SOME RANDOM FIXED POINT THEOREMS AND
RANDOM ALTMAN TYPE INEQUALITY

NING CHEN, BAODAN TIAN, AND JIQIAN CHEN

Abstract. Some new results of common random solution for a class of random operator equations which generalize theorem in [4]-[6], and reference [11] in Banach space. On the other hand, the famous Altman’s inequality is also extended into the type of the determinant form. by comparing some solutions for several examples, we arrive at the main results which are theorem 2.3 and theorem 2.7, theorem 3.1-3.3, theorem 4.1 and theorem 4.2.

Key Words. Random operator, Altman inequality, Comparing equations.

1. Introduction and Preliminaries

In this paper, we shall consider that the common random solution of some random operator equation by random fixed point theorem and the Altman’s type inequality. The random functional analysis developed rapidly in recent years needs that random fixed point theory. Random operators lie at the heart of probabilistic functional analysis and their theory is needed for the study of various type random operator equations. The importance of the application for the random fixed point theorem is very obvious, such as the existence of solution for the equation, multiple solutions problem of the corresponding system and so on.

In this paper, the authors consider the random solution of random operator equation by random fixed point theorem and the Altman’s type inequality, the method is similar as [4],[5] and [6]. And we may further refer to [11,12,13,14,15].

First, we state lemma 1.1(by using of lemma 1 in [5]).

Lemma 1.1 (see lemma 1 in [4]) Assume that $E$ is a separable real Banach space, $X$ is a non-empty closed convex set in $E$, and be a bounded open set in $X$. Let $A : \Omega \times \overline{D} \rightarrow X$ be random semi-closed 1-set contract operator, and $e_0 \in D$ such that

$$A(\omega, x) - \mu e_0 \neq \alpha (x - e_0)$$

where $\alpha$ is a variable that $\alpha > \mu \geq 1, \forall (\omega, x) \in \Omega \times \partial D$, then the operator equation $A(\omega, x) = \mu x$ must have a random solution in $D$. 
2. Some Existence Theorems

Theorem 2.1 Assume that $E$ is a separable real Banach space, $X$ is a non-empty convex and closed set in $E$, $D$ is a bounded open set in $X$. Let $A, B : \Omega \times \overline{D} \to X$ be random semi-closed $1$-set contract operators, $\theta \in D$ and real number $\lambda > 0, \mu \geq 1$ such that for $\forall (\omega, x) \in \Omega \times \partial D$,

$$\|B(\omega, x) - \lambda \mu x\|^6 + \|A(\omega, x)\|^5\|A(\omega, x) + \mu x\| \leq \|A(\omega, x) - \mu x\|^6 + \|\mu x\|^6,$$  \hspace{1cm} (2.1)

then the systems $A(\omega, x) = \mu x, B(\omega, x) = \lambda \mu x$ have a common random solution in $D$.

Proof. By (2.1), we get that

$$\|A(\omega, x)\|^5\|A(\omega, x) + \mu x\| \leq \|A(\omega, x) - \lambda \mu x\|^6 + \|\mu x\|^6, \forall (\omega, x) \in \Omega \times \partial D$$

From theorem 2.2 in [7], we take only $m = 4, \beta = 1, \gamma = 1, \beta_1 = 0$, then this random operator equation $A(\omega, x) = \mu x$ must have a random solution $x^*$ in $D$ such that $A(\omega, x^*) = \mu x^*$, and from (2.1) we have

$$\|B(\omega, x^*) - \lambda \mu x^*\|^6 \leq 0.$$

That is,

$$B(\omega, x^*) = \lambda \mu x^*$$

Hence, we can conclude that system $A(\omega, x) = \mu x, B(\omega, x) = \lambda \mu x$ have a common random solution $x^*$ in $D$. This completes the proof of theorem 2.1.

