STRONG ABSOLUTE STABILITY OF LUR’E DESCRIPTOR SYSTEMS WITH IMPULSIVE MODELS IN THE LINEAR SUBSYSTEM

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Abstract. This paper considers strong absolute stability for Lur’e descriptor systems (LDS) with impulse models in the linear subsystems. First, the system under consideration is transformed into an LDS with uncertainty in the derivative matrix. Then using the existing results on strong absolute stability for LDS and linear matrix inequality technology, a strong absolute stability sufficient condition for LDS with impulse models in the linear subsystems is given. Finally, an example is given to show the effectiveness of the proposed method.

Key Words. Lur’e descriptor systems, impulsive, strong absolute stability, linear matrix inequalities (LMIs).

1. Introduction

Descriptor systems which are also referred to as singular systems are described by differential equations and algebraic equations [1]. It has been widely applied in the areas of power systems, economic systems, biological systems, constrained robots, electronic networking, aerospace, chemical process and astronomy (solar activity)[1-4]. Stability of descriptor systems is a basic and important problem in control theory. Stability of linear descriptor systems has been investigated widely. In [5], necessary and sufficient conditions for robust stability of descriptor interval systems are obtained by using structured singular value theory. [6] studies robust stability of impulse-free uncertain descriptor systems. [7] is concerned with the problem of robust stability for both continuous and discrete singular delay systems, delay-dependent robust stability criteria in terms of strict linear matrix inequality (LMI) are derived. However, the research on nonlinear descriptor systems is premature because of their complex characteristics. In [8], stability problems of a composite descriptor large-scale system are studied by the generalized vector Lyapunov functions. In [9-11], under the assumption that the initial compatible conditions are known, sufficient conditions for stability of the nonlinear descriptor systems are given. [12] is concerned with the quadratic stabilization for a class of switched nonlinear singular systems. [13] gives a sufficient condition for the local stability of nonlinear descriptor systems. [14] studies the solvability and stability of a class of nonlinear descriptor systems. [15] and [16] studies the stability of Lipschitz nonlinear descriptor systems, and sufficient conditions for stability of the systems are given.

Lur’e descriptor system (LDS) is a typical nonlinear descriptor system, which is a feedback system whose feed-forward is linear time-invariant system and feedback
contains nonlinear term. In practice, nonlinear parts of practical nonlinear control systems can usually be separated to form a Lur’e type system. Therefore, the study on stability of LDS are of great significance. Absolute stability theory is an important branch of stability theory, and the study of absolute stability theory is an important way to study stability of nonlinear control systems. For absolute stability theory of normal systems, the most famous criteria are the circle criterion and Popov criterion [17-21]. Recently, LDS has attracted much attention. In [22-23], positive realness of continuous- and discrete-time descriptor systems are investigated, respectively, and absolute stability criteria for LDSs are derived by the obtained positive real lemmas. [24] studies the $H_\infty$ control problem of LDS with delay. However, the results in [22-24] do not consider the impulse behavior of the LDSs. In [25-29], the so-called strongly absolute stability, which concerns not only stability but also impulse behavior of the LDSs, is investigated and some stability criteria are given. However, the above mentioned results for LDS relies on the admissibility of the linear subsystem.

In this paper, we will try to alleviate the restriction that the linear subsystem is admissible. A typical situation for the linear subsystems not to be admissible is that it has impulse modes. We consider strong absolute stability for Lur’e descriptor systems (LDS) with impulse models in the linear subsystems. The system under consideration is transformed into an LDS with uncertainty in the derivative matrix. Then a strong absolute stability criterion is given in the form of linear matrices inequalities. At last, we present an example to show the effectiveness of the proposed method.

2. Basic definitions and lemmas

Consider the system

$$E \dot{x} = Ax + Bu,$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $A, E \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ are constant matrices, and $\text{rank}(E) \leq n$. If $\det(sE - A) \neq 0$ for some complex number $s$, then the pair $(E, A)$ is said to be regular. It is proved in [30] that $(E, A)$ is regular if and only if there exist two nonsingular matrices $M$ and $N$ such that the system (1) can be transformed into the following equivalent form

$$\begin{align*}
\dot{x}_1 &= A_r x_1 + B_1 u, \\
J \dot{x}_2 &= x_2 + B_2 u,
\end{align*}$$

where $J \in \mathbb{R}^{(n-r) \times (n-r)}$ is a nilpotent matrix, $A_r \in \mathbb{R}^{r \times r}$.

