CAPITAL CONTROL AND EFFECTS OF FISCAL AND MONETARY POLICY

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Abstract. This paper numerically analyzes the effectiveness of monetary and fiscal policies, and their associated risks, on growth rate, product volatility, inflation rate, exchange rate under capital controls, domestic interest rate, foreign interest rate and foreign pricing volatility respectively by using an extended equilibrium in a stochastic dynamic optimal model with capital controls. The research results show that the effectiveness of the monetary policy and fiscal policy change with capital controls, domestic interest rate, foreign interest rate and foreign pricing volatility. Capital control has not affected the effects of Monetary and a fiscal policy on growth rate, product volatility, inflation rate and exchange rate for lower interest rate, and has no impact on the effects on growth rate and product volatility for higher interest rate. The impacts of capital controls on the effects of monetary and fiscal policy in lower product volatility country are same as that in higher product volatility country.

Key Words. capital flows, policy effect, and numerical analysis.

1. Introduction

After the classic Fleming(1962)- Mundell(1963) model was published, it has been extensively known that international capital mobility is an important factor for the effectiveness of money policy and fiscal policy in open economy. Mundell-Fleming model implies that the international capital mobility can raise effectiveness of money policy, and diminish the effectiveness of fiscal policy as measured in terms of its short-run effect on output in open economy.

Sutherland (1996) and Senay (2000) presented the key result of the Mundell-Fleming model by using the general dynamic money equilibrium model developed by Obstfeld and Rogoff (1995), which can be used to analyze money policy effect on output. They show that the capital flow changing from imperfect to perfect can improve the effectiveness of money policy by using two countries sticky price model of Sutherland (1996) and Senay (2000). So, as Mundell-Fleming model implies, the higher international capital mobility is, the more effective money policy is, and capital mobility moving from imperfect to perfect diminishes the effectiveness of fiscal policy.

Christian Pierdzioch (2004) use a dynamic general equilibrium two-country optimizing 'new-open economy macroeconomics' model to analyze the consequences
of international capital mobility for the effectiveness of fiscal policy. It shows that a higher degree of capital mobility can also increase the effectiveness of fiscal policy. Christian Pierdzioch (2003) use the same model to show that a higher degree of capital mobility increases the effectiveness of monetary policy only if the Marshall-Lerner condition holds.

This paper extends the equilibrium in the stochastic dynamic optimal model employed by Turnovsky (2000) to a framework of modelling a world economy with capital controls. This paper numerically analyzes the impacts of international capital mobility and interest rate on the effectiveness of money policy, fiscal policy and their uncertainties. We use a downward sloping supply schedule for the foreign bond that reflects a borrowing premium. The results indicate that the capital control and interest rate play an important role in the effects of the monetary, fiscal policy and their uncertainties. As such, the capital movements impact the effects of money policy, fiscal policy and their uncertainties on equilibrium growth rate, product volatility, inflation rate, expected depreciation. Our results generalize the implications of Mundell-Fleming model.

We organize the remainder of the paper as follows. In the Section 2, we lay out the analytical model. In the Section 3, we numerically analyze the effects of domestic monetary and government expenditure policies, and their associated risks, on the equilibrium under the capital controls. The Section 4 is conclusion.

2. Stochastic equilibrium model with capital controls

2.1. Prices and Asset Returns. The basic analytical framework we adopt is an extension of Turnovsky’s (2000) model. There are three prices in the model: P, the domestic price of the traded good; Q, the foreign price level of the traded good; and E, the nominal exchange rate, measured in terms of domestic currency per unit of foreign currency. While Q is assumed to be exogenous, P and E are to be endogenously determined. These prices evolve in accordance with the geometric Brownian motion processes:

\[
\frac{dP}{P} = \pi dt + dp, \quad \frac{dQ}{Q} = \eta dt + dq, \quad \frac{dE}{E} = \varepsilon dt + de
\]

where, \( \pi, \eta \) and \( \varepsilon \) are expected instantaneous rates of change respectively. The terms \( dp, dq \) and \( de \) are temporally independent, normally distributed random variables with zero means and instantaneous variances \( \sigma^2_p dt \), \( \sigma^2_q dt \) and \( \sigma^2_e dt \).