Theorem 2.2 Assume that same as theorem 1. Let $A, B, C : \Omega \times \overline{D} \to X$ be also random semi-closed $1$-set contract operators, $\theta \in D$ and real number $\lambda > 0, \mu \geq 1$ such that

$$\|C(\omega, x) - 5\mu x\|^8 + \|B(\omega, x) - 3\mu x\|^8 + \|A(\omega, x)\|^7\|A(\omega, x) + \mu x\|$$

$$\leq \|A(\omega, x) - \mu x\|^8 + \|\mu x\|^8, \forall (\omega, x) \in \Omega \times \partial D.$$  \hspace{1cm} (2.2)

therefore, $A(\omega, x) = \mu x, B(\omega, x) = 3\mu x, B(\omega, x) = 5\mu x$ must have common random solution $x^*$ in $D$.

Proof. By (2.2), we get

$$\|A(\omega, x)\|^7\|A(\omega, x) + \mu x\| \leq \|A(\omega, x) - \mu x\|^8 + \|\mu x\|^8, \forall (\omega, x) \in \Omega \times \partial D.$$  \hspace{1cm} (2.3)

then we take $m = 6, \beta = 1, \beta_1 = 0$ in theorem 2.2 of [7]. We omit the proof.

Theorem 2.3 Assume that same as theorem 2.1, and let $A, B_i : \Omega \times \overline{D} \to X$ $(i = 1, 2, \cdots, m)$ are random-closed $1$-set contract operators, $\theta \in D$ and real number $\lambda > 0, \mu \geq 1$ such that

$$\sum_{i=1}^{m} \|B_i(\omega, x) - \lambda_i \mu x\|^{10} + \|A(\omega, x)\|^{10} \leq \|A(\omega, x) - \mu x\|^{10} + \|\mu x\|^{10}, \forall (\omega, x) \in \Omega \times \partial D.$$  \hspace{1cm} (2.4)

then the systems $A(\omega, x) = \mu x, B_i(\omega, x) = \lambda_i \mu x (i = 1, 2, \cdots, m)$ have a common random solution in $D$.

Proof. By (2.4), we have

$$\|A(\omega, x)\|^{10} \leq \|A(\omega, x) - \mu x\|^{10} + \|\mu x\|^{10}, \forall (\omega, x) \in \Omega \times \partial D.$$  \hspace{1cm} (2.5)

and let $\varepsilon_0 = \theta \in D$ by lemma 1.1, then we only prove that $A(\omega, x) \neq \alpha x, \forall (\omega, x) \in \Omega \times \partial D$.

Assume the contrary, we take only $A(\omega_0, x_0) = \alpha_0 x_0$. 

\hspace{1cm}
Substituting it in the above inequality and let $\alpha_0 - \mu = \alpha > 0$, we have
$$(\alpha + \mu)^{10} \leq (\alpha)^{10} + (\mu)^{10} < (\alpha + \mu)^{10}$$
This is a contradiction. Then the system $A(\omega, x) = \mu x$ must have a random solution $x^*$ in $D$.
Thus, by lemma 1.1, $A(\omega, x^*) = \mu x^*$, and substitute it in (2.3), we can get
$$\|B_i(\omega, x^*) - \lambda_i\mu x^*\|^{10} \leq 0, (i = 1, 2, \cdots, m) \quad (2.4)$$
Hence, by (2.4) we obtain
$$B_i(\omega, x^*) = \lambda_i\mu x^*, (i = 1, 2, \cdots, m)$$
Therefore, systems $A(\omega, x) = \mu x, B_i(\omega, x) = \lambda_i\mu x(i = 1, 2, \cdots, m)$ have a common random solution in $D$, which completes the proof of this theorem.

**Corollary 2.4** Let $\lambda_1 = \lambda_2 = \cdots = \lambda_m = \mu^{-1}$ or $\lambda_1 = \lambda_2 = \cdots = \lambda_m = \mu$ and $B(\omega, x) = \mu^2 x$, we get the special case.

Also, We have the following theorem, we omit the proof here.

