))P^T B_{1i} \\
* & 0 
\end{bmatrix} + 
\begin{bmatrix} 
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))(C_i + D_{2i}K_j)^T Q_1 \\
\sum_{i=1}^{r} h_i(\xi(t))D^T_{1i} Q_1 \\
0 & \sum_{i=1}^{r} h_i(\xi(t))(C_i + D_{2i}K_j)^T S + D^T_{1i} S
\end{bmatrix} \times 
\begin{bmatrix} 
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))Q_1(C_i + D_{2i}K_j) \\
\sum_{i=1}^{r} h_i(\xi(t))Q_1 D_{1i} \\
0 & 0 & R - \delta I 
\end{bmatrix} - 
\begin{bmatrix} 
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))(C_i + D_{2i}K_j)^T S + D^T_{1i} S + \delta I - R \\
- S^T D_{1i} - D^T_{1i} S + \delta I - R \\
- D^T_{1i} Q_1 
\end{bmatrix} \times 
\begin{bmatrix} 
\sum_{i=1}^{r} h_i^2(\xi(t)) \left( \begin{array}{cc} \Delta_1 & \Omega_1 \\
* & \Sigma_1 
\end{array} \right) + \sum_{i<j} \sum_{i=1}^{r} h_i(\xi(t))h_j(\xi(t)) \left( \begin{array}{cc} \Delta_2 & \Omega_2 \\
* & \Sigma_2 
\end{array} \right) \\
* & -2I 
\end{bmatrix}.
\]

From the above equation and the inequalities (7), it is clear that

\[
\begin{bmatrix} 
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))[A_i + B_{2i}K_j]^T P_+ + \sum_{i=1}^{r} h_i(\xi(t))h_j(\xi(t))[A_i + B_{2i}K_j] \\
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))[P^T B_{1i} + (C_i + D_{2i}K_j)^T S] \\
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))[S^T \times D_{1i} + D^T_{1i} S + \delta I] - \delta I \\
* \\
\end{bmatrix}
< 0,
\]

By Schur complement, the above matrix inequality (9) is equivalent to \( \Xi < 0. \)

From (8) and \( \Xi < 0, \) we have

\[
\dot{V}(x(t)) - \dot{z}^T(t)Qz(t) - 2z^T(t)Sw(t) - w^T(t)Rw(t) + \delta w^T(t)w(t) \leq 0
\]
Note that $V(0)=0$, it follows from the above inequality that

$$V(x(t)) \leq \int_0^T [z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t)]dt - \delta \int_0^T w^T(t)w(t)dt$$

That is, the system (6) is admissible and strictly dissipative.

**Theorem 4.** For system (6) with zero initial condition, if there exist a common nonsingular matrix $X \in \mathbb{R}^{n \times n}$, matrices $N_i$, positive constants $\delta$ such that the following matrix inequalities hold

\begin{align}
(10a) & \quadXE^T = EX^T \geq 0, \\
(10b) & \quad-S^TD_{1i} - D_{1i}^TS + \delta I - R < 0, \\
(10c) & \quad\begin{pmatrix}
\tilde{\Delta}_1 & \tilde{\Omega}_i \\
* & \Theta_i \\
\end{pmatrix} < 0, \quad i = 1, \ldots, r, \\
(10d) & \quad\begin{pmatrix}
\tilde{\Delta}_2 & \tilde{\Omega}_j \\
* & \Theta_j \\
\end{pmatrix} < 0, \quad i < j \leq r,
\end{align}

where

\begin{align*}
\tilde{\Delta}_1 &= XA_i^T + A_iX^T + N_i^2B_{2i}^2 + B_{2i}N_i, \\
\tilde{\Omega}_i &= B_{1i} - XC_i^TS - N_iD_i^2S, \\
\tilde{\Phi}_1 &= XC_i^TQ_i + N_iD_i^2\delta I, \\
\Sigma_1 &= -S^TD_{1i} - D_{1i}^TS + \delta I - R, \\
\Theta_1 &= D_i^2Q_i, \\
\tilde{\Delta}_2 &= XA_i^T + A_iX^T + N_i^2B_{2i}^2 + B_{2i}N_i + XA_j^T + A_jX^T + N_j^2B_{2j}^2 + B_{2j}N_i, \\
\tilde{\Omega}_j &= B_{1j} - XC_j^TS - N_jD_j^2S + B_{1j} - XC_j^TS - N_jD_j^2S, \\
\tilde{\Phi}_2 &= XC_j^TQ_i + N_jD_j^2\delta I + XC_j^TQ_1 + N_jD_j^2\delta I, \\
\Sigma_2 &= -S^TD_{1j} - D_{1j}^TS + 2\delta I - 2R - S^TD_{1j} - D_{1j}^TS, \\
\Theta_2 &= D_i^2Q_1 + D_j^2Q_1,
\end{align*}

then the system (6) is admissible and strictly dissipative. The dissipative controller is

$$u(t) = \sum_{i=1}^r \int_{x(t)}^{x(t)} N_iX^{-1}x(t), \quad X = P^{-1}, N_i = K_iX^T.$$ 

$Q, S$ and $R$ are matrices with appropriate dimension, $Q$ and $R$ are symmetrical.

