STABILITY OF NETWORKED CONTROL SYSTEMS WITH TIME-VARYING SAMPLING PERIODS AND PARTIALLY KNOWN TRANSITION PROBABILITIES

LI YUAN, ZHANG QINGLING, AND QIU ZHANZHI

(Communicated by Hilbert)

Abstract. This paper studies the stability and stabilization problems for networked control systems with partially known transition probabilities by using time-varying sampling period method with both packet losses and time-varying time delay where the main focus is the packet-loss issue, packet-loss process is the Markovian packet-loss process and the transition probabilities do not need completely known. The new time-varying time delay definition is given in this paper. A developed packet-loss dependent Lyapunov functional is used to obtain the stability criteria. The sufficient conditions for stochastic stability are derived via linear matrix inequalities (LMIs) formulation and mode-dependent controller design is also presented correspondingly. Finally, the numerical example and simulations have demonstrated the effectiveness of our result.

Key Words. Networked control systems, time-varying sampling periods; partially known transition probabilities, Markovian packet-loss process.

1. Introduction

Networked control systems (NCSs) are feedback control systems whose feedback paths are implemented by a real-time network. Recently, much attention has been paid to the study of stability and controller design of networked control systems, due to their low cost, reduced weight and power requirements, simple installation and maintenance, high reliability, and so on [1-2]. A basic problem in NCSs is the stability of the systems. In real-time control systems, time delay and packet losses will degrade the performance of control systems and even make systems unstable, so it is significant to overcome the adverse influences of time delay and packet losses [3-8].

In NCSs, constant sampling period is usually adopted, if constant sampling period is adopted, sampling period should be large enough to avoid network congestion when the network is occupied by the most users, so network bandwidth cannot be sufficiently used when the network is idle. Recently, there have been a number of papers considering the problem of time-varying sampling periods of control systems [9-13]. In the work of [9,10], the stability problem of digital feedback control systems with time-varying sampling periods was discussed. In [11], the authors presented an interval model of networked control systems with time-varying sampling periods and time-varying time delay and discussed the problem of stability of networked
control systems. In [12], the stochastic stability of NCSs with time-varying sampling periods and time delay driven by two Markov chains was discussed. [13] studied the problem of designing $H_\infty$ controllers for networked control systems with both time delay and packet losses by using active varying sampling period method. However, the above papers did not study the randomness of packet-loss process.

On the other hand, the Markov chain, a discrete-time stochastic process with the Markov property, can be effectively used to model NCSs with time delay and packet losses, which are modeled as the Markovian jump systems. Recently, there have been considerable research efforts on NCSs (see [14]-[19]). For example, [14] studied the stochastic stability of networked control systems with the presence of time delay and transmitted data dropouts, in which the sequence of transmission interval was driven by a finite state Markov chain. The stability properties of networked linear systems with Markovian packet losses was discussed in [18], in which the packet-loss process of the network was characterized by a binary Markov chain.

In the above references, the transition probabilities are assumed completely accessible and considered as the available knowledge for analyzing and designing the NCSs. In practice, this kind of information including the variation of time delay and packet losses is hard to obtain. The problems of partly unknown transition probabilities were investigated (see [20-22]). The stability of markovian jump linear systems with partly unknown transition probabilities was investigated in [20,21], [22] studied the design problem of networked control systems with partly unknown transition probabilities and [22] did not consider time-varying sampling periods and time-varying time delay.

In this paper, we focus on the problem of stabilization for NCSs with time-varying sampling periods and partially known transition probabilities. Firstly, a new networked control system model with time-varying sampling periods is proposed, in which both time-varying time delay and packet losses are taken into account. The new time-varying time delay definition is given in this paper. The packet-loss process is defined as the sequence of the time intervals between consecutively successfully transmitted data and the packet-loss process is modeled as a discrete-time Markov chain with partially known transition probabilities, which is more general in practical situation. Secondly, according to a developed packet-loss dependent Lyapunov functional, the stability criteria is obtained. Furthermore, the mode-dependent controllers can be designed by solving a set of LMIs. Finally, a numerical example is presented to illustrate the effectiveness and potential of the developed theoretical results.

Notation: The notation used in this paper is standard. The superscript "T" stands for matrix transposition; $\text{eig}(\ldots)$ denotes matrix eigenvalue and $\text{diag}(\ldots)$ stands for a block-diagonal matrix; the symmetric terms in a symmetric matrix are denoted by $\ast$; $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space; the notation $P > 0$ ($\geq 0$) means that $P$ is a real symmetric positive (semi-positive) definite matrix, and $M_i$ is adopted to denote $M(i)$ for brevity; $I$ and $0$ represent, respectively, the identity matrix and zero matrix.

2. Preliminaries and problem statement

The structure of the considered NCSs is shown in Fig.1, where the plant is described by the following continuous-time linear system model

\begin{equation}
\dot{x}(t) = Ax(t) + Bu(t)
\end{equation}

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and control input vector, respectively; $A, B$ are known constant matrices. Throughout this paper, matrices,
if not explicitly stated, are assumed to have appropriate dimensions. Suppose the controller and the actuator are event-driven, the sensor is both clock-driven and event-driven.

Let $\vartheta = \{k_1, k_2, \cdots \}$, a subsequence of $\{1, 2, 3, \cdots \}$, denote the sequence of time points of successful data transmissions from the sensor to the actuator, and $s = \max_{k_j \in \vartheta}(k_{j+1} - k_j - 1)$ be the maximum packet-loss upper bound.

**Definition 1**
Packet-loss process is defined as $\{\eta(k_j) = k_{j+1} - k_j - 1; k_j \in \vartheta\}$, which takes values in the finite set $\chi = \{0, 1, 2, \ldots, s\}$.

For NCSs, the shorter the sampling period, the better the system performance, however, the short sampling period will increase the possibility of network congestion. If the constant sampling period is adopted, the sampling period should be large enough to avoid network congestion, so network bandwidth can not be sufficiently used when the network is idle. In the following, we will choose the time-varying sampling period method to make networked control linear systems model with time-varying time delay and finite data packet losses by using an iterative approach[23]. Thus, not only the network bandwidth can be made full use, but also the analysis and design of NCSs with time delay and packet losses can be simplified.

Suppose $t_k$ is the $k$th sampling instant, $h_k$ is the length of the $k$th sampling period, the instant that the control input $u_k$ reaches actuator is $\hat{k}$, the first sampling instant after $t_k$ is $\hat{k}$. When the network is idle, denote the smallest sampling period as $h_{\text{min}}$, and when the network is occupied by the most users, define the biggest sampling period as $h_{\text{max}}$.