Taking stochastic calculus for PPP relationship, \( \dot{P} = EQ \), results in

\( \pi = \eta + \varepsilon + \sigma_{pe} \)

(1)

\( dp = dq + de \)

(2)

The flow of domestic output, \( dY \), is produced from capital, \( K \), by means of the stochastic constant returns technology \( dY = \alpha K (dt + dy) \). Domestic and foreign bonds are assumed to be short bonds, paying nonstochastic nominal interest rates \( i \) and \( i^* \) respectively over the period \( dt \). Using the Ito calculus, the real rates of return for domestic residents on their holdings of money, domestic bond, and foreign bond respectively are:

\[
\begin{align*}
\frac{dR_K}{K} &= \frac{dY}{K} - \alpha dt + \alpha dy \\
\frac{dR_M}{M} &= r_M dt - dp; r_M \equiv -\pi + \sigma^2_p \\
\frac{dR_B}{B} &= r_B dt - dp; r_B \equiv i - \pi + \sigma^2_p \\
\frac{dR_F}{F} &= r_F dt - dq; r_F \equiv i^* + c(n_F) - \eta + \sigma^2_q
\end{align*}
\]
where, \( c(n_F) \) is the degree of capital control specified by domestic government, and \( n_F \) is the share of portfolio held in foreign bonds:

\[
\begin{align*}
&\text{If } n_F < 0 \text{ (home county is net debtor), then } c(n_F) > 0, c'(n_F) < 0; \\
&\text{If } n_F > 0 \text{ (home county is net creditor), then } c(n_F) < 0, c'(n_F) < 0.
\end{align*}
\]

For simplicity, we take \( c(n_F) \) as a linear function of \( n_F \),

\[
(3) \quad c(n_F) = -\varphi n_F, \varphi > 0
\]

where \( \varphi \) is the specified degree of capital controls. The smaller \( \varphi \) is, the higher international capital mobility is.

2.2. Consumer Optimization. The representative consumer’s asset holdings are subject to the wealth constraint

\[
(4) \quad W = \frac{M}{P} + \frac{B}{P} + \frac{EB^*}{P} + K
\]

where \( W \) denotes real wealth. In addition, the consumer is assumed to consume output over the period \((t, t + dt)\) at a nonstochastic rate \( C(t)dt \) generated by the asset holdings.

The objective of the representative agent is to select his or her rate of consumption and his or her portfolio of asset to maximize the expected value of lifetime utility:

\[
(5) \quad \max E_0 \int_0^\infty \ln[C^\theta(M/P)^{1-\theta}] e^{-\beta t} dt
\]

subject to the wealth constraint (4) and the stochastic wealth accumulation equation, expressed in real terms as

\[
(6) \quad \frac{dW}{W} = \psi dt + dw
\]

where

\[
\begin{align*}
n_M &= \frac{M}{W} = \text{share of portfolio held in money} \\
n_B &= \frac{B}{W} = \text{share of portfolio held in domestic government bonds} \\
n_K &= \frac{K}{W} = \text{share of portfolio held in terms of capital} \\
n_F &= \frac{EB^*}{W} = \text{share of portfolio held in foreign bonds}
\end{align*}
\]

The stochastic optimization problem can be expressed as choosing the consumption-wealth ratio, \( C/W \) and the portfolio shares, \( n_i \), to maximize (5), subject to

\[
\begin{align*}
\frac{dW}{W} &= \psi dt + dw \\
n_M + n_B + n_K + n_F &= 1 \\
\psi &= n_M \gamma (\tilde{M} - \tilde{M}_0) + n_B \tau (\tilde{B} - \tilde{B}_0) + n_K \tau (\tilde{K} - \tilde{K}_0) + n_F \tau (\tilde{F} - \tilde{F}_0) \\
dw &= n_M d\tilde{M} + n_B d\tilde{B} + n_K d\tilde{K} + n_F d\tilde{F}
\end{align*}
\]

Government expenditure policy is specified by the process
\[ dG = g \alpha K dt + \alpha K dz \]

where \( dz \) is an intertemporally independent, normally distributed random variable with zero mean and variance \( \sigma^2_z dt \).