**Proof.** Pre-and Post-multiplying both sides of the above inequality (7c) and (7d) by diag($P^{-1}, I, I$) and diag($P^{-1}, I, I$) respectively, we can obtain the matrices inequalities (10c) and (10d).

We deal with the nonstrict matrix inequality constraint in (10a). Without losing generality, we assume $E = \begin{pmatrix} I_{n_1} & 0 \\ 0 & 0 \end{pmatrix}$. We decompose $P = \begin{pmatrix} P_1 & P_2 \\ P_3 & P_4 \end{pmatrix}$ according to $E$. We can infer that the constraint in (10a) is equivalent to $P = \begin{pmatrix} P_1 & 0 \\ P_3 & P_4 \end{pmatrix}$, where $P_1 > 0, \det P_4 \neq 0$. 


Thus we obtain the design method of dissipative controller by the linear matrix inequality in the following.

**Theorem 5.** For the system (6) with the zero initial condition, if there exist a common nonsingular matrix \( X \in \mathbb{R}^{n \times n} \), matrices \( N_i \) and positive constants \( \delta \) such that the matrix inequalities (10b),(10c),(10d) then the system (6) is admissible and strictly dissipative. The dissipative controller is \( u(t) = \sum_{i=1}^{r} h_i(\xi(t)) N_i X^{-\top} x(t) \).

Where

\[
X = P^{-\top} = \begin{pmatrix}
P_i^{-1} & -P_i^{-1} P_3^{-1} P_4^{-\top} \\
0 & P_4^{-\top}
\end{pmatrix}
\]

\[
:= \begin{pmatrix}X_1 & X_2 \\
0 & X_3
\end{pmatrix},
\]

\( X_1 = P_i^{-1} > 0, N_i = K_i X^\top. \)

4.2. \( H_\infty \) control and Passive control. Two Corollaries of Theorem 3 in the following are the sufficient conditions to check \( H_\infty \) stable and the weak passive separately for T-S fuzzy descriptor system. Therefore robust \( H_\infty \) controller and the passive controller can be designed according to the similar method in theorem 5.

**Corollary 1.** For the system (6) with the zero initial condition, if there exist a common nonsingular matrix \( P \in \mathbb{R}^{n \times n} \) and a positive constant \( \delta \) such that the following matrix inequalities hold

\[
E^T P = P^T E \geq 0,
\]

\[-D_{ii} - D_{ii}^T + \delta I < 0,
\]

\[
\begin{pmatrix}
(A_i + B_2 K_i)^T P^+ + P^T (A_i + B_2 K_i) & P^T B_{1i}^- - (C_i + D_2 K_i)^T \\
0 & -D_{ii}^- + D_{ii}^+ + \delta I
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\]

\[
\begin{pmatrix}
(A_i + B_2 K_j)^T P^+ + P^T (A_i + B_2 K_j) & P^T B_{1i}^- - (C_i + D_2 K_j)^T \\
0 & -D_{ii}^- + D_{ii}^+ + \delta I
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\]

\[
\begin{pmatrix}
(A_i + B_2 K_j)^T P^+ + P^T (A_i + B_2 K_j) & P^T B_{1i}^- - (C_i + D_2 K_j)^T \\
0 & -D_{ii}^- + D_{ii}^+ + \delta I
\end{pmatrix} < 0, \quad i < j \leq r,
\]

\[
\begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
\]
The membership functions are chosen as $h_i(\xi(t)) = M_{i1}$, then the system (6) is admissible and strictly passive.

Let $Q = 0, R = 0, S = I$, we can derive Corollary 1 from Theorem 3.

**Corollary 2.** For the system (6) with the zero initial condition, if there exist a common nonsingular matrix $P \in \mathbb{R}^{n \times n}$ and a positive constant $\gamma$ such that the following matrix inequalities:

$$
E^TP = P^TE \geq 0,
$$

$$
\begin{pmatrix}
A_i^+ & B_2iK_i^+P^+

P^T(A_i^+ & B_2iK_i)

* & -\gamma^2I

* & D_{i1i}^T

\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
$$

$$
\begin{pmatrix}
A_i^+ & B_2iK_i^+P^+

P^T(A_i^+ & B_2iK_i)

* & -2\gamma^2I

* & D_{i11i}^T + D_{i22i}^T

\end{pmatrix} < 0, \quad i < j \leq r,
$$

then the system (6) is $H_\infty$ stable.

Let $Q = -I, R = \gamma^2, S = 0$, we can derive Corollary 2 from Theorem 3 and the properties of linear matrices inequalities.

### 4.3. Numerical Example

In this section, we will give an example to explain that the dissipative control method to unify $H_\infty$ control and the passive control.