**Definition 1.** [31] The non-linear descriptor system $E \dot{x} = F(x)$ is said to be of index one if the constant coefficient system

$$E \dot{\omega} = F_x(\hat{x}) \omega$$

is regular and impulsive-free for any $\hat{x}$ in a neighborhood of the equilibrium point $x = 0$, where $F_x$ is the Jacobian matrix $\partial F/\partial x$.

Consider LDS

$$\begin{align*}
\dot{x}_1 &= A_r x_1 + B_1 u, \\
J \dot{x}_2 &= x_2 + B_2 u, \\
\sigma &= Cx, \\
u &= \phi(\sigma),
\end{align*}$$

(2)
where $\phi$ is sufficiently smooth nonlinear function satisfying

$$\phi(0) = 0$$

and sector constraints

$$\phi^T(\phi - K\sigma) \leq 0, \forall \sigma.$$ 

$K$ is a symmetric matrix of appropriate dimension, and $K > 0$. $\phi$ satisfying the above condition is denoted by $\phi \in F[0, K]$.

The definition of strong absolute stability is recalled.

**Definition 2.** [31] LDS (2) is said to be strongly absolutely stable with respect to $F[0, K]$, if for $\forall \phi \in F[0, K]$, LDS (2) is globally asymptotically stable and is of index one.

This paper considers the following system,

1. $\dot{x}_1 = A_0 x_1 + B_1 \phi(\sigma),$
2. $J \dot{x}_2 = x_2 + B_2 \phi(\sigma),$
3. $\sigma = C_1 x_1 + C_2 x_2,$

where, $J$ satisfaction $J^2 = 0, J \neq 0$, $C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$. $\phi(\sigma)$ is a sufficiently smooth time-invariant real variable and satisfying:

4. $0 < \dot{\phi}(\cdot) \leq k,$
5. $\inf \dot{\phi}(\sigma), \sigma \in \mathbb{R}^m > 0,$
6. $\phi(0) = 0.$

### 3. Main results

We first make a equivalent transformation for system (3-4-5).

Pre- and post-multiplying (4) by $J$, we have

$$J^2 \dot{x}_2 = Jx_2 + JB_2 \phi(\sigma).$$

Since $J^2 = 0, J \neq 0,$

$$Jx_2 = -JB_2 \phi(\sigma).$$

Then

$$J \dot{x}_2 = -JB_2 \dot{\phi}(\sigma) C \dot{\phi}.$$

From (4) and (10), it follows that

$$-JB_2 \dot{\phi}(\sigma) C \dot{\phi} = x_2 + B_2 \phi(\sigma).$$

Then system (3-4) is equivalent to

$$\begin{bmatrix} I_{n_0} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -JB_2 \dot{\phi} C_1 & -JB_2 \dot{\phi} C_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_0 \\ I_{n-n_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \phi(\sigma).$$

From (6), (7), there exists a $\varepsilon_1 > 0$ satisfying $\varepsilon_1 \leq \dot{\phi}(\cdot) \leq k$.

Letting $g = \frac{2}{k+\varepsilon_1}(\dot{\phi} - \frac{k+\varepsilon_1}{2})$, we have $|g| \leq 1$.