Monetary policy is specified by the stochastic growth rule:

\[ \frac{dM}{M} = \phi dt + dx \]

where \( \phi \) is the mean monetary growth rate and \( dx \) is an independently distributed random variable with zero mean and variance \( \sigma^2_x dt \). In this paper, we shall assume that the stochastic component of the monetary growth rate is exogenous.

Debt policy is formulated in terms of ratio of domestic government bonds to money

\[ \frac{B}{M} = \mu \]

where \( \mu \) is a policy parameter set by government.

Define \( \omega \equiv \frac{nK}{(nK + nF)} \) be the share of capital in the consumer’s portfolio. The detail of the optimization is in Appendix A. The optimal shares can be expressed as follows:

\[ C = \theta \beta \]

\[ n_M = \frac{(1 - \theta) \beta}{i} \]

\[ (r_K - r_B) dt = cov(dw, \alpha dy + dp) \]

\[ (r_F - \varphi n_F - r_B) dt = cov(dw, dp - dq) \]

The optimal shares imply that asset portfolios are constant in time, that is,

\[ \frac{d(M/P)}{M/P} = \frac{d(B/P)}{B/P} = \frac{d(EB^*/P)}{EB^*/P} = \frac{dK}{K} = \frac{dW}{W} = \psi dt + dw \]

By current account balance,

\[ d\left(\frac{EB^*}{P}\right) = [dY - dC - dK - dG] + \left(\frac{EB^*}{P}\right)dR_F. \]

we have

\[ d\left(\frac{EB^*}{P} + dK\right) \]

\[ = [\alpha K(1 - g) - C + \left(\frac{EB^*}{P}\right)(i^* - \varphi n_F - \eta + \sigma^2_q)] dt \]

\[ + \alpha K(dy - dz) - \left(\frac{EB^*}{P}\right)dq, \]

which yields

\[ \psi = \left[ \omega \alpha (1 - g) - \frac{\theta \beta}{nK} + (1 + \omega)[i^* - \varphi n_F - \eta + \sigma^2_q] \right] \]

\[ dw = \alpha \omega(dy - dz) - (1 - \omega dq) \]
\[ \sigma_w^2 = \alpha^2 \omega^2 (\sigma_y^2 + \sigma_z^2) + (1 - \omega)^2 \sigma_q^2 \]

Constant asset portfolio implies
\[ \frac{M/P}{K + EB^*/P} = \frac{n_M}{n_K + n_F} \]

Taking stochastic calculus, we have
\[
\frac{dP}{P} = \pi dt + dp = \frac{dM}{M} - \frac{d[EB^*/P + K]}{EB^*/P + K} - (\frac{dM}{M})(\frac{d[EB^*/P + K]}{EB^*/P + K}) + (\frac{d[EB^*/P + K]}{EB^*/P + K})^2
\]
which results in
\[
\pi = \phi - \psi + \sigma_w^2
\]

\[ dp = dx - \alpha \omega (dy - dz) + (1 - \omega) dq \]

By (2),
\[
de = dx - \alpha \omega (dy - dz) - \omega dq
\]

By (1), (7)-(16), we can obtain the following equilibrium relationships
\[
\pi = \eta + \varepsilon - \omega \sigma^2
\]

\[ i = \alpha + \pi - \sigma_z^2 - \alpha^2 \omega \sigma_y^2 \]

\[ i = \alpha + \phi - \sigma_z^2 + \alpha^2 \omega^2 \sigma_y^2 - \alpha \omega \sigma_y^2 + \alpha^2 \omega^2 \sigma_z^2 + (1 - \omega)^2 \sigma_q^2 \]

\[ \varepsilon = i - i^* + \sigma_z^2 + \varphi n_F \]

\[ \omega = \frac{\alpha - i^* + 2 \varphi n_F + \eta}{(\alpha^2 \sigma_y^2 + \sigma_q^2)} \]

3. The effects of domestic policy and policy uncertainty

We now analyze the effects of domestic monetary and government expenditure policies, and their associated risks, on the equilibrium under capital controls.
3.1. The effects of domestic interest rate. To understand the mechanism of policy transform, it is necessary to understand the impacts of changes of domestic nominal interest rate on the equilibrium consumption-wealth ratio, growth rate, and the portfolios. Stochastic equilibrium solutions simplify to the following:

\[
\frac{C}{W} = \theta \beta
\]