Partition $[h_{\text{min}}, h_{\text{max}}]$ into $l$ equidistant small intervals ( $l$ is a positive integer), then the next sampling instant $\hat{k}$ can be chosen as

$$
\hat{k} = \begin{cases} 
    d_1 & \hat{k} \in [d_1, d_2) \\
    t_k + h_{\text{max}} & \hat{k} \geq t_k + h_{\text{max}}
\end{cases}
$$

where

$$
d_1 = t_k + h_{\text{min}} + a(h_{\text{max}} - h_{\text{min}})/l, \\
d_2 = t_k + h_{\text{min}} + (a + 1)(h_{\text{max}} - h_{\text{min}})/l, \\
a = 0, 1, \ldots, l - 1.
$$
Then
\[ h_k = \hat{k} - t_k = h_{\text{min}} + b(h_{\text{max}} - h_{\text{min}})/l, \quad b = 0, 1, \ldots, l \]
that is the sampling period \( h_k \) switches in the finite set
\[ \phi = \{ h_{\text{min}}, h_{\text{min}} + (h_{\text{max}} - h_{\text{min}})/l, \ldots, h_{\text{max}} \} . \]

**Remark 1**
The sampling period of the considered networked control system (1) is time-varying. However, the sampling period of the networked control system considered in [23] is constant. The stability analysis and controller synthesis for networked control system with time-varying sampling periods is very important in both theory and applications, and it is also a very challenging problem.

**Definition 2**
Time delay is defined as
\[ \tau_k = \begin{cases} \hat{k} - \tilde{k} & \hat{k} \in [d_1, d_2) \\ 0 & \hat{k} \geq t_k + h_{\text{max}} \end{cases}, \]
which satisfies 0 ≤ \( \tau_k \) ≤ \( h_k \).

**Remark 2**
As shown in Fig. 2, the control inputs \( u_{k+2} \) and \( u_{k+3} \) do not reach the actuator at the instant \( t_{k+1} + h_{\text{max}} \) and \( t_{k+2} + h_{\text{max}} \), respectively, so the sensor will sample at the instant \( t_{k+1} + h_{\text{max}} \) and \( t_{k+2} + h_{\text{max}} \) and the time delays \( \tau_{k+2} = \tau_{k+3} = 0 \).

**Remark 3**
If \( \hat{k} > t_k + h_{\text{max}} \), the control input \( u_k \) will not be used even if it reaches the actuator eventually, and the latest available control input will be used. The sampling begins only when the former sampled packet reaches or is dropped, so packet disordering cannot occur.

The timing diagram of the considered NCSs with both time delay and packet losses is shown in Fig. 2, in which the two control signals \( u(k+2) \) and \( u(k+3) \) shown in dashed lines are lost in the instants \( t_{k+2} \) and \( t_{k+3} \), respectively. It can be seen from the timing diagram that the control inputs acting on the plant are different from sampling interval to sampling interval, and thus the system models of
the NCSs vary from one sampling interval to another as the packet-loss situations change. So the discrete time representation of (1) can be described as follows:

(i) There is no packet-loss within both the current sampling instant and the previous one, such as the instant \( t_{k+1} \).

\[
x(k+1) = \Phi_k x(k) + \Gamma_{1k} u(k) + \Gamma_{2k} u(k-1)
\]

(ii) There is one packet-loss within the current sampling instant, such as the instant \( t_{k+2} \).

\[
x(k+1) = \Phi_k x(k) + \Gamma_{1b} u(k) + \Gamma_{2b} u(k-1)
\]

(iii) There are \( \eta(k_j) \) successive packet-losses, and \( 2 \leq \eta(k_j) \leq s \), such as the instant \( t_{k+3} \), where \( \eta(k_j) = 2 \).

\[
x(k+1) = \Phi_k x(k) + \Phi_b x(k) + \Gamma_{b} u(k - \eta(k_j)) + 1)
\]

(iv) There is no packet-loss within the current sampling instant, but there are \( \eta(k_j) \) successive packet-losses in the last sampling instant \( 1 \leq \eta(k_j) \leq s \), such as the instant \( t_{k+4} \).

\[
x(k+1) = \Phi_k x(k) + \Gamma_{1k} u(k) + \Gamma_{2k} u(k - \eta(k_j))
\]

where

\[
\Phi_k = e^{Ah_b}, \Phi_b = e^{Ah_{max}}, \Gamma_{1b} = \int_{0}^{h_{max}-\tau_k} e^{As} ds, \Gamma_{2b} = \int_{0}^{h_{max}} e^{As} ds,
\]

\[
\Gamma_{1k} = \int_{h_k-\tau_k}^{h_k} e^{As} ds, \Gamma_{2k} = \int_{h_k}^{h_{max}} e^{As} ds, \Gamma_{b} = \int_{0}^{h_{max}} e^{As} ds.
\]

Suppose that the successive successfully transmitted instants of \( k \) are \( 0 = k_0 < k_1 < \ldots < k_j < \ldots \), then the closed loop system of the NCSs may be described by

\[
x(k_1) = \Phi_b^{k_1-\eta_0} \Phi_{b0} x(k_0) + (\Phi_b^{k_1-\eta_0} - 2) \Phi_{b0} \Gamma_{1b} u(k_0) + \ldots + \Phi_{b0} \Gamma_{b} u(k_{j0}) + (\Phi_b^{k_1-k_0-k_1} \Phi_{b1} \Gamma_{1b} + \Phi_b^{k_1-k_0-k_1} \Phi_{b1} \Gamma_{b} + \ldots + \Phi_{b0} \Gamma_{b} u(k_0))
\]

\[
x(k_2) = \Phi_b^{k_2-k_1} \Phi_{b1} x(k_1) + (\Phi_b^{k_2-k_1} - 2) \Phi_{b1} \Gamma_{1b} + (\Phi_b^{k_2-k_1} \Phi_{b1} \Gamma_{1b} + \Phi_b^{k_2-k_1-k_1} \Phi_{b1} \Gamma_{b} + \ldots + \Phi_{b0} \Gamma_{b} u(k_0))
\]

\[
\vdots
\]

\[
x(k_{j+1}) = \Phi_b^{k_{j+1}-k_{j+1}} \Phi_{b_{j+1}} x(k_j) + (\Phi_b^{k_{j+1}-k_{j+1}} - 2) \Phi_{b_{j+1}} \Gamma_{1b} + \Phi_b^{k_{j+1}-k_{j+1}} \Phi_{b_{j+1}} \Gamma_{2b} + (\Phi_b^{k_{j+1}-k_{j+1}} \Phi_{b_{j+1}} \Gamma_{1b} + \ldots + \Phi_{b_{j+1}} \Gamma_{1b} u(k_{j-1}))
\]

where

\[
\Phi_{b_j} = e^{Ah_{b_j}}, \Phi_b = e^{Ah_{max}}, \Gamma_{1b} = \int_{0}^{h_{max}-\tau_j} e^{As} ds, \Gamma_{2b} = \int_{0}^{h_{max}} e^{As} ds,
\]