**Theorem 2.5** Assume that same as theorem 2.1, and let $A, B, C : \Omega \times \overline{D} \rightarrow X$ are also random semi-closed 1-set contract operators, $\theta \in D$ and real number $\lambda > 0, \mu \geq 1$ such that
$$\|A(\omega, x) - B(\omega, x) - C(\omega, x)\|^{10} + \|A(\omega, x)\|^{10} \leq \|A(\omega, x) - \mu x\|^{10} + \|\mu x\|^{10}, \quad (2.5)$$
then the system $A(\omega, x) = \mu x, A(\omega, x) = B(\omega, x) + C(\omega, x)$ have a common random solution in $D$.
Proof. By (2.5) we have
$$\|A(\omega, x)\|^{10} \leq \|A(\omega, x) - \mu x\|^{10} + \|\mu x\|^{10}, \forall (\omega, x) \in \Omega \times \partial D.$$ and similar as the proof of theorem 2.3, let $e_0 \in D$ in lemma 1.1, then we only prove that $A(\omega, x) \neq \alpha x$, for $\forall (\omega, x) \in \Omega \times \partial D$.
If we assume the contrary, we can take $A(\omega_0, x_0) = \alpha_0 x_0$.
Substituting it in the above inequality and let $\alpha_0 - \mu = \alpha > 0$, we have
$$(\alpha + \mu)^{10} \leq (\alpha)^{10} + (\mu)^{10} < (\alpha + \mu)^{10}$$
This is a contradiction, then this random operator equation $A(\omega, x) = \mu x$ must have a random solution $x^*$ in $D$ such that $A(\omega, x^*) = \mu x^*$, substitute it in (2.5), we can get
$$\|A(\omega, x^*) - B(\omega, x^*) - C(\omega, x^*)\|^{10} \leq 0,$$
which implies the system $A(\omega, x) = \mu x, A(\omega, x) = B(\omega, x) + C(\omega, x)$ have a common random solution $x^* \in D$.

**Theorem 2.6** Assume the same as theorem 2.2, and let $A : \Omega \times \overline{D} \rightarrow X$ are also random semi-closed 1-set contract operators, $\theta \in D$ and real number $\lambda > 0, \mu \geq 1$ such that for $\forall (\omega, x) \in \Omega \times \partial D$,
$$\|A(\omega, x) - \mu x\|^{4(m+\beta+\beta_1)} \geq \|A(\omega, x)\|^{4(m+\beta)}\|A(\omega, x)\| + 2\mu x\|^{4\beta_1} - \|\mu x\|^{4(m+\beta+\beta_1)}, \quad (2.6)$$
then the system $A(\omega, x) = \mu x$ must have a common random solution $x^*$ in $D$ such that $A(\omega, x^*) = \mu x^*$, where $m > 1$ is an integer, $\beta \geq 0, \beta_1 \geq 0$. 

Proof. By (2.6), similar as the proof of theorem 2.3, let \( e_0 = \theta \in D \) in lemma 1.1, then we only prove that \( A(\omega, x) \neq \alpha x \), for \( \forall (\omega, x) \in \Omega \times \partial D \).

If we assume the contrary, we can take \( A(\omega_0, x_0) = a_0 x_0 \).

Substituting it in (2.6) and let \( a_0 - \mu = \alpha > 0 \), we have

\[
\alpha^{4(m+\beta+\beta_1)} \geq (\alpha + \mu)^{4(m+\beta)} \cdot (\alpha + 3\mu)^{4\beta_1} - \mu^{4(m+\beta+\beta_1)}
\]

Let \( l_1 = 4(m + \beta), l_2 = 4\beta_1, l = l_1 + l_2 \), we can get

\[
\alpha^l + \mu^l \geq (\alpha + \mu)^{l_1} \cdot (\alpha + 3\mu)^{l_2} > (\alpha + \mu)^l,
\]

This is a contradiction, then this random operator equation \( A(\omega, x) = \mu x \) must have a random solution \( x^* \) in \( D \) such that \( A(\omega, x^*) = \mu x^* \) by lemma 1.1, which ends the proof of theorem 2.6.

On the other hand, we consider that inequality as (2.7) bellow.