(23) \[\psi = \left[ \omega \alpha(1 - g) - \frac{\theta \beta}{1 - (1 + \mu)n_M} \right] + (1 - \omega)[i^* - \varphi n_F - \eta + \sigma^2_q] \]

(24) \[n_M = \frac{(1 - \theta)\beta}{i} \]

(25) \[n_F = (1 - \omega)[1 - (1 + \mu)n_M] \]

Differentiating the equilibrium set of relationships above (22)-(25) and (21), the following expressions can be derived:

(26) \[\frac{\partial n_M}{\partial i} = -\frac{(1 - \theta)\beta}{i^2} \]

(27) \[\frac{\partial n_F}{\partial i} = (1 - \omega)\frac{(1 - \theta)\beta(1 + \mu)}{i^2} - \frac{(\alpha^2\sigma_y^2 + \sigma_q^2)}{(\alpha^2\sigma_y^2 + \sigma_q^2) + 2[1 - (1 + \mu)n_M] \varphi} \]

(28) \[\frac{\partial \omega}{\partial i} = \frac{2 \varphi}{\alpha^2\sigma_y^2 + \sigma_q^2} \frac{\partial n_F}{\partial i} \]

(29) \[\frac{\partial \psi}{\partial i} = \alpha(1 - g) \frac{\partial \omega}{\partial i} + \frac{\theta \beta(1 + \mu)}{i^2} \frac{(1 - \theta)\beta}{(1 + \mu)n_M} \]

\[\frac{\partial \psi}{\partial i} - (1 - \omega)\varphi \frac{\partial n_F}{\partial i} - (i^* - \varphi n_F - \eta + \sigma^2_q) \frac{\partial \omega}{\partial i} \]

\[= \frac{(1 + \mu)(1 - \theta)\beta}{i^2} \left[ \frac{\theta \beta}{(1 - (1 + \mu)n_M)^2} \right] + (1 - \omega)[(\alpha^2\sigma_y^2 + \sigma_q^2)] \]

3.2. The effects of domestic policy and policy uncertainty. Differentiating the equation (19), we can obtain

(30) \[\frac{\partial i}{\partial \psi}, \frac{\partial i}{\partial \sigma^2_y}, \frac{\partial i}{\partial \sigma^2_q}, \frac{\partial i}{\partial \sigma^2_z}, \frac{\partial i}{\partial \sigma^2_y}, \frac{\partial i}{\partial \sigma^2_q}, \frac{\partial i}{\partial \sigma^2_z} \]

But the expressions of the derivatives are complicated: we have to give some numerical results about the expressions of derivatives in the following sections, in which we shall use software MATLAB7.0. The parameters are taken as following:

\[\alpha = 0.2, \beta = 0.04, \theta = 0.8, i^* = 0.03, \eta = 0.03, \mu = 0.2, \]

\[g = 0.2, \sigma^*_y = 0.1^2, \sigma^*_z = 0.1^2, \sigma^*_q = 0.04^2, \sigma^*_q = 0.2^2.\]
3.2.1. The response of policy effects to capital mobility and domestic interest rate. In this section, the response of policy effects to capital mobility and domestic interest rate are analyzed numerically.

Figure 1 illustrates how international capital mobility and domestic interest rate affect the effectiveness of monetary and fiscal policy on growth rate. As a result, we can see that \( \frac{\partial \psi}{\partial \phi} > 0 \), \( \frac{\partial \psi}{\partial \sigma^2_x} < 0 \), \( \frac{\partial \psi}{\partial g} < 0 \), \( \frac{\partial \psi}{\partial \sigma^2_z} > 0 \) for lower interest rate and that \( \frac{\partial \psi}{\partial \phi} \), \( \frac{\partial \psi}{\partial \sigma^2_x} \), \( \frac{\partial \psi}{\partial g} \), \( \frac{\partial \psi}{\partial \sigma^2_z} \) equal to 0 for higher interest rate that is the effects of monetary and fiscal policy on growth rate are zero.

Figure 1, X axis- \( \phi \), Y axis- interest rate, Z axis- \( \frac{\partial \psi}{\partial \phi} \), \( \frac{\partial \psi}{\partial \sigma^2_x} \), \( \frac{\partial \psi}{\partial g} \), \( \frac{\partial \psi}{\partial \sigma^2_z} \), respectively.