Consider the following T-S fuzzy system

(11) $R_i :$ if $\xi_i(t)$ is $M_{i1}$,

\begin{align*}
\dot{x}(t) &= A_ix(t) + B_{1i}w(t) + B_{2i}v(t), \\
z(t) &= C_ix(t) + D_{1i}w(t) + D_{2i}v(t), \quad i = 1, 2, \ldots, r.
\end{align*}

The membership functions are chosen as $h_i(x_1(t)) = \frac{x_1^2(t)}{2}$, $h_2(x_1(t)) = 1 - \frac{x_1^2(t)}{2}$, where state vector $x(t) = [x_1(t), x_2(t), x_3(t)]^T, x_i(t) \in \mathbb{R}^3, i = 1, 2, 3$. The control parameters are defined as follows

(12) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, A_1 = \begin{pmatrix} 0.3 & -3 & 0.1 \\ 1 & 2 & 1 \\ 2.9 & 2 & 0.4 \end{pmatrix}$,

$A_2 = \begin{pmatrix} 0.1 & -8 & 1 \\ 2 & 0 & 7.1 \\ 9.1 & 2 & 2 \end{pmatrix}, B_{11} = \begin{pmatrix} 1.1 \\ 1 \\ 0 \end{pmatrix}, B_{12} = \begin{pmatrix} 1.9 \\ 0.9 \\ 1 \end{pmatrix},$

$B_{21} = \begin{pmatrix} 1 \\ 1 \\ 0.9 \end{pmatrix}, B_{22} = \begin{pmatrix} 1 \\ 0.1 \\ 0.3 \end{pmatrix},$

$C_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, C_2 = \begin{pmatrix} 0.2 \\ 0.01 \\ -0.4 \end{pmatrix},$

$D_{11} = 0.1, D_{12} = 0.2, D_{21} = 1, D_{22} = 1.$
Let $Q = -1, S = 0.3, R = 0.8$, we obtain the following dissipative controller, and we can achieve $H_\infty$ norm and passive degree if they are required.

$$u(t) = \begin{bmatrix} \frac{x_1^2(t)}{2} & -44.5671 & 27.8380 & 0.4972 \\ + (1 - \frac{x_1^2(t)}{2}) & -60.4979 & 38.5346 & 0.6785 \end{bmatrix} x(t).$$

Let $Q = -1, S = 0, R = 1$, the dissipative controller is degenerated into $H_\infty$ controller with norm 1

$$u(t) = \begin{bmatrix} \frac{x_1^2(t)}{2} & -47.6550 & 35.9829 & 3.4149 \\ + (1 - \frac{x_1^2(t)}{2}) & -40.0300 & 30.1481 & 2.4570 \end{bmatrix} x(t).$$

Let $Q = 0, S = 1, R = 0$, the dissipative controller is degenerated into passive controller with degree 0.0942

$$u(t) = \begin{bmatrix} \frac{x_1^2(t)}{2} & -19.4704 & 10.9680 & 0.3117 \\ + (1 - \frac{x_1^2(t)}{2}) & -33.5354 & 19.8192 & 0.5722 \end{bmatrix} x(t).$$

5. Robust dissipative control

5.1. Robust dissipative control. In this section, an uncertain T-S fuzzy descriptor system is considered, its $i$th fuzzy rule is of the following form

$$R_i : \text{if } \xi_1(t) \text{ is } M_{1i}, \text{ and } \xi_2(t) \text{ is } M_{2i}, \ldots \text{ and } \xi_p(t) \text{ is } M_{pi},$$

then

$$\dot{x}(t) = (A_i + \Delta A_i)x(t) + B_{1i}w(t) + (B_{2i} + \Delta B_{2i})u(t),$$

$$z(t) = (C_i + \Delta C_i)x(t) + D_{1i}w(t) + (D_{2i} + \Delta D_{2i})u(t), \quad i = 1, 2, \ldots, r,$$

where the matrices $\Delta A_i, \Delta B_{2i}, \Delta C_i, \Delta D_{2i}$ represent the time-varying parametric uncertainties with the following structure

$$[\Delta A_i, \Delta B_{2i}, \Delta C_i, \Delta D_{2i}] = H_{1i}F_i[E_{1i}, E_{2i}], \quad [\Delta C_i, \Delta D_{2i}] = H_{2i}F_i[E_{1i}, E_{2i}], \quad i = 1, 2, \ldots, r,$$

where $H_{1i}, H_{2i}, E_{1i}$ and $E_{2i}$ are known constant matrices with appropriate dimensions, and $F_i$ are unknown real matrices satisfying $F_i^T F_i \leq I$. Other parameters are defined in the system (1). By using the fuzzy inference methods with singleton fuzzifier and weighted average defuzzifier, the overall fuzzy model for the system can be inferred as follows

$$\dot{E}\hat{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[(A_i + \Delta A_i)x(t) +$$

$$B_{1i}w(t) + (B_{2i} + \Delta B_{2i})u(t)],$$

$$z(t) = \sum_{i=1}^{r} h_i(\xi(t))[(C_i + \Delta C_i)x(t) + D_{1i}w(t) +$$

$$(D_{2i} + \Delta D_{2i})u(t)], \quad i = 1, 2, \ldots, r,$$
By substituting (5) into (14), we obtain the following closed-loop system

\begin{equation}
\dot{z}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \left\{ \left[(A_i + \Delta A_i) + (B_{2i} + \Delta B_{2i})K_j \right] x_i(t) + B_{1i}w(t) \right\},
\end{equation}

\begin{equation}
z(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \left\{ \left[(C_i + \Delta C_i) + (D_{2i} + \Delta D_{2i})K_j \right] x_i(t) + D_{1i}w(t) \right\}.
\end{equation}