\[
\Gamma_{1k} = \int_{h_k-\tau_j}^{h_k} e^{As} ds, \Gamma_{2k} = \int_{h_k}^{h_{max}} e^{As} ds, \Gamma_{b} = \int_{0}^{h_{max}} e^{As} ds.
\]

\[
A(\eta(k_j)) = \Phi_{b}^{\eta(k_j)} \Phi_{b_j}, B_2(\eta(k_j)) = \Phi_{b}^{\eta(k_j)-1} \Phi_{b_j} \Gamma_{2b} + \Gamma_{2b},
\]

\[
B_1(\eta(k_j)) = \Phi_{b}^{\eta(k_j)-1} \Phi_{b_j} \Gamma_{1b} + \ldots + \Phi_{b}^{\eta(k_j)-1} \Phi_{b_j} \Gamma_{b} + \Gamma_{1b},
\]

then

\[
x(k_{j+1}) = A(\eta(k_j)) x(k_j) + B_1(\eta(k_j)) u(k_j) + B_2(\eta(k_j)) u(k_{j-1})
\]
In this paper, we suppose packet-loss process be a discrete-time homogeneous Markov chain on a complete probability space \((\Omega, F, P)\) which takes values in \(\chi\) with the following mode transition probabilities matrix 

\[
\Pi = (\pi_{\alpha\beta}) \in \mathbb{R}^{(s+1) \times (s+1)}
\]

where \(\pi_{\alpha\beta} \geq 0, \forall \alpha, \beta \in \chi\), and \(\sum_{\beta=0}^{s} \pi_{\alpha\beta} = 1\). Furthermore, the transition probabilities matrix is defined by

\[
\pi = \begin{pmatrix}
\pi_{00} & \pi_{01} & \cdots & \pi_{0s} \\
\pi_{10} & \pi_{11} & \cdots & \pi_{1s} \\
\vdots & \vdots & \ddots & \vdots \\
\pi_{s0} & \pi_{s1} & \cdots & \pi_{ss}
\end{pmatrix}
\]

For \(\eta(k) = \alpha \in \chi\), the system matrices of the \(\alpha\)th mode are denoted by \((A_\alpha, B_{1\alpha}, B_{2\alpha})\), which are assumed known.

In addition, the transition probabilities of the Markov chain in this paper are considered to be partially available, that is, some elements in matrix \(\pi\) are unknown. For example, the system described by (6) with five modes will have the transition probabilities matrix \(\pi\) as

\[
\pi = \begin{pmatrix}
\pi_{00} & ? & ? & \pi_{03} & \pi_{04} \\
? & \pi_{11} & \pi_{12} & ? & \pi_{14} \\
? & \pi_{21} & \pi_{22} & ? & \pi_{24} \\
\pi_{30} & ? & \pi_{32} & ? & \pi_{34} \\
\pi_{40} & \pi_{41} & ? & ? & \pi_{44}
\end{pmatrix}
\]

where ? represents the unavailable elements. For notation clarity, \(\alpha \in \chi\), we denote

\[
\chi^\alpha_K = \{\beta : \text{if } \pi_{\alpha\beta} \text{ is known}\}, \chi^\alpha_U = \{\beta : \text{if } \pi_{\alpha\beta} \text{ is unknown}\}.
\]

Moreover, if \(\chi^\alpha_K \neq \emptyset\), it is further described as

\[
\chi^\alpha_K = \{K_1^\alpha, K_2^\alpha, \cdots, K_m^\alpha\}, 0 \leq m \leq s
\]

where \(K_m^\alpha \in \mathbb{N}^+\) represents the \(m\)th known element with the index \(K_m^\alpha\) in the \(\alpha\)th row of matrix \(\pi\). Also, we denote \(\pi^\alpha = \sum_{\beta \in \chi^\alpha_K} \pi_{\alpha\beta}\) throughout the paper.

**Remark 4**

The jumping process \(\eta(k)\) with partially unknown transition probabilities (9) was first introduced for regular state-space Markovian systems [20,21], and it covers the jumping process of completely available \((\chi^\alpha_U = \emptyset, \chi^\alpha_K = \chi)\) or completely unavailable \((\chi^\alpha_K = \emptyset, \chi^\alpha_U = \chi)\). Moreover, in contrast with the uncertain transition probabilities studied recently, see for example [17,24-25], no structure (polytopic ones), bounds (norm-bounded ones) or “nominal” terms (both) are required for the partially unknown elements in the transition probability matrix. Therefore, our transition probabilities considered here is more natural and reasonable.

Now, for a more precise description of this paper, we introduce the following definition for the underlying system.

**Definition 3**[22]

The system described by (6) is said to be stochastically stable if for every initial
condition \( x(k_0) \) and \( \eta(k_0) \in \chi \), the following holds
\[
E\left( \sum_{j=0}^{\infty} \|x(k_j)\|^2 | x(k_0), \eta(k_0) \right) < \infty
\]

Then, the stochastic stability problem to be discussed is formulated as follows: design a state-feedback stabilizing controller such that the resulting closed-loop system is stochastically stable. A mode-dependent controller is considered here with the form
\[
(11) \quad u(k_j) = K_\alpha x(k_j)
\]
where \( K_\alpha(\eta(k_j) = \alpha \in \chi) \) is the controller gain to be determined. Substituting the formula (11) into (6), we can obtain the following closed-loop NCSs
\[
(12) \quad x(k_{j+1}) = \tilde{A}_\alpha x(k_j) + \tilde{B}_\alpha x(k_{j-1})
\]
where
\[
\tilde{A}_\alpha = \Phi_b^\alpha \Phi_b + (\Phi_b^{\alpha-1} \Phi_b \Gamma_{1b} + \cdots + \Phi_b \Gamma_{b} + \Gamma_{1b}) K_\alpha,
\]
\[
\tilde{B}_\alpha = (\Phi_b^{\alpha-1} \Phi_b \Gamma_{2b} + \Gamma_{2b}) K_{\alpha-1}.
\]

3. Main results

In this section, we will first develop the stability criterion for the system described by (6) with partially known transition probabilities, and further give the corresponding controller design. The following theorem gives the new stability criterion for the system described by (12) with partially known transition probabilities.

