THE MULTI-FREQUENCY STOCHASTIC RESONANCE DETECTION BASED ON WAVELET TRANSFORM IN WEAK SIGNAL

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Abstract. The stochastic resonance method finds its original application on solving the issue of single-frequency signals. In this paper, a multi-frequency stochastic resonance detection method based on wavelet transform weak signal is proposed. The first wavelet transform is carried out for detaching the negative effect of weak noisy signal in multi-frequency to realize the separation of the various frequency bands, and then select the detailed signal and approximation signal of each layer signal as the input signal of stochastic resonance which realizes the detection of multi-frequency weak signal. The experimental results show that the proposed wavelet-based stochastic resonance method has successfully detected the multi-frequency weak signal.

Key Words. wavelet transform, stochastic resonance, multi-frequency, weak signal.

1. Introduction

In recent years, weak signal detection has been an important research field and attracted a growing interest from signal detection community in the use of new weak signal detection methods, such as the theory of stochastic resonance for weak signal detection. In traditional stochastic resonance field, if signal is input, a match is achieved between noise and non-linear system, the noise energy is transferred into signal energy. The weak desired signal could be identified when the output signal to noise ratio (referred as SNR) reaching the maximum. At present stochastic resonance is applied to many different areas of signal processing, for example, chaotic system [1], mechanical engineering failure detection [2],[3],[4], electronic circuit [5],[6] and so on.

The theory of stochastic resonance was put forward by the Italian scholar Benzi firstly [7]. It is a nonlinear two-stable system, when only under the effect of the noise or the small periodic signal are not lead to the system output at the jumps between the two steady-state. While under the common effect of the noise and the small periodic signal, the frequency of the signal appears at a peak in the system output power spectrum. As the noise intensity achieving to an appropriate value, the peak of output power spectrum reaches its maximum. In order to achieve the stochastic resonance of nonlinear system, this paper constructs a stochastic resonance model,
the choice of model parameters has referred a parameter estimation method in [8], which can generate highly accurate parameter estimates. In order to make the system being stable, a new algorithm which have convergence rates and better tracking performance is proposed [9].

Since the original stochastic resonance is only suitable for handling low frequency signals, the author of this paper has constructed a stochastic resonance model and applied it to handle high-frequency signals in another paper [10]. That is, a gain is added into the original model to make the system sampling time multiplied reduced so as to achieve low-frequency mapping from high-frequency signal, enabling the detection of high frequency signal. In order to achieve sampling of different frequency signals, [11] introduces a intersample output estimation for multi-frequent system. When applying stochastic resonance to the actual project, the constructed stochastic resonance model is only applicable to detect single-frequency input signal, and it is difficult to achieve the detection of mixed frequency input signal. Therefore, the wavelet transform is introduced to the stochastic resonance system in this paper.

The frequency separation of multi-frequency signal, and breaking different frequencies into different scales can be obtained by wavelet transform with good characteristic of time-frequency localization. In order to realize the multi-frequency weak signal detection, firstly, do wavelet transform to the input signal, and then choose different scales as the input signal of stochastic resonance, at last adjust various of parameters of the model to satisfy the stochastic resonance.

2. Wavelet transform theory

The concept of wavelet transform is proposed by French geoscientist J. Morlet in the analysis of geophysical materials in 1984 [12]. The mathematical basis of wavelet transform is Fourier transform. Firstly, a displacement $\tau$ is conducted to $\psi(t)$ which is defined as the basic wavelet function. Then do inner product of $\psi(t)$ with analyzed signal $x(t)$ at different scales $a_0$:

$$\text{WT}_f(a_0, \tau) = \int_{-\infty}^{+\infty} x(t) \psi^*(\frac{t-\tau}{a_0}) dt \quad a_0 > 0$$

In 1989, Mallat proposed the concept of multi-resolution analysis and designed a fast wavelet transform algorithm, Mallat algorithm [13]. The filter $\{g_k, h_k\}$ is used in the algorithm, and the corresponding formula is listed as follows:

$$\begin{cases} c_{j+1,k} = \sum_n c_{j,n} h_{n-2k} \\ d_{j+1,k} = \sum_n c_{j,n} g_{n-2k} \quad k \in \mathbb{Z} \end{cases}$$

The reconstruction algorithm formula is:

$$c_{j,k} = \sum_n c_{j+1,n} h_{k-2n} + \sum_n d_{j+1,n} g_{k-2n} \quad k \in \mathbb{Z}$$

Based on the former transformation, the original signal is decomposed into two unrelated sequences as $c_{j,k}$ and $d_{j,k}$ which are on different resolutions. $c_{j,k}$ and $d_{j,k}$ represent low-pass approximation signal and high-pass detailed signal decomposed by sub-coefficient of $j$ respectively. $g_k$ and $h_k$ are low-pass filter and high-pass filter.