**Theorem 6.** For system (15) with zero initial condition, if there exist a common nonsingular matrix $P \in \mathbb{R}^{n \times n}$ and positive constants $\delta, \varepsilon_i$ and $\varepsilon_{ij}$ such that the following matrices inequalities

\begin{equation}
E^T P = P^T E \geq 0,
\end{equation}

\begin{equation}
-S^T D_{11} - D_{11}^T S + \delta I - R < 0,
\end{equation}

\begin{equation}
\begin{pmatrix}
\Delta_1 & \Omega_1 & \Phi_1 & P^T H_{11} & \Psi_1 \\
* & \Sigma_1 & \Theta_1 & -S^T H_{2i} & 0 \\
* & * & -I & Q_{1i} H_{2i} & 0 \\
* & * & * & -\varepsilon_i I & 0 \\
* & * & * & * & -\varepsilon_i I
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\end{equation}

\begin{equation}
\begin{pmatrix}
\Delta_2 & \Omega_2 & \Phi_2 & P^T H_{1j} & P^T H_{1j} & \Psi_{21} & \Psi_{22} \\
* & \Sigma_2 & \Theta_2 & -S^T H_{2i} & -S^T H_{2j} & 0 & 0 \\
* & * & -2I & Q_{1i} H_{2i} & Q_{1j} H_{2j} & 0 & 0 \\
* & * & * & -\varepsilon_{ij} I & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_{ij} I & 0 & 0 \\
* & * & * & * & * & -\varepsilon_{ij} I & -\varepsilon_{ij} I
\end{pmatrix} < 0,
\end{equation}

where

\begin{align*}
\Delta_1 &= (A_i + B_{2i}K_i)^T P + P^T (A_i + B_{2i}K_i), \\
\Omega_1 &= P^T B_{1i} - (C_i + D_{2i}K_i)^T S, \\
\Phi_1 &= (C_i + D_{2i}K_i)^T Q_1, \\
\Psi_1 &= E_{1i}^T + K_i^T E_{2i}^T, \\
\Sigma_1 &= -S^T D_{11} - D_{11}^T S + \delta I - R, \\
\Theta_1 &= D_{1i}^T Q_1, \\
\Delta_2 &= (A_i + B_{2j}K_i)^T P + P^T (A_i + B_{2i}K_j) + (A_j + B_{2j}K_i)^T P + P^T (A_j + B_{2j}K_i), \\
\Omega_2 &= P^T B_{1j} - (C_i + D_{2j}K_j)^T S + P^T B_{1i} - (C_j + D_{2j}K_i)^T S, \\
\Phi_2 &= (C_j + D_{2j}K_j)^T Q_1 + (C_j + D_{2j}K_i)^T Q_1, \\
\Psi_{21} &= E_{1i}^T + K_i^T E_{2j}^T, \\
\Psi_{22} &= E_{1j}^T + K_j^T E_{2j}^T, \\
\Sigma_2 &= -S^T D_{11} - D_{11}^T S - S^T D_{1j} - D_{1j}^T S + 2\delta I - 2R, \\
\Theta_2 &= D_{1i}^T Q_1 + D_{1j}^T Q_1.
\end{align*}
then system (15) is admissible and strictly dissipative. $Q, S$ and $R$ are matrices with suitable dimension, $Q$ and $R$ are symmetrical.

**Proof.** When $w(t) = 0$, the system (15) is admissible by Theorem 1 and the matrix inequalities (16). Consider the nonnegative functional $V(x(t))$ in form $V(x(t)) = x^T(t)E^TPx(t)$. Along the trajectory of the closed-loop system (15), we have

$$
\dot{V}(x(t)) - z^T(t)Qz(t) - 2z^T(t)Sw(t) - w^T(t)Rw(t) + \delta w^T(t)w(t)
$$

where

$$
\dot{z} = \left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))[(A_i + \Delta A_i) + (B_{2i} + \Delta B_{2i})K_j]P + \\
P[(A_i + \Delta A_i) + (B_{2i} + \Delta B_{2i})K_j] \\
\sum_{i=1}^{r} h_i(\xi(t))P^TB_{1i}
\end{array} \right) + \\
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))[(C_i + \Delta C_i) + (D_{2i} + \Delta D_{2i})K_j]^TQ_1 \\
\sum_{i=1}^{r} h_i(\xi(t))D_{1i} \\
\sum_{i=1}^{r} h_i(\xi(t))[D_{1i}^T, S + S^TD_{1i}]
\end{array} \right) \times \\
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))[C_i + \Delta C_i] \\
\sum_{i=1}^{r} h_i(\xi(t))[C_i + \Delta C_i] \\
\sum_{i=1}^{r} h_i(\xi(t))[D_{1i}^T, S + S^TD_{1i}]
\end{array} \right) \\
0
$$