**Theorem 1**

Consider the system described by (12) with partially known transition probabilities (8). The corresponding system is stochastically stable if there exist matrices \( P_\alpha > 0, \alpha \in \chi, Q > 0, Z > 0 \) and matrices \( N_1, N_2 \) such that the LMIs (13) and (14) hold for every feasible values of \( \eta(k_j) \) and \( h_{k_j}(\eta(k_j) \in \chi, h_{k_j} \in \phi) \).

\[
(13) \begin{bmatrix}
-P_\alpha & 0 & P_\alpha \tilde{A}_\alpha & P_\alpha \tilde{B}_\alpha & 0 \\
* & -\pi_\alpha^\alpha Z & \pi_\alpha^\alpha Z (\tilde{A}_\alpha - I) & \pi_\alpha^\alpha Z \tilde{B}_\alpha & 0 \\
* & * & \pi_\alpha^\alpha \Psi_{\alpha 1} & \pi_\alpha^\alpha \Psi_{\alpha 2} & \pi_\alpha^\alpha N_1 \\
* & * & * & \pi_\alpha^\beta \Psi_{\alpha 3} & \pi_\alpha^\beta N_2 \\
* & * & * & * & -\pi_\rho^\beta Z
\end{bmatrix} < 0, \quad \beta \in \chi^\alpha
\]

\[
(14) \begin{bmatrix}
-P_\beta & 0 & P_\beta \tilde{A}_\alpha & P_\beta \tilde{B}_\alpha & 0 \\
* & -Z & Z (\tilde{A}_\alpha - I) & Z \tilde{B}_\alpha & 0 \\
* & * & \Psi_{\alpha 1} & \Psi_{\alpha 2} & N_1 \\
* & * & * & \Psi_{\alpha 3} & N_2 \\
* & * & * & * & -Z
\end{bmatrix} < 0, \quad \beta \in \chi^\alpha
\]

where
\[
\Psi_{\alpha 1} = -P_\alpha + Q + N_1 + N_2^T \]
\[
\Psi_{\alpha 2} = -N_1 + N_2^T \]
\[
\Psi_{\alpha 3} = -Q - N_2 - N_2^T
\]

with \( P_\alpha = \sum_{\beta \in \chi^\alpha} \pi_{\alpha \beta} P_\beta \).

**Proof**
Consider the system described by (12) and construct a stochastic Lyapunov functional as

\[ V(x(k_j), \eta(k_j)) = \sum_{i=1}^{3} V_i(x(k_j), \eta(k_j)) \]

where \( \forall \eta(k_j) = \alpha \in \chi \)

\begin{align*}
V_1(x(k_j), \eta(k_j)) &= x^T(k_j) P_\alpha x(k_j), \\
V_2(x(k_j), \eta(k_j)) &= x^T(k_{j-1}) Q x(k_{j-1}), \\
V_3(x(k_j), \eta(k_j)) &= \zeta^T(k_{j-1}) Z \zeta(k_{j-1}),
\end{align*}

with \( \zeta(k_j) = x(k_{j+1}) - x(k_j) \) and \( P_\alpha, Q, Z \) satisfying (13). Then, for \( \eta(k_j) = \alpha, \eta(k_{j+1}) = \beta \), we denote \( \Delta V(x(k_j), \eta(k_j)) = \sum_{i=1}^{3} \Delta V_i \), where

\begin{align*}
\Delta V_1 &= E[V_1(x(k_{j+1}), \eta(k_{j+1}) | x(k_j), \eta(k_j))] - V_1(x(k_j), \eta(k_j)) \\
&= x^T(k_{j+1}) \sum_{\beta \in \chi} \pi_{\alpha \beta} P_\beta x(k_{j+1}) - x^T(k_j) P_\alpha x(k_j) \\
&= x^T(k_{j+1}) \left( \sum_{\beta \in \chi} \pi_{\alpha \beta} P_\beta + \sum_{\beta \in \chi^{\alpha}} \pi_{\alpha \beta} P_\beta \right) x(k_{j+1}) - x^T(k_j) P_\alpha x(k_j) \\
\Delta V_2 &= E[V_2(x(k_{j+1}), \eta(k_{j+1}) | x(k_j), \eta(k_j))] - V_2(x(k_j), \eta(k_j)) \\
&= x^T(k_j) Q x(k_j) - x^T(k_{j-1}) Q x(k_{j-1}) \\
\Delta V_3 &= E[V_3(x(k_{j+1}), \eta(k_{j+1}) | x(k_j), \eta(k_j))] - V_3(x(k_j), \eta(k_j)) \\
&= \zeta^T(k_{j+1}) Z \zeta(k_j) - \zeta^T(k_{j-1}) Z \zeta(k_{j-1}) \\
&= \xi^T(k_j) \Theta \xi(k_j) - \xi^T(k_{j-1}) \Theta \xi(k_{j-1})
\end{align*}

where

\[ \Theta = \begin{bmatrix} (\tilde{A}_\alpha - I)^T Z (\tilde{A}_\alpha - I) & (\tilde{A}_\alpha - I)^T Z \tilde{B}_\alpha \\ \ast & \tilde{B}_\alpha^T Z \tilde{B}_\alpha \end{bmatrix}. \]

Note that for any \( Z > 0 \) and \( N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \), we have

\[-\zeta^T(k_{j-1}) Z \zeta(k_{j-1}) \leq \xi^T(k_j) N^{-1} N^T \xi(k_j) + 2 \xi^T(k_j) N_\alpha \xi(k_{j-1}),\]

then we have

\[ \Delta V(x(k_j), \eta(k_j)) \leq \xi^T(k_j) (\Pi_\alpha + \sum_{\beta \in \chi^{\alpha}} \pi_{\alpha \beta} \Pi_\beta^{\alpha}) \xi(k_j) \]

where

\[ \Pi_\alpha = \begin{bmatrix} \Pi_{11}^{\alpha \beta} & \Pi_{12}^{\alpha \beta} \\ \ast & \Pi_{22}^{\alpha \beta} \end{bmatrix}, \Pi_\alpha^{\prime} = \begin{bmatrix} \Pi_{11}^{\alpha \beta} & \Pi_{12}^{\alpha \beta} \\ \ast & \Pi_{22}^{\alpha \beta} \end{bmatrix}, \xi(k_j) = \begin{bmatrix} x^T(k_j) \\ x^T(k_{j-1}) \end{bmatrix}^T \]
\[ \Pi_{\alpha,\gamma} = \tilde{A}_\alpha P_\alpha \tilde{A}_\alpha + \pi^\alpha_\gamma (\Phi_\beta + Q + (\tilde{A}_\alpha - \tilde{I})^T Z (\tilde{A}_\alpha - \tilde{I}) + N_1^T + N_1 Z^{-1} N_1^T) \]
\[ \Pi_{\alpha,\gamma} = \tilde{A}_\alpha P_\alpha \tilde{A}_\alpha + \pi^\alpha_\gamma ((\tilde{A}_\alpha - \tilde{I})^T Z \tilde{B}_\alpha - N_1 + N_2^T + N_1 Z^{-1} N_1^T) \]
\[ \Pi_{\alpha,\gamma} = \tilde{B}_\alpha P_\alpha \tilde{B}_\alpha + \pi^\alpha_\gamma (-Q + \tilde{B}_\alpha^T Z \tilde{B}_\alpha - N_2 - N_2^T + N_2 Z^{-1} N_2^T) \]
\[ \Pi_{\alpha,\gamma} = \tilde{A}_\alpha P_\beta \tilde{A}_\alpha - P_\alpha + Q + (\tilde{A}_\alpha - \tilde{I})^T Z (\tilde{A}_\alpha - \tilde{I}) + N_1 + N_1^T + N_1 Z^{-1} N_1^T \]
\[ \Pi_{\alpha,\gamma} = \tilde{A}_\alpha P_\beta \tilde{B}_\alpha + \tilde{B}_\alpha^T Z \tilde{B}_\alpha - N_1 + N_2^T + N_2 Z^{-1} N_2^T \]