**Theorem 3.7** Assume that \( E \) is a separable real Banach space, \( X \) is a non-empty convex and closed set in \( E, D \) is a bounded open set in \( X \), Let \( A : \Omega \times \overline{D} \rightarrow X \)

are also random semi-closed 1-set contract operators, \( \theta \in D \) and real number \( \lambda > 0, \mu \geq 1 \) such that for \( \forall (\omega, x) \in \Omega \times \partial D \),

\[
\| A(\omega, x) - \mu x \| + \| A(\omega, x) + \mu x \| \geq 2(\| A(\omega, x) \|^3 + 3\| A(\omega, x) \|\| \mu x \|)^3), \quad (2.7)
\]

then the system \( A(\omega, x) = \mu x \) must have a common random solution \( x^* \) in \( D \).

Proof. By (2.7), similar as the proof of theorem 2.6, let \( e_0 = \theta \in D \) in lemma 1.1, then we only prove that \( A(\omega, x) \neq \alpha x \), for \( \forall (\omega, x) \in \Omega \times \partial D \).

If we assume the contrary, we can take \( A(\omega_0, x_0) = a_0 x_0 \).

Substituting it in (2.6) we have

\[
(a_0 - \mu)^3 + (a_0 - \mu)^3 \neq 2(a_0^3 + 3a_0\mu^2)
\]

This is a contradiction, then this random operator equation \( A(\omega, x) = \mu x \) must have a random solution \( x^* \) in \( D \) such that \( A(\omega, x^*) = \mu x^* \), and we completes the proof of this theorem.

**3. Altman Type Inequality**

We can extend this Altman’s inequality into the type of determinant form (also see [4]) as following theorem 3.1.

Let \( n \)-order determinate (as symmetry form)

\[
D_n = \begin{vmatrix}
\| A(\omega, x) \| & \| \mu x \| & \cdots & \| \mu x \| \\
\| \mu x \| & \| A(\omega, x) \| & \cdots & \| \mu x \| \\
\vdots & \vdots & \ddots & \vdots \\
\| \mu x \| & \| \mu x \| & \cdots & \| A(\omega, x) \| \\
\end{vmatrix}
\]

**Theorem 3.1** Suppose that same as theorem 2.2, and substituting (2.2) in the following form,

\[
\| A(\omega, x) - \mu x \|^{2n+1} \geq D_n D_{n+1}, \forall (\omega, x) \in \Omega \times \partial D, \quad (3.1)
\]

then the systems \( A(\omega, x) = \mu x \) have a common random solution in \( D \).

Proof. By calculating that we easy have

\[
D_n = (\| A(\omega, x) \| + (n - 1)\| \mu x \|)(\| A(\omega, x) \| - \| \mu x \|)^{n-1}
\]

and

\[
D_{n+1} = (\| A(\omega, x) \| + n\| \mu x \|)(\| A(\omega, x) \| - \| \mu x \|)^n
\]

Therefore, by lemma 1.1 that we take \( A(\omega_0, x) = a_0 x \),
Substitute it in (3.1), we get that
\[(\alpha_0 - \mu)^{2n+1} \geq (\alpha_0 + (n-1)\mu)(\alpha_0 - \mu)^{n-1}(\alpha_0 + n\mu)(\alpha_0 - \mu)^n\]

And let \(\alpha_0 - \mu = \alpha > 0\), we have
\[\alpha^{2n+1} \geq (\alpha + (n-1)\mu)(\alpha + n\mu)\alpha^{2n-1}\]

Therefore, we get a contradiction. Then the random operator equation \(A(\omega, x) = \mu x\) have a common random solution in \(D\) by lemma 1.1.

Thus, we complete the proof of this theorem 3.1.

Corollary 3.2 Let \(n = 2\), then we get random Altman type inequality,
\[
\|A(\omega, x) - \mu x\|^2 \geq \|A(\omega, x)\|^2 - \|\mu x\|^2.
\]

Now We consider also that similar as conclusion for the inequality,
\[
\|A(\omega, x) - \mu x\|^{n+m} \geq D_n D_m,
\]
or the case
\[
\|A(\omega, x) - \mu x\|^{n+m+l} \geq D_n D_m D_l,
\]
where \(l, m, n\) are positive integers and we omit the proof of it.