By the properties of standardization membership function, we can get the following equation

$$
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) \\
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) \\
\sum_{i=1}^{r} h_i(\xi(t))
\end{array} \right) + \\
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))[(A_i + \Delta A_i) + (B_{2i} + \Delta B_{2i})K_j]P + \\
P[(A_i + \Delta A_i) + (B_{2i} + \Delta B_{2i})K_j] \\
\sum_{i=1}^{r} h_i(\xi(t))P^TB_{1i}
\end{array} \right) + \\
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))[(C_i + \Delta C_i) + (D_{2i} + \Delta D_{2i})K_j]^TQ_1 \\
\sum_{i=1}^{r} h_i(\xi(t))D_{1i} \\
\sum_{i=1}^{r} h_i(\xi(t))[D_{1i}^T, S + S^TD_{1i}]
\end{array} \right) \times \\
\left( \begin{array}{c}
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))[(C_i + \Delta C_i)] \\
\sum_{i=1}^{r} h_i(\xi(t))[C_i + \Delta C_i] \\
\sum_{i=1}^{r} h_i(\xi(t))[D_{1i}^T, S + S^TD_{1i}]
\end{array} \right) \\
0
$$

$$
= \sum_{i=1}^{r} h_i^2(\xi(t))G_{ii} + \sum_{i<j} h_i(\xi(t))h_j(\xi(t))(G_{ij} + G_{ji}),
$$
Thus the above inequality is equivalent to the matrices inequality (16c) by using Schur complement.

By Schur complement and the above equation, the matrix inequality \( \tilde{\Xi} < 0 \) holds if \( G_{ii} < 0 \) (i = 1, 2, ..., r) and \( G_{ij} + G_{ji} < 0 \) (i < j ≤ r). Since

\[
G_{ii} = \begin{pmatrix}
A_i^T P + & P^T A_i + & C_i^T Q_1 + \\
K_i^T B_{2i} P + & C_i^T S - & K_i^T D_{21} S \\
P^T B_{2i} K_i & -D_{1i}^T S - S^T D_{1i} - R + \delta I & D_{1i}^T Q_1
\end{pmatrix} +
\begin{pmatrix}
P^T H_{1i} & -S H_{2i} & 0 \\
0 & Q_1 H_{2i}
\end{pmatrix} F_i \begin{pmatrix}
E_{1i} + E_{2i} K_i & 0 & 0
\end{pmatrix} +
\begin{pmatrix}
E_{1i}^T + K_i^T E_{2i}^T & 0
\end{pmatrix} F_i^T \begin{pmatrix}
H_{1i}^T P & -H_{2i}^T S & H_{2i}^T Q_1
\end{pmatrix}.
\]

By Lemma 1, the inequality \( G_{ii} < 0 \) holds if and only if there exists some constant \( \varepsilon_i \) such that

\[
\begin{pmatrix}
A_i^T P + & P^T A_i + & C_i^T Q_1 + \\
K_i^T B_{2i} P + & C_i^T S - & K_i^T D_{21} S \\
P^T B_{2i} K_i & -D_{1i}^T S - S^T D_{1i} - R + \delta I & D_{1i}^T Q_1
\end{pmatrix} +
\begin{pmatrix}
P^T H_{1i} & -S H_{2i} & 0 \\
0 & Q_1 H_{2i}
\end{pmatrix} \varepsilon_i \begin{pmatrix}
H_{1i}^T P & -H_{2i}^T S & H_{2i}^T Q_1
\end{pmatrix} +
\begin{pmatrix}
E_{1i}^T + K_i^T E_{2i}^T & 0
\end{pmatrix} \varepsilon_i^{-1} \begin{pmatrix}
E_{1i} + E_{2i} K_i & 0 & 0
\end{pmatrix} < 0.
\]

Thus the above inequality is equivalent to the matrices inequality (16c) by using Schur complement.
And the following equation holds:

\[
G_{ij} + G_{ji} = \begin{pmatrix}
A_i^T P + \\
P^T A_i + \\
K_i^T B_{2i} P + \\
P^T B_{2i} K_j \\
A_j^T P + \\
P^T A_j + \\
K_j^T B_{2j} P + \\
P^T B_{2j} K_i
\end{pmatrix}
\begin{pmatrix}
P^T B_{1i} - C_i^T S - \\
K_i^T D_{1j} S + \\
P^T B_{1j} - C_j^T S - \\
K_j^T D_{2j} S
\end{pmatrix}
\begin{pmatrix}
C_i^T Q_i + \\
K_i^T D_{2j} Q_i + \\
C_j^T Q_j + \\
K_j^T D_{2j} Q_j
\end{pmatrix}
\]