By Schur complement, (13) and (14) guarantees \( \Pi_{\alpha,\gamma} < 0 \). Therefore, we have

\[ \Delta V(x(k_j), \eta(k_j)) \leq -\lambda_{\min}(\Pi_{\alpha,\gamma}) \xi^T(j) \xi(j) - \sum_{\beta \in X} \pi_{\alpha,\gamma} \min_{\beta} \lambda_{\min}(\Pi'_{\alpha,\gamma}) \xi^T(j) \xi(j) \]

\[ = -\lambda_{\min}(\Pi_{\alpha,\gamma}) \xi^T(j) \xi(j) - (1 - \tilde{P}_\alpha) \min_{\beta} \lambda_{\min}(\Pi'_{\alpha,\gamma}) \xi^T(j) \xi(j) \]

\[ \leq - (\delta_1 + \delta_2) \| x(k_j) \|^2, \]

where \( \lambda_{\min}(\Pi_{\alpha,\gamma}) \) denotes the minimal eigenvalue of \( -\Pi_{\alpha,\gamma} \) and \( \delta_1 = \inf \{ -\lambda_{\min}(\Pi_{\alpha,\gamma}) \} \), \( \delta_2 = \inf \{ (1 - \tilde{P}_\alpha) \min_{\beta} \lambda_{\min}(\Pi'_{\alpha,\gamma}) \} \). Let \( \delta = \delta_1 + \delta_2 \), then \( \delta > 0 \). From (15), we obtain that for any \( T \geq 1 \),

\[ \sum_{j=0}^{T} \Delta V(x(k_j), \eta(k_j)) = E\{ V(x(k_T), \eta(k_T)) \} - E\{ V(x(k_0), \eta(k_0)) \} \]

\[ \leq - \delta \sum_{j=0}^{T} E\{ \| x(k_j) \|^2 \} \]

yielding the following holds for any \( T \geq 1 \),

\[ \sum_{j=0}^{T} E\{ \| x(k_j) \|^2 \} \leq \frac{1}{\delta} E\{ V(x(k_0), \eta(k_0)) \} - E\{ V(x(k_T), \eta(k_T)) \} \]

\[ \tilde{A} \leq \frac{1}{\delta} E\{ V(x(k_0), \eta(k_0)) \}, \]

implying

\[ E\{ \sum_{j=0}^{\infty} \| x(k_j) \|^2 \} < \frac{1}{\delta} E\{ V(x(k_0), \eta(k_0)) \} < \infty, \]

that is, the system described by (12) is stochastically stable. This completes the proof.

Remark 5

If we assume \( P_\alpha = \sum_{\beta \in X} \pi_{\alpha,\gamma} P_\beta, \pi_{\alpha,\gamma} = \sum_{\beta \in X} \pi_{\alpha,\gamma} \) in LMIs (13), the system with completely known transition probabilities is stochastically stable, it is obvious that the system with completely known transition probabilities is just a special case of our considered systems.

Remark 6

If we aggregate to LMIs (13) the coefficient matrix \( \Phi_k = \Phi_k \) and \( \Gamma_k = \Gamma_k \) in Theorem 1, then we recover Theorem 16 of the literature [23] exactly.

Now let us consider the stabilizing controller design. In the following, we will give a stabilization condition of the NCSs with partially known transition probabilities.
Theorem 2
Consider the system described by (12) with partially known transition probabilities (8). There exists a controller (11) such that the resulting closed-loop system is stochastically stable if there exist matrices \(X_\alpha > 0, \alpha \in \chi, Q > 0, W > 0\), for any nonsingular matrix \(G\) and matrices \(\tilde{N}_1, \tilde{N}_2\) and \(Y_\alpha\) such that the LMs (16) and (17) hold for every feasible values of \(\eta(k_j)\) and \(h_{k_j}(\eta(k_j)) \in \chi, h_{k_j} \in \phi\).

\[
\begin{bmatrix}
-\hat{\Phi} & 0 & \psi_{a4} & \mathcal{L}_\beta(\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1} & 0 \\
* & -W & \psi_{a5} & (\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1} & 0 \\
* & * & \psi_{a6} & \psi_{a7} & \tilde{N}_1 \\
* & * & * & \psi_{a8} & \tilde{N}_2 \\
* & * & * & * & \psi_{a9}
\end{bmatrix} \prec 0, \\
\forall \beta \in \chi^\alpha
\]

(16)

\[
\begin{bmatrix}
-P_b & 0 & \psi_{a4} & (\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1} & 0 \\
* & -W & \psi_{a5} & (\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1} & 0 \\
* & * & \psi_{a6} & \psi_{a7} & \tilde{N}_1 \\
* & * & * & \psi_{a8} & \tilde{N}_2 \\
* & * & * & * & \psi_{a9}
\end{bmatrix} \prec 0, \\
\forall \beta \in \chi^\alpha
\]

(17)

where

\[
\psi_{a4} = \mathcal{L}(\Phi_b^\alpha\Phi_b^\alpha G + (\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1}),
\psi_{a5} = \Phi_b^\alpha\Phi_b^\alpha G + (\Phi_b^{-1}\Phi_b\Gamma_{2b} + \Gamma_{2b})Y_{\alpha - 1} - G,
\psi_{a6} = X_\alpha - G - G^T + \tilde{Q} + \tilde{N}_1 + \tilde{N}_2^T,
\psi_{a7} = -\tilde{N}_1 + \tilde{N}_2^T,
\hat{\Phi} = \pi_{\alpha}^\alpha diag\{X_{k_{\alpha}}^\alpha, \cdots, X_{k_{\alpha}}^\alpha\},
\mathcal{L}_\beta = [\sqrt{\pi_{\alpha_{\alpha}}} I \cdots \sqrt{\pi_{\alpha_{\alpha}}} I]^T
\]

Moreover, if (16) and (17) has solutions, the controller gain is given by \(K_{\alpha} = Y_\alpha G^{-1}\).