Let \(n\)-order determinate (as symmetry form)
\[
D_n = \begin{vmatrix}
\|A(\omega, x)\|^2 & \|\mu x\|^2 & \cdots & \|\mu x\|^2 \\
\|\mu x\|^2 & \|A(\omega, x)\|^2 & \cdots & \|\mu x\|^2 \\
\vdots & \vdots & \ddots & \vdots \\
\|\mu x\|^2 & \|\mu x\|^2 & \cdots & \|A(\omega, x)\|^2
\end{vmatrix}
\] (3.2)
then we easy have following similar conclusion.

Theorem 3.3 Assume that same as theorem 3.1, and satisfies following inequality,
\[
\|A(\omega, x) - \mu x\|^{2n} \geq D_n, \forall (\omega, x) \in \Omega \times \partial D,
\]
then the systems \(A(\omega, x) = \mu x\) have a random solution in \(D\).

Proof. By calculating (3.2), we can easily have
\[
D_n = (\|A(\omega, x)\|^2 + (n-1)\|\mu x\|^2)(\|A(\omega, x)\|^2 - \|\mu x\|^2)^{n-1}
\]

Therefore, by lemma 1.1 that we take \(A(\omega_0, x) = \alpha_0 x\),
Substitute it in (3.3), we get that
\[
(\alpha_0 - \mu)^{2n} \geq (\alpha_0^2 + (n-1)\mu^2)(\alpha_0^2 - \mu^2)^{n-1}
\]

And let \(\alpha_0 - \mu = \alpha > 0\), we have
\[\alpha^{2n} \geq [(\alpha + \mu)^2 + (n-1)\mu^2][(\alpha + 2\mu)^n - \alpha^{n-1}] \geq \alpha^{2n},\]

Therefore, we get a contradiction. Then the random operator equation \(A(\omega, x) = \mu x\) have a common random solution \(x^*\) in \(D\) by lemma 1.1. Thus, we complete the proof of this theorem 3.3.

We consider that following \(n\)-order determinate (as non-symmetry form) in the same way,
\[
D_n = \begin{vmatrix}
\|A(\omega, x) + \mu x\| & \|\mu x\| & \cdots & \|\mu x\| \\
\|A(\omega, x) + \mu x\| & \|A(\omega, x)\| & \cdots & \|\mu x\| \\
\vdots & \vdots & \ddots & \vdots \\
\|A(\omega, x) + \mu x\| & \|A(\omega, x)\| & \cdots & \|A(\omega, x)\|
\end{vmatrix}
\]
Theorem 3.4 Assume that same as theorem 3.1, and satisfies following inequality,

\[ \|A(\omega, x) - \mu x\|^{n} \geq D_{n}, \forall (\omega, x) \in \Omega \times \partial D, \]  

(3.4)

then the systems \(A(\omega, x) = \mu x\) have a random solution in \(D\).

Proof. By calculating we can easily have

\[ D_{n} = (\|A(\omega, x) + \mu x\|)(\|A(\omega, x)\| - \|\mu x\|)^{n-1}, \]

then similar as the proof of theorem 3.1, we take

\[ A(\omega_{0}, x_{0}) = \alpha_{0}x_{0}, \]

Substitute it in (3.4), we get that

\[ (\alpha_{0} - \mu)^{n} \geq (\alpha_{0} + \mu)(\alpha_{0} - \mu)^{n-1}, \]

And let \( \alpha_{0} - \mu = \alpha > 0 \), we have

\[ \alpha^{n} \geq (\alpha + 2\mu)\alpha^{n-1} > \alpha^{n}, \]

Therefore, we get a contradiction. And by lemma 1.1, \(A(\omega, x) = \mu x\) must have a common random solution in \(D\).

Thus, we complete the proof of this theorem 3.4.

Theorem 3.5 Assume that same as theorem 3.3, and satisfies following inequality,

\[ \|A(\omega, x) - \mu x\|^{n} \geq (1 - \varepsilon)D_{n}, \forall (\omega, x) \in \Omega \times \partial D, \]  

(3.5)

then the systems \(A(\omega, x) = \mu x\) have a random solution in \(D\) for \(0 < \varepsilon < 1\).