Figure 1-1 explains that for lower interest rate, higher degree of capital controls may cause sharp decline in effect of policy \( \frac{\partial \psi}{\partial \phi} \) but then a modest increase follows. Figure 1-2 presents that for lower interest rate, higher degree of capital controls may cause sharp rise (negative effects decline) in effect of policy \( \frac{\partial \psi}{\partial \sigma^2_x} \), then gradually decreases (negative effects rise). Figure 1-3 shows that for lower interest rate, increasing degree of capital controls may cause sharp rise in effect of policy \( \frac{\partial \psi}{\partial g} \) from negative to zero. Figure 1-4 indicates that for lower interest rate, increasing degree of capital controls may cause sharp decline in effect of policy \( \frac{\partial \psi}{\partial \sigma^2_z} \) from positive to zero.

Figure 2 interprets how international capital mobility and domestic interest rate affect the effectiveness of monetary and fiscal policy on product volatility. It follows that \( \frac{\partial \sigma^2_w}{\partial \phi} < 0 \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_x} > 0 \), \( \frac{\partial \sigma^2_w}{\partial g} > 0 \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_z} < 0 \) for lower interest rate and that \( \frac{\partial \sigma^2_w}{\partial \phi} \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_x} \), \( \frac{\partial \sigma^2_w}{\partial g} \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_z} \) equal to 0 for higher interest rate that is the effects of monetary and fiscal policy on product volatility are zero.

Figure 2, X axis- \( \phi \), Y axis- interest rate, Z axis- \( \frac{\partial \sigma^2_w}{\partial \phi} \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_x} \), \( \frac{\partial \sigma^2_w}{\partial g} \), \( \frac{\partial \sigma^2_w}{\partial \sigma^2_z} \).

Figure 3 shows how international capital mobility and domestic interest rate affect the effectiveness of monetary and fiscal policy on inflation rate. It follows
that $\frac{\partial \pi}{\partial \phi} > 0$, $\frac{\partial \pi}{\partial \sigma^2} < 0$ for lower interest rate and the effects of monetary and fiscal policy on inflation rate equal to zero for higher interest rate. The effects of fiscal policy on inflation rate are $\frac{\partial \pi}{\partial g} < 0$, $\frac{\partial \pi}{\partial \sigma^2} > 0$ for all levels of interest rate.

Figure 3, X axis-$\phi$, Y axis- interest rate, Z axis- $\frac{\partial \pi}{\partial \phi}$, $\frac{\partial \pi}{\partial \sigma^2}$, $\frac{\partial \pi}{\partial g}$, $\frac{\partial \pi}{\partial \sigma^2}$, respectively.

The behavior that international capital mobility and domestic interest rate affect the effectiveness of monetary and fiscal policy on exchanged rate are the same as the behavior that the international capital mobility and domestic interest rate affect the effectiveness of monetary and fiscal policy on inflation rate (hence details are omitted).

It can be seen from the numerical results that monetary and fiscal policies have significant effects and remain the same sign for lower interest rate. But the effects vary with the change of degree of capital controls.

For higher interest rate, monetary and fiscal policies have no effects on growth rate, product volatility. With the change of capital controls, the effects of fiscal and monetary policies on inflation rate and exchange rate do not change.

3.2.2. The response of policy effects to capital mobility and product volatility. In this section, how the domestic capital flow and product volatility affect the effects of monetary and fiscal policies will be analyzed numerically.

It can be seen from Figure 6 that domestic product volatility has no impact on effectiveness of monetary policy and fiscal policy. The increase of the degree of capital controls causes monetary policy effect (positive and negative) declines sharp and then modestly rises. The increase of the degree of capital controls makes fiscal policy effect decreases sharp.

Figure 6, X axis-$\phi$, Y axis- product volatility, Z axis- $\frac{\partial \pi}{\partial \phi}$, $\frac{\partial \pi}{\partial \sigma^2}$, $\frac{\partial \pi}{\partial g}$, $\frac{\partial \pi}{\partial \sigma^2}$, respectively.