Thus, by using Schur complement, the above inequality is equivalent to the matrices

\[
\begin{pmatrix}
-P^T H_{1i} \\
-P^T H_{1j} \\
-S H_{2i} \\
-S H_{2j}
\end{pmatrix}
\begin{pmatrix}
F_i & 0 \\
0 & F_j
\end{pmatrix}
\begin{pmatrix}
E_{1i} + E_{2i} K_j \\
E_{1j} + E_{2j} K_i
\end{pmatrix} < 0
\]

By Lemma 1, the inequality \(G_{ij} + G_{ji} < 0\) holds if and only if there exists some constant \(\varepsilon_{ij}\) such that

\[
\begin{pmatrix}
A_i^T P + \\
P^T A_i + \\
K_i^T B_{2i} P + \\
P^T B_{2i} K_j \\
A_j^T P + \\
P^T A_j + \\
K_j^T B_{2j} P + \\
P^T B_{2j} K_i
\end{pmatrix}
\begin{pmatrix}
P^T B_{1i} - C_i^T S - \\
K_i^T D_{1j} S + \\
P^T B_{1j} - C_j^T S - \\
K_j^T D_{2j} S
\end{pmatrix}
\begin{pmatrix}
C_i^T Q_i + \\
K_i^T D_{2j} Q_i + \\
C_j^T Q_j + \\
K_j^T D_{2j} Q_j
\end{pmatrix}
\]

Thus, by using Schur complement, the above inequality is equivalent to the matrices inequality (16d).
Thus from equations (16) and (17), we have
\[
\dot{V}(x(t)) - z^T(t)Qz(t) - 2z^T(t)Sw(t) - w^T(t)Rw(t) + \delta w^T(t)w(t) \leq 0
\]
Note that \(V(0)=0\), it follows from the above inequality that
\[
V(x(t)) \leq \int_0^t [z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t)]dt - \delta \int_0^t w^T(t)w(t)dt
\]
That is, the system (15) is admissible and strictly dissipative.

**Theorem 7.** For system (15) with zero initial condition, if there exist a common nonsingular matrix \(X \in \mathbb{R}^{n \times n}\), matrices \(N_i\), positive constants \(\delta, \varepsilon_i, \varepsilon_{ij}\) such that the following matrix inequalities

\[
\begin{align*}
XE^T &= EX^T \geq 0, \\
-SD_{ii} - D_{ii}^T S + \delta I - R &< 0, \\
\begin{pmatrix}
\tilde{\Delta}_1 & \tilde{\Omega}_1 & \tilde{\Phi}_1 & \tilde{\Psi}_1 \\
\ast & \Sigma_1 & \Theta_1 & -\varepsilon_i H_{1i} & \varepsilon_i H_{1j} & \varepsilon_{ij} H_{1j} \\
\ast & \ast & -I & \varepsilon_i Q_1 H_{2i} & 0 & 0 \\
\ast & \ast & \ast & \ast & \varepsilon_i Q_1 H_{2j} & 0 \\
\ast & \ast & \ast & \ast & \ast & \varepsilon_i I \\
\ast & \ast & \ast & \ast & \ast & \ast & \varepsilon_i I
\end{pmatrix} &< 0, \quad i = 1, 2, \ldots, r, \\
\begin{pmatrix}
\tilde{\Delta}_2 & \tilde{\Omega}_2 & \tilde{\Phi}_2 & \tilde{\Psi}_{21} & \tilde{\Psi}_{22} \\
\ast & \Sigma_2 & \Theta_2 & -\varepsilon_{ij} S^T H_{2i} & -\varepsilon_{ij} S^T H_{2j} & 0 & 0 \\
\ast & \ast & -2I & \varepsilon_i Q_1 H_{2i} & 0 & 0 & 0 \\
\ast & \ast & \ast & \varepsilon_i Q_1 H_{2j} & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -\varepsilon_{ij} I & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \varepsilon_{ij} I \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \varepsilon_{ij} I
\end{pmatrix} &< 0, \quad i < j \leq r,
\end{align*}
\]

where
\[
\begin{align*}
\tilde{\Delta}_1 &= XA_i^T + A_i X^T + N_i^T B_{2i}^T + B_{2i} N_i, \\
\tilde{\Omega}_1 &= B_{1i} - XC_i^T S - N_i^T D_{2i}^T S, \\
\tilde{\Phi}_1 &= X C_i^T Q_1 + N_i^T D_{2i}^T Q_1, \\
\tilde{\Psi}_1 &= X E_i^T + N_i^T E_{2i}, \\
\Sigma_1 &= -S^T D_{1i} - D_{1i}^T S + \delta I - R, \\
\Theta_1 &= D_{1i}^T Q_1, \\
\tilde{\Delta}_2 &= XA_i^T + A_i X^T + N_i^T B_{2j}^T + B_{2j} N_i + XA_j^T + A_j X^T + N_j^T B_{2j}^T + B_{2j} N_i, \\
\tilde{\Omega}_2 &= B_{1i} - XC_i^T S - N_j^T D_{2i}^T S + B_{1j} - XC_j^T S - N_j^T D_{2j}^T S, \\
\tilde{\Phi}_2 &= X C_j^T Q_1 + N_j^T D_{2j}^T Q_1 + X C_j^T Q_1 + N_j^T D_{2j}^T Q_1, \\
\tilde{\Psi}_{21} &= X E_i^T + N_j^T E_{2i}, \\
\tilde{\Psi}_{22} &= X E_j^T + N_j^T E_{2j}, \\
\Sigma_2 &= -S^T D_{1i} - D_{1i}^T S + 2\delta I - 2R - S^T D_{1j} - D_{1j}^T S, \\
\Theta_2 &= D_{1i}^T Q_1 + D_{1j}^T Q_1,
\end{align*}
\]
then the system (15) is admissible and strictly dissipative. The dissipative controller is
\[ u(t) = \sum_{i=1}^{r} h_i(\xi(t))N_iX^{-\top}x(t), \]
where \( X = P^{-\top}N_i = K_iX^\top \).