Substituting $\phi = \frac{\varepsilon_1}{k+\varepsilon_1} g + \frac{k+\varepsilon_1}{2}$ into the above equation, we obtain
\[
\begin{align*}
&\begin{bmatrix}
I_r & 0 \\
-JB_2 \frac{k-\varepsilon_1}{2} C_1 & -JB_2 \frac{k-\varepsilon_1}{2} C_2
\end{bmatrix} \\
&+ \begin{bmatrix}
0 & 0 \\
-JB_2 \frac{k-\varepsilon_1}{2} C_1 & -JB_2 \frac{k-\varepsilon_1}{2} C_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} \\
&= \begin{bmatrix}
A_r & 0 \\
0 & I_{n-r_0}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} \phi(\sigma).
\end{align*}
\]

(13)

For convenience, we denote (13) by

\[(14)\]

\[
(E_0 + gE)\dot{x} = A_0 x + B_0 \phi(\sigma)
\]

where, \(gE = \begin{bmatrix} 0 & 0 \\ -\frac{k-\varepsilon_1}{2} JB_2 \end{bmatrix} g \begin{bmatrix} C_1 & C_2 \end{bmatrix} = M g N.\)

There exit two nonlinear matrices \(P, Q\) such that

\[
E_{0r} = PE_0 Q = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},
\]

\[
A_{0r} = PA_0 Q = \begin{bmatrix} A_{r1} & A_{r2} \\ A_{r3} & A_{r4} \end{bmatrix}.
\]

Correspondingly

\[
M_{0r} = PM = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix},
\]

\[
N_{0r} = NQ = \begin{bmatrix} N_1 & N_2 \end{bmatrix},
\]

\[
B_{0r} = PB_0, C_{0r} = C_0 Q.
\]

Suppose \(N_2 = 0, \|N_1 M_1\| = \|NQPM\| < 1.\) Then from [32], (14) is equivalent to the following system

\[(15)\]

\[
\begin{cases}
\dot{\tilde{E}}_r \dot{x} = \tilde{A}_r x + \tilde{B}_r \phi, \\
y = \tilde{C}_r x,
\end{cases}
\]

where

\[
\begin{align*}
\tilde{E}_r &= E_{0r}, \tilde{A}_r = P g A_{0r} Q g, \\
\tilde{B}_r &= P g B_{0r}, \tilde{C}_r = C_{0r} Q g.
\end{align*}
\]

Because \(N_2 = 0,\) we get

\[
P g = I_n - M_{0r} \tilde{g} N_{0r}, Q g = I_n,
\]

\[
\tilde{g} = g(I + N_1 M_1 g)^{-1}.
\]

From (6), we get

\[
0 < \lim_{\sigma \to 0} \frac{\phi(\sigma) - \phi(0)}{\sigma} \leq k,
\]

then \(0 < \frac{\phi(\sigma)}{\sigma} \leq k\) because of (8).

The following lemmas will be used in this paper.

**Lemma 1.** [33]: Let \(I - KTK > 0,\) and define the set

\[
Y = \{g(I - Kg)^{-1}, g^T g \leq I\}.
\]

Then,

\[
Y = \{K^T(I - KK^T)^{-1} + \Pi^T(I - KK^T)^{-1/2}, \Pi^T \Pi \leq (I - K^T K)^{-1}\}.
\]
Lemma 2. [34]: Let $\Omega$, $H$, $F$ and $R > 0$ be real matrices with appropriate dimensions, and the matrix $\Pi$ satisfies $\Pi^T \Pi \leq R$. Then, for all $\Pi^T \Pi \leq R$, the matrix inequality

$$\Omega + H \Pi F + F^T \Pi^T H^T < 0$$

holds if and only if there exists a scalar $\varepsilon > 0$, such that

$$\begin{bmatrix} \Omega & H \\ H^T & 0 \end{bmatrix} + \varepsilon \begin{bmatrix} F^T \Pi F & 0 \\ 0 & -I \end{bmatrix} < 0.$$ 

Lemma 3. [31]: If there are matrixes, $P \in R^{n \times n}$, $Q \in R^{n \times n}$, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$ and $\Gamma = \text{diag}(\tau_1, \tau_2, \ldots, \tau_m) \geq 0$ satisfy

$$AC = QE,$$

$$E^T P = P^T E \geq 0,$$

$$\begin{bmatrix} A^T P + P^T A & CT \Gamma + A^T \Pi^T P \\ \Gamma K C + QA - B^T P & -2I - QB - B^T \Pi^T P \end{bmatrix} < 0.$$ 

Then the LDS (2) about $\phi$ satisfies the condition $(8-9-10)$, the system $(4-5-6)$ is strongly absolutely stable.

