IDENTIFICATION OF LINEAR TIME-VARYING SYSTEMS USING HAAR BASIS FUNCTIONS

VINAYAK G. ASUTKAR, BALASAHEB M. PATRE AND T. K. BASU

Abstract Most of the physical systems exhibit some degree of time-varying behavior. Physical phenomena exhibit time-varying behavior for a number of reasons. Some of the systems are inherently time-varying and can not effectively be modeled using time invariant models. This paper deals with the identification of time-varying systems using Haar basis functions. Basis functions approach involves expanding the time-varying parameters onto a set of basis functions and then estimating the resulting expansion coefficients. In this paper we employed first order Time-Varying Auto-Regressive with eXogenous input (TVARX) model and its parameters are estimated using Haar basis functions with forgetting factor approach. The input to the time-varying system is chosen to be a Pseudo-Random Binary Sequence (PRBS) which is frequency rich signal and excites complete system. Results obtained from a simulation study of time-varying systems are presented to demonstrate the performance of the Haar basis functions approach. Parameters are estimated for noisy and noiseless conditions. Comparisons of Haar basis functions with Walsh, Cosine and Legendre basis functions, for estimation of the time-varying parameters are presented and it is found that Haar Basis functions produce best results.

Key Words, Parameter estimation, Haar basis functions, System identification, Time-varying systems, TVARX model.

1. Introduction
All most all the practical systems exhibit some degree of time-varying behavior. Over short time intervals most of the processes can be satisfactorily approximated by linear dynamic time invariant models, but over the longer time intervals they reveal time-varying features or characteristics [7]. For modeling this type of systems, models with time-dependent coefficients are needed. Physical phenomena exhibit time-varying behavior for a number of reasons, mainly due to the variation of internal (aging, fatigue) and external (set point changes, time dependent disturbances) operating conditions [7]. Time-varying signals are present in many applications. In chemical process control, heat transfer rates may be altered by fouling or corrosion. Such phenomena may induce, for example, thermocouple drift or change the characteristics of a heat exchanger resulting in a time-varying process description. Moreover, the dynamics of several chemical reactions are highly nonlinear. Consequently, in processes involving such reactions, varying set points or feed-stream composition and other disturbances in general may cause a significant change in operating conditions and the parameters of the process model [12]. The speech processing, communication, biomedical signal processing,
dynamic weighing systems, crack monitoring etc are a few examples where time-varying nature of signals is present.

System identification is an experimental approach for determining the dynamic model of a system to serve certain purposes. A dynamical mathematical model in this context is a mathematical description of the dynamic behavior of a system or process. Essentially it is done by adjusting parameters within a given model until its output coincides as well as possible with the measured output. An advantage of system identification is that it takes a lot less time than physical modeling since it is only concerned with input and output signals from the system [2], [5], [6].

Different approaches for model identification and parameter estimation of time-varying systems can be classified as follows:
1. Assume a time-varying process is locally stationary and Least Square (LS) techniques are applied. This method gives good results for stationary process but not suitable for rapidly time-varying systems [5].
2. A Kalman filtering algorithm is applied to time-varying system estimation. This approach can track the rapidly varying process but often overestimates the parameters and the variances of estimates are high for an inappropriate model [10].
3. Nonparametric approaches for time-varying spectrum estimation such as local evolving spectrum, Short Time Fourier Transform (STFT) and Wigner-Ville Distribution (WVD) are also developed to characterize time-varying signals. However, they cannot achieve high-resolution modeling and the computation burden is heavy [9], [3].

In this paper the method of basis functions which is based on an explicit model of parameter variation is used. In this case it is assumed that the parameter trajectory can be approximated by a linear combination of known functions of time. A key advantage of using basis functions is that a considerable reduction in the number of parameters needed to track each time-varying coefficient can be obtained. Furthermore, identification of both slow and fast-varying processes can be more properly handled by the use of basis functions [14], [11].