Proof. From

\[ D_{n} = (\|A(\omega, x) + \mu x\|)(\|A(\omega, x)\| - \|\mu x\|)^{n-1}, \]

By calculating, and we take \(A(\omega_{0}, x_{0}) = \alpha_{0}x_{0}, \)

And substitute it in (3.5), we get that

\[ (\alpha_{0} - \mu)^{n} \geq (1 - \varepsilon)(\alpha_{0} + \mu)(\alpha_{0} - \mu)^{n-1}, \]

Hence, we have

\[ (\alpha_{0} - \mu)^{2} \geq (1 - \varepsilon)(\alpha_{0} + \mu)(\alpha_{0} - \mu), \]

Then we get a contradiction. By lemma 1.1 \(A(\omega, x) = \mu x\) must have a random solution in \(D\), and we complete the proof of this theorem.

4. Some random solutions

Theorem 4.1 Suppose that same as theorem 2.2, substituting (2.2) in the following form, for \(\forall (\omega, x) \in \Omega \times \partial D, \)

\[ \|A(\omega, x)\|^{2} \leq \|\mu x + \delta A(\omega, x)\|^{2} + \|\mu x - \delta A(\omega, x)\|^{2} + \|\mu x + \delta A(\omega, x)\| \]

(4.1)

then the equation \(A(\omega, x) = \mu x\) have a random solution in \(D\) for \(0 \leq \delta < 1\).

Proof. Similar as the proof of theorem 3.1, we only take \(A(\omega_{0}, x_{0}) = \alpha_{0}x_{0}, \)

And substitute it in (4.1), we get that

\[ \alpha_{0}^{2} \leq (\mu x + \delta \alpha_{0})^{2} + (\mu^{2} - \delta^{2} \alpha_{0}^{2}), \]

that is,

\[ 2\mu^{2} + 2\delta \alpha_{0} \mu - \alpha_{0}^{2} \geq 0, \]

while the discriminate,

\[ \Delta = 4[\delta \alpha_{0}^{2} + 2\alpha_{0}^{2}] > 0, \]

Then we get a contradiction. Hence, by lemma 1.1 \(A(\omega, x) = \mu x\) must have a random solution in \(D\).
Theorem 4.2 Suppose that same as theorem 2.2, substituting (2.2) in the following form, for \( \forall (\omega, x) \in \Omega \times \partial D \),
\[
(1 - \varepsilon)\|A(\omega, x)\|^2 \leq (1 + 5\varepsilon)\|\mu x - \delta A(\omega, x)\|\|\mu x + \delta A(\omega, x)\|,
\]
then equation \( A(\omega, x) = \mu x \) have a random solution in \( D \) for \( 0 \leq \delta < 1 , 0 < \varepsilon < 1 \).
Proof. Similar as the proof of theorem 4.1, we only take \( A(\omega_0, x_0) = \alpha_0 x_0 \),
And substitute it in (4.2), we have
\[
(1 - \varepsilon)\alpha_0^2 \leq (1 + 5\varepsilon)(\mu^2 - \delta^2\alpha_0^2),
\]
that is,
\[
(1 + 5\varepsilon)\mu^2 - [(1 - \varepsilon) + (1 + 5\varepsilon)\delta^2]\alpha_0^2 \geq 0,
\]
while the discriminate is that,
\[
\Delta = 4[(1 + 5\varepsilon)(1 - \varepsilon) + (1 + 5\varepsilon)\delta^2]\alpha_0^2 > 0,
\]
Then we get a contradiction.
Hence, by lemma 1.1 the equation \( A(\omega, x) = \mu x \) have a random solution in \( D \) for \( 0 \leq \delta < 1 , 0 < \varepsilon < 1 \).

Corollary 4.3 Let \( \delta = 0, \varepsilon = 0 \), then in theorem 4.2 holds condition
\[
\|A(\omega, x)\|^2 \leq \|\mu x\|^2. \tag{*}
\]

5. Comparing several random operator’s equations

From corollary 4.5 of theorem 4.3(see [5] or [6]), we can easily get following examples(see [6]).