Theorem 1. If there exist $\varepsilon > 0$, $P_1 \in R^{n \times n}$, $Q_1 \in R^{m \times n}$, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$ and $\Gamma = \text{diag}(\tau_1, \tau_2, \ldots, \tau_m) \geq 0$ satisfy

$$AC_{01} = Q E_{01},$$

$$P_{01}^T P_1 = P_1^T E_{01} \geq 0,$$

$$\begin{bmatrix} A_{01}^T \Theta^T P_1 & + P_1^T \Theta A_{01} \\ \Gamma K C_{01} + Q_1 \Theta A_{01} - B_{01}^T \Theta P_1 & 1/\varepsilon \Xi_2 N_{01} A_{01} \\ -2I - Q_1 \Theta B_{01} - B_{01}^T \Theta^T Q_1 & -1/\varepsilon I \\ 1/\varepsilon \Xi_2 N_{01} A_{01} & -1/\varepsilon I \\ -1/\varepsilon \Xi_3 & -1/\varepsilon \Xi_3 \end{bmatrix} < 0.$$ 

where

$$\Theta = I + M_{01} \Xi_1 N_{01},$$

$$\Xi_1 = M_{01}^T N_{10} (I - N_1 M_{01} M_{10}^T N_{11})^{-1},$$

$$\Xi_2 = (I - N_1 M_{01} M_{10}^T N_{11})^{-1/2},$$

$$\Xi_3 = I - M_{01}^T N_{10}^T N_1 M_{10}.$$

Then, when $\phi$ satisfies the condition $(8-9-10)$, the system $(4-5-6)$ is strongly absolutely stable.

Proof. Note

$$\Omega_0 = \begin{bmatrix} A_{01}^T \Theta^T P_1 & + P_1^T \Theta A_{01} \\ \Gamma K C_{01} + Q_1 \Theta A_{01} - B_{01}^T \Theta P_1 & -2I - Q_1 \Theta B_{01} - B_{01}^T \Theta^T Q_1 \end{bmatrix},$$

$$H_{01}^T = \begin{bmatrix} -\Xi_2 N_{01} A_{01} & \Xi_2 N_{01} B_{01} \\ -\Xi_2 N_{01} A_{01} & \Xi_2 N_{01} B_{01} \end{bmatrix},$$