In this paper the Time-Varying Auto-Regressive with eXogenous input (TVARX) model is employed and its parameters are estimated using Haar basis functions. For experimental simulation, the first order TVARX model is used and its parameters are estimated for noiseless and noisy conditions. The parameters of the model are varied abruptly manner. The input signal to the system is chosen to be a Pseudo-Random Binary Sequence (PRBS) which is frequency rich signal. The performance is evaluated by calculating different performance measures of errors between true and estimated parameters values.

The rest of the paper is organized as follows. In section II, the model employed and its time-varying parameters are explained. In section III, the basis functions for identification of parameters of the time-varying systems are discussed. Details of Haar basis functions are presented in section IV and in section V some simulation results are presented while conclusions drawn from simulation results are given in section VI.
2. Models of time-varying systems

There are many forms of models for time-varying systems are available [7]. Consider a TVARX model given by,

\[ y(t) = \sum_{i=1}^{n_p} a_i(t) y(t-i) + \sum_{i=1}^{n_r} b_i(t) u(t-i) + \nu(t) \]

where signal \( y(t) \in \mathbb{R} \) is the output, \( u(t) \in \mathbb{R} \) is the measurable input signal, and \( \nu(t) \) denotes a white noise sequence, i.e. a sequence of zero mean uncorrelated random variables of constant variance. \( a_i(t) \) and \( b_i(t) \) are (for each \( i \)) the time-varying model parameters which depend on discrete time \( t \). \( n_a \) and \( n_b \) are the orders of the TVARX model. From the above (1), we are mostly concerned with the identification of parameters \( a_i(t) \) and \( b_i(t) \) using Haar basis functions.

The model in (1) can be written down in the following shorthand form,

\[ y(t) = \varphi^T(t) \theta(t) + \nu(t) \]

where

\[ \varphi(t) = [y(t-1),...,y(t-n_a),u(t-1),...,u(t-n_b)]^T \]

and \( \theta(t) \) is the vector of model parameters [7], [8].

3. Method of basis functions

Let a set of linearly independent functions, called the basis functions be defined by

\[ \{ f_k(t), k = 1, ..., q \} \]

where \( q \) is the expansion dimension. For selection of basis functions, if some prior knowledge about the nature of process time variation is available, then one may choose the basis functions so as to capture the dominant trends in coefficient changes. If no such insights are available, then any general linearly independent basis functions can be selected. Assume that each time-varying parameter \( a_i(t) \) and \( b_i(t) \) of parameter vector \( \theta(t) \) in (2) can be expressed as a linear combination of these basis functions [7], [1], so that

\[ \theta_i(t) = \sum_{k=1}^{q} c_{ik} f_k(t) \]

where \( i = 1, ..., p \) and \( p = n_a + n_b \), here \( n_a \) and \( n_b \) are as defined in (1).
By expanding parameter vector \( \theta(t) \) as in (6), we can express \( \phi(t) \) in (2) as the generalized regression vector \( \psi(t) \) associated with the analyzed time-varying process,

\[
\psi(t) = \phi(t) \otimes f(t) = [\phi_1(t) f^T(t), \ldots, \phi_q(t) f^T(t)]^T
\]

where \( f(t) = [f_1(t), \ldots, f_q(t)]^T \) is the basis functions vector and \( \phi_j(t) \) is the \( j \)th component of \( \phi(t) \) and \( A \otimes B \) is the Kronecker product of two arbitrarily sized matrices \( A \) and \( B \).

Now let \( \gamma = [\gamma_1^T, \ldots, \gamma_p^T]^T, \gamma_j = [c_{j1}, \ldots, c_{jn}]^T \) be the vector of all coefficients used to describe the process time variation. The system (2) can be written as,

\[
y(t) = \psi^T(t) \gamma + \nu(t)
\]

Since, in practice, (6) is adopted as a local model of parameter variation rather than as a global model valid for all time instants \( t \), the parameter vector \( \gamma \) is usually regarded as slowly time-varying.