3. The principle of stochastic resonance

3.1. The basic principle of stochastic resonance. The idea of stochastic resonance is proposed by Italian scholar Benzi in 1981, they had done meteorological
studies to the cyclical appearance of ancient glaciers, and established a stochastic resonance model in which the earth’s climate is expressed by a double-well potential function. In 1983 Fauve and Heslot confirmed that the phenomenon of stochastic resonance for the first time by studying the noise depending on the spectrum in AC drive Schmitt trigger [14]. Later in the threshold detector also found the effect of stochastic resonance, indicating the existence and application of non-dynamic stochastic resonance. Collins proposed aperiodic stochastic resonance based on Fitzhugh-Nagumo neural network, trying the new integration of stochastic resonance and information theory [15]. Subsequently, stochastic resonance has been widely used in different areas.

Stochastic resonance is consisted of three basic elements: input weak signal, noise and bistable nonlinear system. Suppose $s(t)$ is a useful weak periodic signal, $n(t)$ is a noise signal. Under the synergy of $s(t)$ and $n(t)$, the output signal of the system will produce the phenomenon of stochastic resonance. Bistable nonlinear system can be described by a Langevin equation [16]:

$$\dot{x}(t) = ax(t) - bx^3(t) + s(t) + n(t)$$

Where $s(t) = A \sin(2\pi ft + \tau)$, $E[n(t) = 0]$, $E[n(t)n(t-\tau)] = \sigma^2 \delta(\tau)$. The corresponding potential function of (1) is as follows:

$$U(t) = \frac{1}{2}ax^2 - \frac{1}{4}bx^4 + [s(t) + n(t)]x$$

The potential function which can be described in a pair of potential well curve is shown in Fig.1.

![Potential function curve](image)

Fig. 1  Potential function curve

The potential function curve [17] is composed of two potential trap points ($-x_m, x_m$) and a barrier point (coordinate origin). When the input signal amplitude $A$ and noise intensity $D$ are zero, the potential well point is $x_m = \pm \sqrt{a/b}$, the potential barrier height is $\Delta U = a^2/4b$. The curve describes an overdamped particle movement: When $A = 0$, the particle flip between the two potential well driven by noise; when $A > 0$, the cycle change of signal and the flip between potential well driven by system noise are likely to be synchronized, so as to occur the phenomenon of stochastic resonance.

3.2. Stochastic resonance model of high-frequency signal detection. From the adiabatic approximation theory and the linear corresponding theory we can draw the following conclusions, when there is only noise as input of the bistable stochastic resonance system, the spectrum energy of output mainly concentrates on the low-frequency band. And when the signal frequency falls in this band, the noise energy will be transferred to signal, making the cycle component protrude.
Therefore, the original stochastic resonance model is only suitable for dealing with low-frequency signals. In this respect, we choose a stochastic resonance model aiming to detecting high-frequency signal.

In the bistable non-linear system, time is discretized by simpler. Then let \( t_1 = K \cdot t \), \( K > 1 \). Thus the sampling time could be expand several times, and the signal frequency could also be decreased. The following model can be obtained:

\[
\dot{x}(t_1) = K \left[ ax(t_1) - bx^3(t_1) + s(t_1) + n(t_1) \right]
\]

In this model, the low-frequency mapping from the original signal frequency is \( f_0 = f/K \). The size of \( K \) determines the extent of the original signal frequency \( f \) changing into the low-frequency \( f_0 \). An appropriate choice of \( K \) could accurately map the high signal frequency to the low frequency suitable for stochastic resonance occurring. As the role of the gain \( K \) in the model, the sample period of noise signal will increase. In order to avoid the influence of the noise signal intensity, a proportional amplifier module is introduced into the model. Therefore, the system will still generate stochastic resonance in the case that the system parameters do not change.

4. The multi-frequency stochastic resonance detection based on wavelet transform

In order to realize the multi-frequency weak signal detection, wavelet transform is introduced to stochastic resonance system. Construct stochastic resonance system based on wavelet transform shown in Fig.2.

\[
\begin{align*}
  s_0(t) & \rightarrow z_j(t) \\
  n_0(t) & \rightarrow z_1(t) \\
  c_j(t) & \rightarrow \text{Sampling} \\
  d_j(t) & \rightarrow \text{Wavelet Transform} \\
  d_j(t) & \rightarrow \text{Stochastic Resonance System} \\
  x(t) & \rightarrow d_j(t)
\end{align*}
\]

Fig. 2 Stochastic resonance system based on wavelet transform

In the system, \( s_0(t) \) is a weak period signal of multi-frequency, and \( n_0(t) \) is a white noise signal with the mean zero and noise intensity \( D \). \( s_k(t) \) and \( n_k(t) \) are discretized after sampling. Do \( j \)-layer wavelet decomposition to the noisy signal composed of two discrete signals, could obtaining approximation signal \( c_j(t) \) and detailed signal \( d_j(t) \). According to wavelet theory, low-frequency signal is located in the \( j \)-layer of approximation signal, while high-frequency signal and white noise are distributed in the layers of detailed signal. As the frequency energy of white noise is uniformly distributed, it will focus on the low-frequency regional after the processing of stochastic resonance system. Therefore, selecting the appropriate detailed signal and approximation signal as the input of stochastic resonance system can also produce the phenomenon of stochastic resonance.