$$F_{01} = \begin{bmatrix} M_{01}^T P_1 & M_{01}^T Q_1 \end{bmatrix},$$

then (18) can be rewritten as

$$\Omega_\varepsilon = \begin{bmatrix} \Omega_0 & \varepsilon^{-1} H_{01}^T & \varepsilon^{-1} F_{01} \\ \varepsilon^{-1} H_{01} & \varepsilon^{-1} I & \Xi_2 N_{01} B_{01} \\ \varepsilon^{-1} F_{01} & \Xi_2 N_{01} B_{01} & \Xi_3 \end{bmatrix} < 0.$$
From the Schur complement,
\[
\begin{bmatrix}
\Omega_0 & \varepsilon^{-1}H_\varepsilon \\
\varepsilon^{-1}H_\varepsilon^T & -\varepsilon^{-1}I
\end{bmatrix} - \begin{bmatrix}
F_\varepsilon^T \\
0
\end{bmatrix} \begin{bmatrix}
-\varepsilon^{-1}\Xi_3 \\
0
\end{bmatrix}^{-1} \begin{bmatrix}
F_\varepsilon \\
0
\end{bmatrix} < 0
\]
which means
\[
\begin{bmatrix}
\Omega_0 & \varepsilon^{-1}H_\varepsilon \\
\varepsilon^{-1}H_\varepsilon^T & -\varepsilon^{-1}I
\end{bmatrix} + \begin{bmatrix}
F_\varepsilon^T\Xi_3^{-1}F_\varepsilon \\
0 \\
0 \\
-\varepsilon^{-1}I
\end{bmatrix} < 0
\]
with \(\varepsilon > 0\),
\[
\begin{bmatrix}
\varepsilon\left(\varepsilon^{-1}\Omega_0 + F_\varepsilon^T\Xi_3^{-1}F_\varepsilon\right) \\
\varepsilon^{-1}H_\varepsilon \\
-\varepsilon^{-1}I
\end{bmatrix} < 0.
\]
Pre- and post- multiplying the above inequality by \(\varepsilon\), we have
\[
\begin{bmatrix}
\Omega_0 \\
H_\varepsilon \\
0
\end{bmatrix} + \varepsilon \begin{bmatrix}
F_\varepsilon^T\Xi_3^{-1}F_\varepsilon \\
0 \\
0 \\
-\varepsilon^{-1}I
\end{bmatrix} < 0
\]
Then from Lemma 2, it equivalent to
\[
\Omega_0 + H_\varepsilon\Pi F_\varepsilon + F_\varepsilon^T\Pi^T H_\varepsilon < 0, \forall \Pi^T\Pi \leq \Xi_3^{-1},
\]
i.e.
\[
\Gamma(KC_0 + Q_1\Theta A_0) - B_0^T\Theta^T P_1 - 2\Xi - Q_1\Theta B_0 \leq B_0^T\Theta^T Q_1
\]
\[
+ \begin{bmatrix}
-A_0^T N_0^T\Xi_2 \Pi M_0^T P_1 \\
B_0^T N_0^T\Xi_2 \Pi M_0^T P_1 \\
-B_0^T N_0^T\Xi_2 \Pi M_0^T Q_1^T \\
B_0^T N_0^T\Xi_2 \Pi M_0^T Q_1^T
\end{bmatrix}^T < 0.
\]
Denoted by
\[
(20) T = \begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{bmatrix} < 0,
\]
where
\[
T_1 = A_0^T\Theta^T P_1 + P_1^T\Theta A_0 - A_0^T N_0^T\Xi_2 \Pi M_0^T P_1 \\
- P_1^T M_0^T \Xi_2 N_0 A_0,
\]
\[
T_2 = \Gamma(KC_0 + Q_1\Theta A_0) - B_0^T\Theta^T P_1 \\
+ B_0^T N_0^T\Xi_2 \Pi M_0^T P_1 - Q_1 M_0^T \Xi_2 N_0 A_0,
\]
\[
T_4 = -2\Xi - Q_1\Theta B_0 \leq B_0^T\Theta^T Q_1 + B_0^T N_0^T\Xi_2 \Pi M_0^T Q_1^T \\
+ Q_1 M_0^T \Xi_2 N_0 B_0.
\]
Considering \(\Theta = I + M_0^T \Xi_2 N_0\), we obtain
\[
\Theta = M_0^T \Xi_2 N_0 = I - M_0^T(\Pi^T \Xi_2 - \Xi_1)N_0.
\]
Because of \(\|M_1 N_1\| < 1\),
\[
\Xi_3 = I - M_1^T N_1^T N_1 M_1 > 0.
\]
From Lemma 1 with \(K = -N_1 M_1\), we have
\[
\Xi_1 = M_1^T N_1^T (I - N_1 M_1 M_1^T N_1^T)^{-1},
\]
which shows
\[ \Pi^T \Xi_2 - \Xi_1 = g(I + N_1 M_1 g)^{-1}, \]
then we have
\[ \Theta - M_{or} \Pi^T \Xi_{2 or} = I - M_{or} (\Pi^T \Xi_2 - \Xi_1) N_{or} = P_g. \]
From (20) we get
\[ T_1 = \tilde{A}_r^T P_1 + P_1^T \tilde{A}_r, \]
\[ T_2 = \Gamma K C_{0r} + Q_1 \tilde{A}_r - \tilde{B}_r^T P_1, \]
\[ T_4 = -2\Gamma - Q_1 \tilde{B}_r - \tilde{B}_r^T Q_1^T. \]
Then,
\[ \begin{bmatrix} \tilde{A}_r^T P_1 + P_1^T \tilde{A}_r & * \\ \Gamma K C_{0r} + Q_1 \tilde{A}_r - \tilde{B}_r^T P_1 & -2\Gamma - Q_1 \tilde{B}_r - \tilde{B}_r^T Q_1^T \end{bmatrix} < 0. \]
From (16), (17)
\[ \Lambda C_r = Q_1 \tilde{E}_r, \]
\[ \tilde{E}_r^T P_1 = P_1^T \tilde{E}_r \geq 0. \]
Therefore, by Lemma 3 system (15) is strongly absolutely stable. Thus, system (3-4-5) is strongly absolutely stable.