Other numerical results for effects of monetary and fiscal policy show that domestic product volatility has no impact to the effectiveness of monetary and fiscal
policy on inflation rate, interest rate, exchange rate. (Numerical results are omitted)

3.2.3. The response of policy effects to capital mobility and foreign interest rate. In this section, how capital controls and foreign interest rate affect the effective of monetary and fiscal policy is numerically analyzed for domestic interest rate $i = 0.01$. It can be seen from the numerical results that monetary and fiscal policy have significant effects and keep the same sign for low foreign rate. But the effects vary with the change of degree of capital controls. The behavior of the effects is almost the same as that in domestic interest rate, but the behavior of the effects of monetary policy on inflation rate and exchange rate has opposite trends.

For higher foreign rate, fiscal policy has no effects on growth rate, product volatility, inflation rate, exchange rate. And monetary policy only has no effects on product volatility, inflation rate. The effects of monetary policy on growth rate, inflation rate do not change with the change of degree of capital controls.

3.2.4. The response of policy effects to capital mobility and foreign price volatility. In this section, how capital controls and foreign product volatility affect the effectiveness of monetary and fiscal policy is numerically analyzed for domestic interest rate $i = 0.01$. It can be seen from the numerical results that monetary and fiscal policy have significant effects and keep the same sign for low foreign price volatility. But the effects change with the change of degree of capital controls. Behavior of the effects is almost the same as before, but the behavior of the effects of monetary policy on inflation rate and exchange rate has opposite trends.

For higher foreign price volatility, fiscal policy has no effects on growth rate, product volatility, inflation rate, exchange rate. And monetary policy only has no effects on product volatility. The effects of monetary policy on growth rate, inflation rate and monetary demand do not vary with the change of degree of capital controls, but the effects of monetary policy on exchange rate vary with the change of degree of capital controls.

4. Conclusion

This paper yields some interesting insights that can account for the effectiveness of monetary and fiscal policies, and their uncertainties on growth rate, product volatility, inflation rate, exchange rate under capital controls, domestic interest rate, foreign interest rate and foreign pricing volatility respectively by using an extended equilibrium in a stochastic dynamic optimal model with capital controls.

Our results generalize the Mundell-Fleming model. The research results show that the effectiveness of the monetary and fiscal policies and their uncertainties change with capital controls, domestic interest rate, foreign interest rate and foreign pricing volatility respectively by using an extended equilibrium in a stochastic dynamic optimal model with capital controls.

Capital control has no impact on the effectiveness of Monetary and fiscal policies for lower interest rate. The effects of the monetary policy and fiscal policy change with capital controls. It follows from our model that, with the increase of capital mobility, positive effect of monetary growth rate on growth rate is increased and negative effect of monetary growth volatility on growth rate is magnified, and negative effect of fiscal expenditure on growth rate is magnified and the positive effect of fiscal expenditure volatility on growth rate is enhanced.
For higher domestic interest rate, capital mobility has not affected the effects of monetary and fiscal policies on growth rate, product volatility, but has affected the effects of monetary and fiscal policies on inflation rate and exchange rate.

For low foreign rate and low foreign price volatility, capital control has affected the effects of monetary and fiscal policies. But the effects are changed with different degrees of capital controls. The behavior of the effects is almost the same as that for lower domestic interest rate, but the behavior of the effects of monetary policy on exchange rate has opposite trends.

For higher foreign rate, capital mobility has not affected the effects of fiscal policy on growth rate, product volatility, inflation rate, exchange rate.

For higher foreign price volatility, capital mobility has not affected the effects of fiscal policy on growth rate, product volatility, inflation rate, exchange rate. And capital mobility has not affected the effects of monetary policy on product volatility.