Proof
By Schur complement, (13) and (14) is equivalent to

\[
\Xi_{\alpha1} + \Xi_{\alpha2} < 0, \forall \beta \in \chi^\alpha, \pi_{\alpha}^\alpha \neq 0
\]

(18)

\[
\begin{bmatrix}
-P_b & 0 & P_b & \tilde{A}_\alpha & P_b & B_\alpha & 0 \\
* & -Z & Z & (\tilde{A}_\alpha - I) & Z & B_\alpha & 0 \\
* & * & \psi_{a1} & \psi_{a2} & N_1 & \psi_{a2} & N_1 \\
* & * & * & \psi_{a3} & N_2 & \psi_{a3} & N_2 \\
* & * & * & * & -Z
\end{bmatrix} \prec 0, \forall \beta \in \chi^\alpha
\]

(19)

where

\[
\Xi_{\alpha1} = \begin{bmatrix}
-Z^{-1} & \tilde{A}_\alpha - I & \tilde{B}_\alpha & 0 \\
* & \psi_{a1} & \psi_{a2} & N_1 \\
* & * & \psi_{a3} & N_2 \\
* & * & * & -Z
\end{bmatrix}
\]
\[
\Xi_{\alpha 2} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
* & \frac{1}{\pi^2} \tilde{A}_\alpha^T \tilde{P}_\alpha \tilde{A}_\alpha & \frac{1}{\pi^2} \tilde{A}_\alpha^T \tilde{P}_\alpha \tilde{B}_\alpha & 0 \\
* & * & \frac{1}{\pi^2} \tilde{B}_\alpha \tilde{P}_\alpha \tilde{A}_\alpha & 0 \\
* & * & * & 0 
\end{bmatrix}
\]

Bearing the notations \(\chi_\alpha^\circ = \{\kappa_1^\alpha, \ldots, \kappa_m^\alpha\}\) and \(\tilde{P}_\alpha = \sum_{\beta \in \chi_\alpha^\circ} \pi_{\alpha \beta} P_\beta\) in mind and by Schur complement again, we have (18) is equivalent to

\[
(20)
\begin{bmatrix}
\Xi_{\alpha 3} & \Xi_{\alpha 4} \\
* & \Xi_{\alpha 1}
\end{bmatrix} < 0.
\]

where

\[
\Xi_{\alpha 3} = \text{diag}(-\pi_{\alpha \kappa_1^\alpha}^{-1}P_{\kappa_1^\alpha}, -\pi_{\alpha \kappa_2^\alpha}^{-1}P_{\kappa_2^\alpha}, \ldots, -\pi_{\alpha \kappa_m^\alpha}^{-1}P_{\kappa_m^\alpha}),
\]

\[
\Xi_{\alpha 4} = \begin{bmatrix}
0 & \sqrt{\pi_{\alpha \kappa_1^\alpha}} \tilde{A}_\alpha & \sqrt{\pi_{\alpha \kappa_1^\alpha}} \tilde{B}_\alpha & 0 \\
0 & \sqrt{\pi_{\alpha \kappa_2^\alpha}} \tilde{A}_\alpha & \sqrt{\pi_{\alpha \kappa_2^\alpha}} \tilde{B}_\alpha & 0 \\
\vdots & \vdots & \vdots & \vdots \\
0 & \sqrt{\pi_{\alpha \kappa_m^\alpha}} \tilde{A}_\alpha & \sqrt{\pi_{\alpha \kappa_m^\alpha}} \tilde{B}_\alpha & 0
\end{bmatrix}
\]

Note that if \(\pi_{\alpha \kappa}^\circ = 0\), (14) will be just equivalent to (19). Then, consider the system with the control input (11), replace \(\tilde{A}_\alpha\) and \(\tilde{B}_\alpha\) in (19) and (20) by \(\Phi_b^\alpha \Phi_b + (\Phi_b^{\alpha - 1} \Phi_b \Gamma_b + \cdots + \Phi_b \Gamma_b + \Gamma_{1k_b})K_\alpha\) and \((\Phi_b^\alpha \Gamma_{2b})K_{\alpha - 1}\), and set \(X_\alpha = P_\alpha^{-1}, W = Z^{-1}\), we can obtain

\[
(21)
\begin{bmatrix}
-\tilde{T}_\beta & 0 & L_\beta \tilde{\Psi}_{\alpha 4} & L_\beta (\Phi_b^{\alpha - 1} \Phi_b \Gamma_{2b} + \Gamma_{2b})K_{\alpha - 1} & 0 \\
* & W & \tilde{\Psi}_{\alpha 5} & 0 & 0 \\
* & * & \tilde{\Psi}_{\alpha 1} & 0 & \tilde{N}_1 \\
* & * & * & \tilde{N}_2 \\
* & * & * & * & -Z
\end{bmatrix} < 0,
\]

\(\forall \beta \in \chi_\alpha^\circ\)

\[
(22)
\begin{bmatrix}
-P_b & 0 & \tilde{\Psi}_{\alpha 4} & (\Phi_b^{\alpha - 1} \Phi_b \Gamma_{2b} + \Gamma_{2b})K_{\alpha - 1} & 0 \\
* & W & \tilde{\Psi}_{\alpha 5} & 0 & 0 \\
* & * & \tilde{\Psi}_{\alpha 1} & 0 & \tilde{N}_1 \\
* & * & * & \tilde{N}_2 \\
* & * & * & * & -Z
\end{bmatrix} < 0,
\]

\(\forall \beta \in \chi_\mu^\alpha\)

where

\[
\tilde{\Psi}_{\alpha 4} = \Phi_b^\alpha \Phi_b + (\Phi_b^{\alpha - 1} \Phi_b \Gamma_b + \cdots + \Phi_b \Gamma_b + \Gamma_{1k_b})K_\alpha,
\]

\[
\tilde{\Psi}_{\alpha 5} = \Phi_b^\alpha \Phi_b + (\Phi_b^{\alpha - 1} \Phi_b \Gamma_b + \cdots + \Phi_b \Gamma_b + \Gamma_{1k_b})K_{\alpha - 1} - I.
\]