Firstly, low-frequency signal should be detected when the stochastic resonance system based on wavelet transform is adopted to detect multi-frequency signal. In general, low-frequency signal is located in the higher layer of the approximation signal. Taking high-layer approximation signal as input of stochastic resonance system and selecting the appropriate stochastic resonance system parameters can make the system produce stochastic resonance, which can detect the low-frequency signal in noisy multi-frequency signal. In the detection of high-frequency signal, the same system parameters are adopted and the appropriate value of \( K \) is selected, which can also make the system to produce the stochastic resonance. Mapping the
frequency of output signal which at this time is low-frequency to high-frequency can detect the high-frequency signal in noisy multi-frequency signal.

5. Simulation experiment

5.1. The simulink model of stochastic resonance based on wavelet transform. The simulink model of stochastic resonance based on wavelet transform is shown in Fig.3.

![Fig. 3 The simulink model of stochastic resonance based on wavelet transform](image)

In this model, the detailed signal and approximation signal after wavelet transform are introduced by From Workspace module. Both of the two signals are imposed into mix input signal by Add module. Gain represents a gain module which can reduce the approximation signal intensity so as to the noise signal intensity. Gain1 and Gain2 are the system parameter modules. Gain3 is the system gain module. Both of them are added with mix input signal by Add1 module. When detecting the low-frequency signal, let \( K = 1 \). The input signal and the output signal which feed back from the one-time item and three-time item could obtain new output signal after passing through the Integrator module. The output is displayed by Scope module.

5.2. The simulation of multi-frequency weak signal detection. The simulation of multi-frequency weak signal detection can be divided into the following three steps:

1. The construction of multi-frequency noisy signal
   Select a superimposed sinusoidal signal, which is mixed by two frequencies of 0.01Hz and 1Hz. The signal amplitude is respectively 0.3 and 0.5. The multi-frequency signal is added to the white noise with noise intensity \( D = 0.0024 \). Numerical step size is \( \Delta t = 0.006s \). The time and frequency domain spectrum of multi-frequency noisy signal are shown in Fig.4.
   The diagram shows that the useful signal spectrum submerged in the noise spectrum is difficult to identify.

2. The low-frequency detection of multi-frequency noisy signal
   Do 6-layer wavelet decomposition to multi-frequency signal by using db5 as mother wavelet. Choose the 6-layer approximation signal as the input of stochastic resonance model. According to the system parameter principle \([18],[19]\) of stochastic resonance, select the system parameters as \( a = 0.07 \) and \( b = 1 \). The system will produce stochastic resonance to obtain time and frequency domain spectrum of low-frequency output signal as shown in Fig.5.
(a) The time-domain spectrum (b) The frequency-domain spectrum

Fig. 4 The time and frequency domain spectrum of multi-frequency noisy signal

(a) The time-domain spectrum (b) The frequency-domain spectrum

Fig. 5 The time and frequency domain spectrum of low-frequency noisy signal

From Fig. 5 we can see that, the low-frequency output signal can be identified, that is, \( f_1 = 0.01 \text{Hz} \).

(3) The high-frequency detection of multi-frequency noisy signal

The 1,2-layer detailed signals added with 6-layer approximation signal is chosen as the input signal of high-frequency stochastic resonance model. While system parameters remain unchanged and let \( K = 100 \), the system will product stochastic resonance. The time and frequency domain spectrum of high-frequency output signal are shown in Fig. 6.

(a) The time-domain spectrum (b) The frequency-domain spectrum

Fig. 6 The Time and frequency domain spectrum of high-frequency noisy signal

The frequency spectrum of output signal can be identified to be the frequency which is mapping from useful signal frequency, that is, \( f_0 = 0.01 \text{Hz} \). Thus calculate the original high-frequency \( f_2 = K \times f_0 = 100 \times 0.01 = 1 \text{Hz} \).
On the basis of the above three detection steps, when detecting the low-frequency and high-frequency of input noisy weak signal, the time-domain diagram of output signal shows a clear cycle change and frequency-domain diagram can also be clearly identified to the frequency of useful signal proving that the new model can lead to a better detection results. As can be seen from the above simulation effect, the stochastic resonance model based on wavelet transform has accurately detected the high-frequency and low-frequency of input signal, reaching the purposes of multi-frequency weak signal detection.

6. Conclusion

In order to detect multi-frequency weak signal, wavelet transform is introduced to the stochastic resonance system in this paper. In the new detection method, firstly do wavelet transform to the original multi-frequency noisy signal, and then choose different scales of signals as input signal of the stochastic resonance to achieve the detection of each frequency band of weak signal. The simulation results show that the stochastic resonance model based on wavelet transform has accurately realized the detection of multi-frequency signal.

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References


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