\( Q, S \) and \( R \) are matrices with suitable dimension, \( Q \) and \( R \) are symmetrical.

**Proof.** Pre-multiplying and Post-multiplying both sides of (16c) by \( \text{diag}(P^{-\top}, I, I, \varepsilon, I, I, I, I) \) and \( \text{diag}(P^{-1}, I, I, I, I, I, I, I) \), Pre-multiplying and Post-multiplying both sides of (16d) by \( \text{diag}(P^{-\top}, I, I, \varepsilon, I, \varepsilon, I, I, I) \) and \( \text{diag}(P^{-1}, I, I, I, \varepsilon, I, \varepsilon, I, I, I, I) \), we can obtain the matrices inequalities (18c) and (18d).

We decompose the non-singular matrix \( P \) and the matrix \( X = P^{-1} \), as described above. We can obtain the design method of the dissipative controller by the linear matrix inequality in the following.

**Theorem 8.** For the system (15) with the zero initial condition, if there exist a common nonsingular matrix \( P \in \mathbb{R}^{n \times n} \), matrices \( N_i, \) positive constants \( \delta, \varepsilon_i, \varepsilon_{ij} \) such that the matrix inequalities (18b),(18c),(18d) then the system (15) is admissible and strictly dissipative. The dissipative controller is
\[ u(t) = \sum_{i=1}^{r} h_i(\xi(t))N_iX^{-\top}x(t), \]
where \( X = P^{-1} \), \( X_1 = P_1^{-1} > 0, \) \( N_i = K_iX^\top \).

5.2. Robust \( H_\infty \) control and passive control. Two Corollaries of Theorem 6 in the following are the sufficient conditions to check robust \( H_\infty \) stable and the strict passive separately for uncertain T-S fuzzy descriptor system. Therefore robust \( H_\infty \) controller and the passive controller can be designed according to the similar method in theorem 8.

**Corollary 3.** For system (15) with zero initial condition, if there exist a common nonsingular matrix \( P \in \mathbb{R}^{n \times n} \) and a positive constant \( \delta \) such that the following matrix inequalities
\[ E^TP = P^TE \geq 0, \]
\[ -D_1 - D_1^T + \delta I < 0, \]
\[ \begin{pmatrix} \triangle_1 & \breve{\Omega}_1 & P^TH_{11} & \Psi_1 \\ * & \breve{\Lambda}_1 & -H_{21} & 0 \\ * & * & -\varepsilon_i^{-1}I & 0 \\ * & * & * & -\varepsilon_i^{-1}I \end{pmatrix} < 0, \]
\[ \begin{pmatrix} \triangle_2 & \breve{\Omega}_2 & P^TH_{11} & P^TH_{1j} & \Psi_{21} & \Psi_{22} \\ * & \breve{\Lambda}_2 & -H_{21} & -H_{2j} & 0 & 0 \\ * & * & -\varepsilon_i^{-1}I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{ij}I & 0 & 0 \\ * & * & * & * & -\varepsilon_{ij}I & 0 \\ * & * & * & * & * & -\varepsilon_{ij}I \end{pmatrix} < 0, \]
\[ i < j \leq r, \]
where
\[
\begin{align*}
\Delta_1 &= (A_i + B_{2i}K_i)^T P + P^T(A_i + B_{2i}K_i), \\
\hat{\Omega}_1 &= P^T B_{1i} - (C_i + D_{2i}K_i)^T, \\
\Psi_1 &= E_{1i}^T + K_i^T E_{2i}, \\
\hat{\Sigma}_1 &= -D_{1i} - D_{1i}^T + \delta I, \\
\Delta_2 &= (A_i + B_{2i}K_j)^T P + P^T(A_i + B_{2i}K_j) + \\
&\quad (A_j + B_{2j}K_i)^T P + P^T(A_j + B_{2j}K_i), \\
\hat{\Omega}_2 &= P^T B_{1i} - (C_i + D_{2i}K_j)^T + P^T B_{1j} - (C_j + D_{2j}K_i)^T, \\
\hat{\Sigma}_2 &= -D_{1i} - D_{1j} - D_{1j}^T + 2\delta I, \\
\Psi_{21} &= E_{1i}^T + K_i^T E_{2j}, \\
\Psi_{22} &= E_{1j}^T + K_j^T E_{2j},
\end{align*}
\]

then the system (15) is admissible and strictly passive.