4. Computational issues and example

In this section, we will address the computational issues and show how to deal with the conditions (16) and (17). Strict LMI-based algorithms for Theorem 1 is obtained without any additional conservatism. One numerical example is given to illustrate the effectiveness of the obtained results. Matlab 7.0 is used to check the LMI feasibility problems.

Consider the solving of the conditions of the inequality theorem. Let \( U = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \) and \( V = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \) be such that orthogonal matrix, satisfying
\[ E = U \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix} V^T. \]
It can be seen that \( EV_2 = 0, U_2^T E = 0 \). Here, \( \Sigma_r \in R^{r \times n} \) is positive definite and diagonal.

By [6, 22], we have
(i)
\[ Z_1 = Z_2 \]
where
\[ Z_1 = \{(A, Q)|\Lambda C = QE\} \]
and
\[ Z_2 = \{(A, Q)|\Lambda CV_2 = 0, Q_1 = \Lambda CV_1 \Sigma_r^{-1} Q_2 \} U^T, Q_2 \in R^{m \times (n-r)}. \]
(ii)
\[ Y_1 = Y_2 \]
where
\[ Y_1 = \{P \in R^{n \times n}|E^T P = P^T E\} \]
and
\[ Y_2 = \{P = XE + U_2 S|X \in R^{n \times n}, X = X^T, S \in R^{(n-r) \times n}\}. \]
Thus, without any additional conservatism, the conditions in Theorem 1 can be performed by the following algorithm, respectively.

Algorithm 1:

Step 1: Determine the set

\[ \Psi = \{ \Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m) | \Lambda CV_2 = 0 \} \].

Step 2: Find \( \varepsilon > 0 \), and matrices

\[ \Lambda \in \Psi, \]
\[ X \in \mathbb{R}^{n \times n}, X > 0, \]
\[ S \in \mathbb{R}^{(n-r) \times n}, \]
\[ Q_2 \in \mathbb{R}^{m \times (n-r)} \]
\[ \Gamma = \text{diag}(\tau_1, \tau_2, \cdots, \tau_m) \geq 0 \]

satisfying

\[
\begin{pmatrix}
A_\varepsilon^T \Theta^T (XE + U_2 S) + (XE + U_2 S)^T \Theta A_{0r} \\
\Gamma K C_{0r} + Q_1 \Theta A_{0r} - B_{0r}^T \Theta^T (XE + U_2 S) \\
-1/\varepsilon \Xi_2 N_{0r} A_{0r} \\
M_{0r}^T (XE + U_2 S) \\
-2\Gamma - Q_1 \Theta B_{0r} - B_{0r}^T \Theta^T Q_1 \\
1/\varepsilon \Xi_2 N_{0r} B_{0r} \\
M_{0r}^T Q_1^T \\
\end{pmatrix}
\begin{pmatrix}
\ast \\
\ast \\
\ast \\
\ast \\
\ast \\
\ast \\
1/\varepsilon \Xi_3 \\
-1/\varepsilon \Xi_3 \\
0 \\
\end{pmatrix}
< 0,
\]

where

\[ \Theta = I + M_{0r} \Xi_1 N_{0r}, \]
\[ \Xi_1 = M_{0r}^T N_{0r}^T (I - N_1 M_1 M_{0r}^T N_{0r}^T)^{-1}, \]
\[ \Xi_2 = (I - N_1 M_1 M_{0r}^T N_{0r}^T)^{-1/2}, \]
\[ \Xi_3 = I - M_{0r}^T N_{0r}^T N_1 M_1, \]
\[ Q_1 = \left[ \begin{array}{c}
\Lambda CV_1 \Sigma_r^{-1} \\
Q_2
\end{array} \right] U^T = \Lambda CV_1 \Sigma_r^{-1} U_1^T + Q_2 U_2^T. \]

Remark 1. The above algorithm includes two steps. The first step to determine the matrix \( \Lambda CV_2 = 0 \) to meet the variable \( \Lambda \). Since \( \Lambda \) is diagonal, and \( C, V_2 \) is known, the collection \( \Psi \) is very easy to determine. The second step is the feasibility of LMI. LMI feasibility problem can be globally convergent interior point method and other algorithms to solve it. Therefore, the Algorithm 1 is convergent.