Note that \((G^{-1} - P_\alpha)T P_\alpha^{-1}(G^{-1} - P_\alpha) \geq 0\) implies that \(-P_\alpha \leq G^{-T} P_\alpha^{-1} G^{-1} - G^{-1} - G^{-T} = \Lambda_1\), for any nonsingular matrix \(G\). Similarly, \(-Z \leq G^{-T} \tilde{T} Z^{-1} G^{-1} - \tilde{T}\).
\(G^{-1} - G^{-T} = \Lambda_2\). Hence the above inequality holds if
\[
\begin{bmatrix}
-\tilde{\Psi}_a & 0 & L_3 \Psi_{a4} & L_3(\Phi_b^{\alpha-1} \Gamma_{k_b} + \Gamma_{2k_b})Y_{\alpha-1} & 0 \\
* & -W \Psi_{a5} & (\Phi_b^{\alpha-1} \Gamma_{k_b} + \Gamma_{2k_b})Y_{\alpha-1} & 0 \\
* & * & \Psi_{a6} & \psi_{a2} & \tilde{N}_1 \\
* & * & * & \psi_{a3} & \tilde{N}_2 \\
* & * & * & * & \Lambda_2
\end{bmatrix} < 0,
\]
\(\forall \beta \in \chi_\alpha^a\)
(23)

\[
\begin{bmatrix}
-P_\beta & 0 & \tilde{\Psi}_{a4} & (\Phi_b^{\alpha-1} \Gamma_{k_b} + \Gamma_{2k_b})Y_{\alpha-1} & 0 \\
* & -W \tilde{\Psi}_{a5} & (\Phi_b^{\alpha-1} \Gamma_{k_b} + \Gamma_{2k_b})Y_{\alpha-1} & 0 \\
* & * & \tilde{\Psi}_{a6} & \psi_{a2} & \tilde{N}_1 \\
* & * & * & \psi_{a3} & \tilde{N}_2 \\
* & * & * & * & \Lambda_2
\end{bmatrix} < 0,
\]
\(\forall \beta \in \chi_{\mu_\alpha}^a\)
(24)

where \(\tilde{\Psi}_{a6} = \Lambda_1 + Q + N_1 + N_2^T\). Pre- and post-multiply this inequality by \(\text{diag}(I \ I \ G^T \ G^T \ G^T)\) and its transpose, and define \(P_a^{-1} = X_\alpha, G^T Q G = \tilde{Q}, G^T N_1 G = \tilde{N}_1, G^T N_2 G = \tilde{N}_2, Y_\alpha = K_\alpha G\), we can readily obtain (16)and(17). This completes the proof.

If the sampling period \(h_{k_j}\) is constant, to stabilize the system described by (12), the LMIs (16) and (17) should be satisfied for the specific sampling period \(h_{k_j}\), which can be described as the following corollary.

**Corollary 1**

Consider the system described by (12) with partially known transition probabilities (8). There exists a controller (11) such that the resulting closed-loop system is stochastically stable if there exist matrices \(X_\alpha > 0, \alpha \in \chi, Q > 0, W > 0, \) for any nonsingular matrix \(G\) and matrices \(N_1, N_2\) and \(Y_\alpha\) such that the LMIs (25) and (26) hold for every feasible values of \(\eta(k_j)\) and the specific sampling period \(h_{k_j} (\eta(k_j) \in \chi)\).

\[
\begin{bmatrix}
-\tilde{\Psi}_a & 0 & L_3 \Psi_{a4} & L_3(\Phi_b^{\alpha} + I) \Gamma_{2k_b} Y_{\alpha-1} & 0 \\
* & -W \Psi_{a5} & (\Phi_b^{\alpha} + I) \Gamma_{2k_b} Y_{\alpha-1} & 0 \\
* & * & \Psi_{a6} & \psi_{a7} & \tilde{N}_1 \\
* & * & * & \psi_{a8} & \tilde{N}_2 \\
* & * & * & * & \Psi_{a9}
\end{bmatrix} < 0, \quad \forall \beta \in \chi_\alpha^a
\]
(25)

\[
\begin{bmatrix}
-P_\beta & 0 & \Psi_{a4} & (\Phi_b^{\alpha} + I) \Gamma_{2k_b} Y_{\alpha-1} & 0 \\
* & -W \Psi_{a5} & (\Phi_b^{\alpha} + I) \Gamma_{2k_b} Y_{\alpha-1} & 0 \\
* & * & \Psi_{a6} & \psi_{a7} & \tilde{N}_1 \\
* & * & * & \psi_{a8} & \tilde{N}_2 \\
* & * & * & * & \Psi_{a9}
\end{bmatrix} < 0, \quad \forall \beta \in \chi_{\mu_\alpha}^a
\]
(26)

where
\[
\begin{align*}
\psi_{a4} &= \Phi_b^{\alpha+1} G + (\Phi_b^{\alpha} \Gamma_{1k_b} + \cdots + \Phi_b^{\alpha} \Gamma_{k_b + \Gamma_{1k_b}}) Y_{\alpha}, \\
\psi_{a5} &= \Phi_b^{\alpha+1} G + (\Phi_b^{\alpha} \Gamma_{1k_b} + \cdots + \Phi_b^{\alpha} \Gamma_{k_b + \Gamma_{1k_b}}) Y_{\alpha}, \\
\psi_{a6} &= X_\alpha - G - G^T + \tilde{Q} + \tilde{N}_1 + \tilde{N}_2^T, \quad \psi_{a7} = \tilde{N}_1 + \tilde{N}_2^T,
\end{align*}
\]
\[ \Psi_{a8} = -\tilde{Q} - \tilde{N}_2 - \tilde{N}^T, \quad \Psi_{a9} = W - G - G^T, \quad \Gamma_{kj} = \int_0^{h_k} e^{A_s}dsB. \]

\[ \tilde{\gamma}_\beta = \sqrt{\pi \alpha \pi} \text{diag}\{X_{\kappa_1}, \ldots, X_{\kappa_m}\}, \quad \mathcal{L}_\beta = [\sqrt{\pi \alpha \pi_1} 1 \cdots \sqrt{\pi \alpha \pi_m} 1]^T. \]

Moreover, if (25) and (26) has solutions, the controller gain is given by \( K_\alpha = Y_\alpha G^{-1} \).

### 4. Numerical example

In this section, a numerical example and simulations are used to illustrate the usefulness of the developed synthesis methods.

Let us consider the nominal continuous-time system with no disturbance input [23]:

\[ \dot{x}(t) = \begin{bmatrix} -1 & 0 & 0.5 \\ 1 & -0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \]

Suppose the minimum sampling period is 0.1s, the maximum sampling period is 0.5s and suppose the feasible values of sampling period are \( h_1 = 0.1s, h_2 = 0.3s, h_3 = 0.5s \), we discretize the system (27) and obtain

\[
\Phi_1 = \begin{bmatrix}
0.9048 & 0 & -0.0488 \\
0.0928 & 0.9512 & -0.0024 \\
0 & 0 & 1.0513
\end{bmatrix},
\]

\[
\Phi_2 = \begin{bmatrix}
0.7408 & 0 & -0.1403 \\
0.2398 & 0.8607 & -0.0204 \\
0 & 0 & 1.1618
\end{bmatrix},
\]

\[
\Phi_3 = \begin{bmatrix}
0.6065 & 0 & -0.2258 \\
0.3445 & 0.7788 & -0.0536 \\
0 & 0 & 1.2840
\end{bmatrix}.
\]