Example 1. We consider the following equation similar as example 2 in [6]or7,\[
\sin(x + 3\omega) + \frac{1}{6}\sin(x + \omega) - 2x = 0, \ \omega \in \Omega = [0, 1], \tag{5.1}
\]
which must have a random solution in \([-\pi, \pi]\).

Example 2. We consider the following equation similar as example in [6],
\[
\sin(x + 3\omega) + \frac{1}{5}\sin(x + \omega) - 2x = 0, \ \omega \in \Omega = [0, 1], \tag{5.2}
\]
which must have a random solution in \([-\pi, \pi]\).

Example 3. We consider again the following equation,
\[
\sin(x + 3\omega) + \frac{1}{2}\sin(x + \omega) - 2x = 0, \ \omega \in \Omega = [0, 1], \tag{5.3}
\]
which must have a random solution in \([-\pi, \pi]\).

Example 4. We consider again the following equation,
\[
\sin(x + 3\omega) + \frac{1}{3}\sin(x + \omega) - 2x = 0, \ \omega \in \Omega = [0, 1], \tag{5.4}
\]
which must have a random solution in \([-\pi, \pi]\).

Example 5. We consider again the following system,
\[
\sin(x + y + \omega) - 2x = 0, \cos(x - y + \omega) - 3y = 0, \tag{5.5}
\]
where \( \omega \in \Omega = [0, 1], (x, y) \in D = \{(x, y)| |x| < \pi, |y| < \pi\}. \) It must have a random solution in \( D \)(see example 1 in [6]).

We will calculate these for \( \omega = 1/2, 1/3, 1/4, 1 \in [0, 1] \) that shows some behavior of solution for equation by table 1-5 as follow.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>( \sin(x + 3\omega) + \frac{1}{3}\sin(x + \omega) - 2x = 0 )</th>
</tr>
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<tr>
<td>( \omega )</td>
<td>1/2</td>
</tr>
<tr>
<td>Solution(x)</td>
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</tr>
</tbody>
</table>

<table>
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<th>( \sin(x + 3\omega) + \frac{1}{5}\sin(x + \omega) - 2x = 0 )</th>
</tr>
</thead>
<tbody>
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<td>( \omega )</td>
<td>1/2</td>
</tr>
<tr>
<td>Solution(x)</td>
<td>0.5333583804</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Equation 3</th>
<th>( \sin(x + 3\omega) + \frac{1}{2}\sin(x + \omega) - 2x = 0 )</th>
</tr>
</thead>
<tbody>
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<td>( \omega )</td>
<td>1/2</td>
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<tr>
<td>Solution(x)</td>
<td>0.6471300728</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 4</th>
<th>( \sin(x + 3\omega) + \frac{1}{3}\sin(x + \omega) - 2x = 0 )</th>
</tr>
</thead>
<tbody>
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<td>( \omega )</td>
<td>1/2</td>
</tr>
<tr>
<td>Solution(x)</td>
<td>0.5830525072</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 5</th>
<th>( \sin(x + y + \omega) - 2x = 0 ) ( \cos(x - y + \omega) - 3y = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>1/2</td>
</tr>
<tr>
<td>Solution (x, y)</td>
<td>( x = 0.496373799, y = 0.2508733866 )</td>
</tr>
<tr>
<td>Equation 5</td>
<td>( \sin(x + y + \omega) - 2x = 0 ) ( \cos(x - y + \omega) - 3y = 0 )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>1/4</td>
</tr>
<tr>
<td>Solution (x, y)</td>
<td>( x = 0.4145476935, y = 0.3129400846 )</td>
</tr>
</tbody>
</table>

**Remark.** Comparing this equation (in examples) that exists solution in theory that we prove these theorems are true, and look at these numerical calculating to other some examples more models of phenomenon shows some behavior of solution for some other equations intimately.

### 6. Concluding Remarks

In this paper, we give out some results of that random common solution for the operators systems, and combining Altman inequality for determinant, and extend some results in [5].

Recently, the impulsive stochastic differential delay equations is also a very interesting topic, and we may see [8] and [9] etc. That application of the random fixed point theorem of random draw or compression for some integral equations.
In our future work, we may try to do some research in this field and maybe could obtain some better results.

References