Example:

Consider the following nonlinear descriptor system

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} =
\begin{pmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} +
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
\phi(y)
\]

\[ y = \begin{pmatrix}
1 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} \]

where \( \phi(y) = \frac{1}{8} \sin(y) + \frac{3}{8} y \). Then \( k = 0.5, \varepsilon_1 = 0.25 \).

We will use the proposed method by the following steps:

1. System equivalent transformation.
Corresponding to (14), we have

\[
E_0 = \begin{bmatrix}
I_{r_0} & 0 \\
-JB_2 \frac{k + \varepsilon}{2} C_1 & -J B_2 \frac{k + \varepsilon}{2} C_2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & 0 \\
-0.375 & -0.375 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
gE = \begin{bmatrix}
0 & -\frac{k + \varepsilon}{2} \sqrt{2} \\
0 & 0
\end{bmatrix} g \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = MgN
\]

\[
= \begin{bmatrix}
0 \\
-\frac{1}{8}
\end{bmatrix} g \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},
\]

\[
A_0 = \begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, B_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ C_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

Find \( P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -\frac{8}{7} & 0 \\ 0 & 0 & 0 \end{bmatrix}, Q = I, \) such that,

\[
E_{0r} = PE_0Q = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
A_{0r} = PA_0Q = \begin{bmatrix}
-1 & 0 & 0 \\
1 & -\frac{8}{7} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Corresponding

\[
M_{0r} = PM = \begin{bmatrix} 0 \\ \frac{1}{3} \\ 0 \end{bmatrix}, N_{0r} = NQ = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

\[
B_{0r} = PB = \begin{bmatrix} 1 \\ -\frac{1}{3} \\ 1 \end{bmatrix}, C_{0r} = C_0Q = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.
\]

Obviously \( K = 0.5, N_2 = 0, ||N_1 M_1|| = ||N Q P M|| < 1.\)

**Step 2.** The solutions to the LMIs of Algorithm 1 are as follows,

\[
\varepsilon = 1.7617
\]

\[
\Lambda = 0.1852
\]

\[
X = \begin{bmatrix}
3.0275 & 0.4292 & -0.0000 \\
0.4292 & 1.1739 & -0.0000 \\
-0.0000 & -0.0000 & 3.3637
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
0.6612 & 0.7665 & -1.3518 \\
Q_2 &= -0.7819 \\
\Gamma &= 5.0875
\end{bmatrix}
\]

\[
Q_1 = \begin{bmatrix} 0.1852 & 0.1852 & -0.7819 \\
3.0275 & 0.4292 & 0 \\
0.4292 & 1.1739 & 0 \\
0.6612 & 0.7665 & -1.3518
\end{bmatrix}
\]

\[
P_1 =XE + U_2S = \begin{bmatrix}
3.0275 & 0.4292 & 0 \\
0.4292 & 1.1739 & 0 \\
0.6612 & 0.7665 & -1.3518
\end{bmatrix}
\]
Then, from Theorem 1, the system is strongly absolutely stable. Figure 1 illustrates the state responses of the nonlinear descriptor system, which shows that the system is stable as expected.

5. Conclusions

The existing results on strong absolute stability for Lur’e descriptor systems (LDS) are mostly based on the admissibility of the linear parts of the systems. In this paper, the linear parts of the systems are allowed to be impulsive. The system under consideration is transformed into an LDS with uncertainty in the derivative matrix and an LMI-based strong absolute stability criterion is derived. In our future work, stabilization problem for the systems under consideration will be considered.

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