The estimation of \( \gamma \) can be obtained from (8) using the method of recursive least square with forgetting factor approach [7], [8], [13].

\[
\hat{\gamma}(t) = \hat{\gamma}(t-1) + L(t)e(t),
\]

\[
e(t) = y(t) - \psi^T(t) \hat{\gamma}(t-1),
\]

\[
L(t) = \frac{Q(t-1)\psi(t)}{\lambda + \psi^T(t)Q(t-1)\psi(t)},
\]

\[
Q(t) = \frac{1}{\lambda} \left[ Q(t-1) - \frac{Q(t-1)\psi(t)\psi^T(t)Q(t-1)}{\lambda + \psi^T(t)Q(t-1)\psi(t)} \right],
\]

where \( e(t) \) is the a priori prediction error, \( L(t) \) is the filtering gain, \( Q(t) \) is the estimation covariance matrix, and \( \lambda \) is the forgetting factor which plays the major roll in the identification of \( \gamma \). The parameter vector can then be derived from (6).

4. Haar basis functions

Here we focus on Haar basis functions for the problem of tracking the time-varying characteristics of the system as wavelet can capture the local and the global behaviors of the transient parameters of the underlying TVARX model. Haar basis functions consist of enough information regarding the changes of the parameters at different resolutions although different wavelet basis should be evaluated and chosen for different kinds of systems. In general, wavelet basis are much better for modeling the time-varying parameters than a series common basis functions.
The distinct feature of Haar function is its multiresolution characteristics that are very suitable for time-varying system identification. Since a wavelet is a localized function both in time and frequency domain, it can be used to represent an abrupt variation or a local function vanishing outside a short time interval adaptively.

The function \( h_0(t) \) which is called the Haar scaling function is defined as,

\[
\begin{align*}
h_0(t) &= 1, \quad t \in [0, 1] 
\end{align*}
\]

The function called Haar mother wavelet defined by,

\[
\begin{align*}
h(t) &= \begin{cases} 
1, & t \in [0, \frac{1}{2}] \\
-1, & t \in [\frac{1}{2}, 1]
\end{cases}
\end{align*}
\]

We derive the normalized Haar functions from (11) as [4],

\[
\begin{align*}
h_n(t) &= 2^{n/2} h(2^j t - k), \quad t \in [0, 1]
\end{align*}
\]

with \( n = 1, 2, 3,... \) and \( n = k + 2^j \). We define the Haar basis functions from (10) and (12) as, \( f_i(t) = h_i(t) \) and \( f_{n+1}(t) = h_n(t) \). The first eight Haar basis functions \( f_1(t),..., f_8(t) \) are shown in Fig 1.

![Fig 1. Haar basis functions](image_url)

5. Simulation results

For the purpose of simulation, we have considered following first order TVARX model,

\[
y(t) = a_1(t)y(t-1) + b_1(t)u(t-1) + \nu(t)
\]
The process parameters $a_i(t)$ and $b_i(t)$ are varied suddenly and the output $y(t)$ is observed for system input $u(t)$ which is PRBS. Then the system parameters are estimated using Haar basis functions with forgetting factor approach.

In this simulation, the process time-varying parameters $a_i(t)$ and $b_i(t)$ in (13) are varied abruptly as,

$$a_i(t) = \begin{cases} 0.2 & 0 \leq t \leq 0.2 \\ 0.8 & 0.2 < t \leq 0.5 \\ 0.4 & 0.5 < t \leq 1 \end{cases}$$

$$b_i(t) = \begin{cases} 0.4 & 0 \leq t \leq 0.3 \\ 0.8 & 0.3 < t \leq 0.6 \\ 0.4 & 0.6 < t \leq 1 \end{cases}$$

With these time-varying parameters the system (13) is then excited by PRBS signal. Fig.2. shows the PRBS input, which is frequency rich signal and excites complete system for 1 second. The output is shown in Fig.3 for noise of 18.5797 dB Signal to Noise Ratio (SNR). The system time-varying parameters are estimated using Haar basis functions with forgetting factor approach.