**Appendix**

The consumer’s stochastic optimization problem is to choose his or her consumption-wealth ratio and portfolio shares to

\[ \text{max} \, E_0 \int_0^{\infty} \ln[C^\theta(M/P)^{1-\theta}]e^{-\beta t}dt \]

Subject to the stochastic wealth accumulation

\[ \frac{dW}{W} = \psi dt + dw \]

\[ n_M + n_B + n_K + n_F = 1 \]

\[ \psi = n_Mr_M + n_Br_B + n_Kr_K + n_Fr_F - \frac{C}{W} \]

\[ = \rho - \frac{C}{W} \]

where \( \rho \equiv n_Mr_M + n_Br_B + n_Kr_K + n_Fr_F, dw \equiv n_M du_M + n_B du_B + n_K du_K + n_F du_F. \)

We define the differential generator of the value function \( V(W, t) \) by

\[ L[V(W, t)] = \frac{\partial V}{\partial t} + \psi W \frac{\partial V}{\partial W} + \frac{1}{2}\sigma_w^2 W^2 \frac{\partial^2 V}{\partial W^2} \]

Given the exponential time discounting, \( V \) can be assumed to be of the time-separable form

\[ V(W, t) = \Omega(W)e^{-\beta t} \]

The formal optimization problem is now to choose \( C, n_M, n_K, n_B \) and \( n_F \) to maximize the Lagrange expression

\[ e^{-\beta t}\ln[C^\theta(n_MW)^{1-\theta}] + L[e^{-\beta t}\Omega(W)] + \frac{\lambda}{\beta}e^{-\beta t}(1 - n_M - n_K - n_B - n_F) \]

Taking partial derivatives of this expression and canceling \( e^{-\beta t} \) yields

\[ \theta C^{-1} = \Omega_W \]

\[ \frac{1-\theta}{n_M} + r_M W \frac{\partial \Omega}{\partial W} dt + \text{cov}(dw, du_M)W^2 \frac{\partial^2 \Omega}{\partial W^2} = \frac{\lambda}{\beta} dt \]

\[ r_B W \frac{\partial \Omega}{\partial W} dt + \text{cov}(dw, du_B)W^2 \frac{\partial^2 \Omega}{\partial W^2} = \frac{\lambda}{\beta} dt \]

\[ r_K W \frac{\partial \Omega}{\partial W} dt + \text{cov}(dw, du_K)W^2 \frac{\partial^2 \Omega}{\partial W^2} = \frac{\lambda}{\beta} dt \]
These equations determine the optimal values for $C, n_M, n_B, n_K, n_F, \lambda$, as functions of $\Omega_W, \Omega_{WW}$ of the value function. In addition, the value function must satisfy the Bellman equation

\[
\max_{C,n_M,n_B,n_K,n_F} \left[ \ln[C^\theta(n_MW)^{1-\theta}]e^{-\beta t} + L[e^{-\beta t}\Omega(W)] \right] = 0
\]

\[
\ln[C^\theta(n_MW)^{1-\theta}] - \beta \Omega(W) + \hat{\psi}W\Omega_W + \frac{1}{2}\sigma^2_w W^2 \Omega_{WW} = 0
\]

The solution is by trial and error, finding a function $\Omega(W)$ that satisfies both the optimality condition and the Bellman equation. We postulate a solution of the form

\[
\Omega(W) = a + b \ln W
\]

where the coefficients $a$ and $b$ are to be determined. This equation immediately implies

\[
\Omega_w = b/W
\]
\[
\Omega_{WW} = -b/W^2
\]

Substituting the expressions for $C$ and $\Omega_w$ back into (A4a) yields

\[
C = (\theta/b)W
\]

Next, substituting (A8) into the Bellman equation (A6), we obtain

\[
\theta[\ln \theta - \ln b + \ln W] + (1 - \theta)[\ln n_M + \ln W] - \beta[a + b \ln W] + \hat{\psi}b - \frac{1}{2}\sigma^2_w b = 0
\]

This equation consists of constants and terms involving $\ln W$. The function (A7) will be a viable solution provided $a$ and $b$ are chosen to satisfy

\[
b = 1/\beta
\]
\[
a = [\theta \ln(\theta\beta) + (1 - \theta) \ln n_M + (\psi - \frac{1}{2}\sigma^2_w)(1/\beta)]/\beta
\]

Substituting the expressions for $\Omega_w$ and $\Omega_{ww}$ back into (A4) yields the corresponding optimality conditions

\[
\frac{C}{W} = \theta \beta
\]
\[
n_M = \frac{(1 - \theta) \beta}{\lambda}
\]
\[
(r_K - r_B)dt = \text{cov}(dw, ady + dp)
\]
\[
(r_F - \varphi n_F - r_B)dt = \text{cov}(dw, dp - dq)
\]

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