For simplicity, suppose the maximum packet-loss upper bound \( s = 4 \) and \( \{\eta_k, k_j = 0, 1, 2, \cdots\} \) is a Markov chain with the three different cases of transition probability described by

\[
R_1 = \begin{bmatrix}
0.5 & 0.2 & 0.1 & 0.1 & 0.1 \\
0.2 & 0.5 & 0.3 & 0 & 0 \\
0 & 0 & 0.2 & 0.5 & 0.3 \\
0.1 & 0.1 & 0.1 & 0.2 & 0.5
\end{bmatrix},
\]

\[
R_2 = \begin{bmatrix}
0.5 & ? & ? & 0.1 & 0.1 \\
? & 0.5 & 0.3 & ? & 0 \\
? & 0.2 & 0.5 & ? & 0 \\
0 & ? & 0.2 & ? & 0.3 \\
0.1 & 0.1 & ? & ? & 0.5
\end{bmatrix},
\]

\[
R_3 = \begin{bmatrix}
\end{bmatrix}.
\]

Such \( R_i (i = 1, 2, 3) \) exhibit the bursty nature of packet losses. The bursty nature is modeled with \( \pi_{a\alpha} > \pi_{a\beta} \) for all \( \alpha, \beta \in \chi, \beta \neq \alpha \), which says that the likelihood
of losing a packet after a lost packet transmission is higher than after a successful packet transmission[26].

Our purpose here is to design a stabilizing controller of the form (11) for the three different cases of transition probabilities. First of all, given \( s = 0 \), even if all the transition probabilities are known, both the continuous-time system and the discretized system are unstable because of \( e^i(A) = -0.5, -1, 0.5 ; e^i(\Phi_2) = 0.9512, 0.9048, 1.0513; e^i(\Phi_2) = 0.8607, 0.7408, 1.1618 \) and \( e^i(\Phi_3) = 0.7788, 0.6065, 1.2840 \). It implies that the underlying system will be unstable starting from \( s = 0 \). Then, we assume that the packet-loss upper bound \( s = 4 \), which means that up to 80% of the packets can be lost during the network transmissions when the sampling period is 0.5s. Applying Theorem 2 and Corollary 1, we obtain the stabilizing networked controller gains for the three different cases of transition probabilities, respectively, as shown in Table I. To simulate, we take the initial state

<table>
<thead>
<tr>
<th></th>
<th>( h_3 ) adopted (Corollary3.1)</th>
<th>( h_3 ) and ( h_3 ) adopted (Theorem3.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Completely known</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_0 )</td>
<td>([ 0.0548 - 0.0222 - 1.7503 ])</td>
<td>([ 0.1587 - 0.0254 - 2.3557 ])</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>([ 0.0052 - 0.0149 - 1.0715 ])</td>
<td>([ 0.0340 - 0.0005 - 1.1721 ])</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>([-0.0032 - 0.0101 - 0.7870 ])</td>
<td>([ 0.0082 - 0.0012 - 0.8200 ])</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>([-0.0087 - 0.0113 - 0.7107 ])</td>
<td>([-0.0026 - 0.0007 - 0.7024 ])</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>([-0.0106 - 0.0119 - 0.6649 ])</td>
<td>([-0.0063 - 0.0071 - 0.6647 ])</td>
</tr>
<tr>
<td><strong>Partially known</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_0 )</td>
<td>([ 0.0686 - 0.0143 - 1.8274 ] )</td>
<td>([ 0.2963 - 0.0053 - 2.4048 ])</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>([ 0.0122 - 0.0108 - 1.1001 ] )</td>
<td>([ 0.0379 - 0.0002 - 1.1894 ])</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>([-0.0011 - 0.0091 - 0.8004 ] )</td>
<td>([ 0.0103 - 0.0011 - 0.7878 ])</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>([-0.0071 - 0.0103 - 0.7170 ] )</td>
<td>([-0.0018 - 0.0054 - 0.7250 ])</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>([-0.0095 - 0.0111 - 0.6682 ] )</td>
<td>([-0.0061 - 0.0072 - 0.6673 ])</td>
</tr>
<tr>
<td><strong>Completely unknown</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_0 )</td>
<td>([ 0.0852 - 0.0088 - 1.7689 ] )</td>
<td>([ 0.0550 - 0.0510 - 1.6346 ])</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>([ 0.0173 - 0.0026 - 1.0711 ] )</td>
<td>([ -0.0022 - 0.0180 - 1.1828 ])</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>([ 0.0012 - 0.0064 - 0.7850 ] )</td>
<td>([ -0.0008 - 0.0018 - 0.7799 ])</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>([-0.0055 - 0.0089 - 0.7120 ] )</td>
<td>([-0.0013 - 0.0021 - 0.6972 ])</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>([-0.0083 - 0.0103 - 0.6545 ] )</td>
<td>([-0.0011 - 0.0013 - 0.6545 ])</td>
</tr>
</tbody>
</table>

as \( x_0 = [ -5 \quad 0 \quad 5 ]^T \). Fig.3 depicts the trajectory of the system state for the three different cases in Table I when the constant sampling period \( h_3 \) is adopted and the maximum packet-loss upper bound \( s = 4 \). If the initial sampling period is \( h_1 \), then at the instant 5s the sampling period switches to \( h_3 \), the trajectory of the system state for the three different cases in Table I is pictured in Fig.4. It should be noticed that our maximum sampling period is the same as the choice in [23], where \( T_s = 0.5s \). Just as shown in Fig.3-Fig.4, if there exists the switching of sampling periods, time varying sampling period may provide faster convergent speed than constant sampling period \( h_3 \) does. It is obvious that the designed controller is feasible and ensures the stability of the closed-loop system despite the partially known transition probabilities.

**Remark 7**

In the simulation process, the chosen specific elements are treated as unknown transition probabilities in [20-22]; however, we choose the random numbers as unknown
transition probabilities. In practically, this kind of information including the variation of time delay and the packet losses is hard to obtain. Therefore, our unknown transition probabilities considered here is more natural and reasonable.

5. Conclusion

The stability and stabilization problem for NCSs with partially known transition probabilities and time varying sampling period has been discussed in this paper. A new switched linear system model of NCSs with packet losses has been proposed. The packet-loss process has been considered as the Markovian process with partially known transition probabilities. The corresponding stability conditions have been derived via applying a constructed Lyapunov functional. Furthermore, the state feedback controllers have been designed by solving a set of LMIs. Numerical example has illustrated the effectiveness and utility of the proposed approaches.

References


Figure 4. State response ($h_1$ and $h_3$ adopted).


School of Science, Shenyang University of Technology, Shenyang, liaoning ,China ;Institute of Systems Science, Northeastern University , Shenyang, liaoning ,China .

E-mail: syliyy@163.com

Institute of Systems Science, Northeastern University, Shenyang, liaoning, China;Software technology institute, Dalian Jiaotong University, Dalian, Liaoning 116028, China

E-mail: qizhang@mail.neu.edu.cn and qiuzhanzhi@163.com