The Fig.4 and Fig.5 show the true and estimated values of parameters $a_i(t)$ and $b_i(t)$ respectively for noise of 18.5797 dB SNR. These parameters are estimated using Haar basis functions and with forgetting factor $\lambda = 0.7$ in which, the estimated parameters follow the true parameters’ variations. It is clearly seen that the estimated parameters are same as the true parameters except small noise at high frequencies.
Fig. 3 Output signal

Fig. 4 Estimated and true parameter $a_1(t)$ using Haar basis functions

Fig. 5 Estimated and true parameter $b_1(t)$ using Haar basis functions
Fig. 6 (a) MSE, (b) SAE, and (c) SSE in estimated parameters for 18.5797 dB SNR.
The mean square error (MSE), sum of absolute error (SAE) and sum of square error (SSE) between true and estimated parameters \( a_i(t) \) and \( b_i(t) \), estimated using cosine, Legendre, Walsh and Haar basis functions for noise of 18.5797 dB SNR are shown in Fig.6 (a), (b) and (c) respectively. From this comparative bar graph it is cleared that Haar basis functions are giving the best results. Legendre basis functions are also giving good results.

![Fig.7 (a) MSE in estimated parameter \( a_i(t) \), (b) Zoom in of (a)](image_url)

Fig.7 (a) MSE in estimated parameter \( a_i(t) \), (b) Zoom in of (a)
Parameters are estimated using four basis functions with different noise SNR levels and the MSE, SAE and SSE are calculated. Fig. 7 (a) shows the MSE in parameters $a(t)$ estimated using four basis functions. Fig. 7 (b) shows the zoom in version of Fig. 7 (a). These results depict that Haar basis functions give minimum MSE. Similarly Fig. 8 (a) shows the MSE in parameter $b(t)$ estimated using four basis functions. Fig. 8 (b) shows the zoom in version of Fig. 8 (a). Here also results depict that Haar basis functions give minimum MSE and good estimation accuracy. We see that the basis functions approach is robust up to the noise level of 15 dB SNR.

6. Conclusions

Time-varying parameters of TVARX model are estimated using Haar basis functions with forgetting factor. The comparison is given with Walsh, Cosine and Legendre basis
functions for estimation of the time-varying parameters. Parameters variation is considered with abruptly changing manner. For comparison of different basis functions, MSE, SAE and SSE between true and estimated parameters \( a_t(t) \) and \( b_t(t) \) are calculated for different noise levels. Even in the present of more noise estimated parameters follow the variations of true parameters. The Haar basis functions model time-varying parameters variations effectively and give best results. From the overall results presented here, it is concluded that the time-varying systems can be modeled using TVARX model effectively and the identification of model with fast time-varying parameters is possible with good accuracy using Haar basis functions with forgetting factor approach.

REFERENCES


Vinayak G. Asutkar is Assistant Professor in the department of Instrumentation Engineering, Shri Guru Gobind Singhji Institute of Engineering and Technology, Nanded, India. E-mail: vgasutkar@sggs.ac.in. He received the BE and ME from Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, India in 1989 and 1994 respectively. His main areas of research interests include time-varying systems, wavelets and power electronics.

Balasaheb M. Patre is Professor and head of the department of Instrumentation Engineering, Shri Guru Gobind Singhji Institute of Engineering and Technology, Nanded, India. E-mail: bmpatre@sggs.ac.in. He received the BE and ME from Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, India in 1986 and 1990 respectively. He received Ph. D. from Indian Institute of Technology, Bombay, Mumbai, India in 1998. He has published 75 papers in National/ International Conferences and Journals. He is a member of IEEE, IET, ISTE etc. He is a fellow of IE(India). He is reviewer of IEEE, IET and many other Journals. His main areas of research interests include sliding mode control, model reduction and fuzzy control.

T. K. Basu is Professor in the Department of Electrical Engineering, Indian Institute of Technology, Kharagpur, WB, India. E-mail: tkb@ee.iitkgp.ernet.in. His main areas of interests include digital signal and image processing, speech processing and network theory.