Let \( Q = 0, R = 0, S = I \), we can get Corollary 3 from Theorem 6.

**Corollary 4.** For the system (15) with the zero initial condition, if there exist a common nonsingular matrix \( P \in R^{n \times n} \) and a positive constant \( \gamma \) such that the following matrix inequalities

\[
E^T P = P^T E \succeq 0,
\]

\[
\begin{pmatrix}
\Delta_1 & \hat{\Omega}_1 & \hat{\Phi}_1 & P^T H_{1i} & \Psi_1 \\
* & -\gamma^2 I & D_{1i}^T & 0 & 0 \\
* & * & -I & H_{2i} & 0 \\
* & * & * & -\varepsilon_i^{-1} I & 0 \\
* & * & * & * & -\varepsilon_i I
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\]

\[
\begin{pmatrix}
\Delta_2 & \hat{\Omega}_2 & \hat{\Phi}_2 & P^T H_{1i} & P^T H_{1j} & \Psi_{21} & \Psi_{22} \\
* & -2\gamma^2 I & D_{1i}^T + D_{1j}^T & 0 & 0 & 0 & 0 \\
* & * & -2I & H_{1i} & H_{2j} & 0 & 0 \\
* & * & * & -\varepsilon_{ij}^{-1} I & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_{ij}^{-1} I & 0 & 0 \\
* & * & * & * & * & -\varepsilon_{ij} I & 0 \\
* & * & * & * & * & * & -\varepsilon_{ij} I
\end{pmatrix} < 0,
\]

\[
i < j \leq r,
\]

where

\[
\begin{align*}
\Delta_1 &= (A_i + B_{2i}K_i)^T P + P^T(A_i + B_{2i}K_i), \\
\hat{\Omega}_1 &= P^T B_{1i}, \\
\hat{\Phi}_1 &= (C_i + D_{2i}K_i)^T, \\
\Psi_1 &= E_{1i}^T + K_i^T E_{2i}, \\
\Delta_2 &= (A_i + B_{2i}K_j)^T P + P^T(A_i + B_{2i}K_j) + \\
&\quad (A_j + B_{2j}K_i)^T P + P^T(A_j + B_{2j}K_i), \\
\hat{\Omega}_2 &= P^T B_{1i} + P^T B_{1j}, \\
\hat{\Phi}_2 &= (C_i + D_{2i}K_j)^T + (C_j + D_{2j}K_i)^T, \\
\Psi_{21} &= E_{1i}^T + K_j^T E_{2i}, \\
\Psi_{22} &= E_{1j}^T + K_i^T E_{2j},
\end{align*}
\]
then the system (15) is robust $H_\infty$ stable.

Let $Q = -I, R = \gamma^2, S = 0$, we can get Corollary 2 from Theorem 6 and the properties of linear matrix inequalities.

5.3. Numerical Example. In this section, we still considered the control system (11) with controlled variable (13), the uncertainty matrix parameters are defined as follows

\[
\begin{align*}
H_{11} &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, H_{12} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix}, H_{21} = \begin{pmatrix} 1 \\ 1.1 \end{pmatrix}, \quad H_{22} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\
E_{11} &= \begin{pmatrix} 0 & 9 & 0 & 1 \end{pmatrix}, E_{12} = \begin{pmatrix} 1 & 0 & 1 & 1 \end{pmatrix}, \\
E_{21} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, E_{22} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\end{align*}
\]

We can obtain the following dissipative controller, which achieve $H_\infty$ norm or passive degree when the need arises

\[
u(t) = \begin{cases}
\frac{x_1^2(t)}{2} \begin{bmatrix} -676.6038 & 254.8724 & 0.5976 \\
+ (1 - \frac{x_1^2(t)}{2}) \begin{bmatrix} -724.0655 & 272.9710 & 0.6362 
\end{cases}
\end{cases}
\]

and robust $H_\infty$ controller

\[
u(t) = \begin{cases}
\frac{x_1^2(t)}{2} \begin{bmatrix} -67.4656 & 29.3005 & 0.5087 \\
+ (1 - \frac{x_1^2(t)}{2}) \begin{bmatrix} -84.8548 & 37.1543 & 0.6310 
\end{cases}
\end{cases}
\]

and passive controller

\[
u(t) = \begin{cases}
\frac{x_1^2(t)}{2} \begin{bmatrix} -41.6189 & 19.2736 & 0.2991 \\
+ (1 - \frac{x_1^2(t)}{2}) \begin{bmatrix} -65.1124 & 30.6429 & 0.4469 
\end{cases}
\end{cases}
\]

6. Conclusion

This paper discussed the problem of robust dissipative control for a kind of uncertain T-S fuzzy descriptor system. The proposed dissipative control approaches successfully unify the existing results on $H_\infty$ control and passive control. Thus the method can potentially cut down the cost and time in actual physical system design.

References


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