Abstract. A major drawback of multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems is its high peak-to-average power ratio (PAPR). A decomposed selected mapping algorithm is proposed to overcome this problem. The candidate OFDM frames transmitted on antennas are decomposed to be real and imaginary part, thereby, a sufficiently large number of alternative transmit signals are provided, thus better PAPR performance can be achieved compared with conventional SLM algorithm. Moreover, the computational complexity of proposed algorithm retain unchanged by using the conjugate symmetry property of real sequence. Simulations results demonstrate that the proposed algorithm outperforms the conventional SLM algorithm in MIMO-OFDM systems.

Key Words. MIMO, OFDM, PAPR, SLM.

1. Introduction

As an attractive technique for high data rate wireless communication systems, multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems exhibit a large peak-to-average power ratio (PAPR) due to the superposition of the individual signal components (the carriers) [1]. The high PAPR brings on the OFDM signal distortion in the nonlinear region of high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER).

In OFDM systems, several PAPR reduction schemes have been proposed to solve this problem [2]–[8]. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level, but it causes both in-band distortion and out of band radiation. Block coding, the encoding of an input data into a codeword with low PAPR is another well-known technique to reduce PAPR, but it incurs the rate decrease. Among these methods, selective mapping (SLM) scheme is an efficient approach for PAPR reduction by generating some statistically independent sequences from same information and transmitting the sequence with the lowest PAPR.

In MIMO-OFDM systems, a straightforward way for PAPR reduction is to apply existing algorithms, such as SLM, separately on each transmit antenna. It is effective to reduce PAPR, but requires high complexity and large amount of side information (SI). Then the concurrent algorithm is investigated to solve this problem, a concurrent SLM algorithm was proposed in document [9], which selects the transmitted sequence with the minimum average PAPR over all transmit antennas, thus decreases side information (SI) significantly.

Received by the editors September 15, 2008 and, in revised form, November 15, 2008.
In this paper, a decomposed individual selected mapping (D-ISLM) algorithm is proposed to improve the PAPR performance of MIMO-OFDM systems. In the proposed algorithm, a sufficiently large number of alternative transmit signals are provided by treating the real and imaginary part of the transmitted signals separately, thereby, the significant PAPR reduction performance can be achieved without increasing computational complexity. Moreover, a decomposed concurrent selected mapping (D-CSLM) is proposed to reduce the side information at the expense of a slight degradation of the PAPR performance compared with the D-ISLM algorithm.

The organization of the article is as follows: Section 2 outlines the data model of the MIMO-OFDM systems and the PAPR reduction by SLM algorithm. In Section 3, we review the SLM algorithms briefly, and propose the D-ISLM and D-CSLM algorithms. The simulation results illustrating the proposed algorithms’ performance are provided in Section 4. Finally, Section 5 is the conclusions.

2. PAPR and SLM algorithm

We consider the MIMO-OFDM systems with \( N \) transmit antennas that uses \( N \) subcarriers. With OFDM modulation, a block of \( N \) data symbols (one OFDM symbol), \( \{X_n, n = 0, 1, \ldots, N - 1\} \) will be transmitted in parallel such that each modulates a different subcarrier from a set \( \{f_n, n = 0, 1, \ldots, N - 1\} \). The \( N \) subcarriers are orthogonal, i.e. \( f_n = n\Delta f \), where \( \Delta f = 1/NT \) and \( T \) is the symbol period. The resulting baseband OFDM signal \( x_m(t) \) of a block can be expressed as

\[
x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \quad 0 \leq t \leq NT
\]

Where \( X_n \) is the transmitted OFDM signal at the \( n \)th subcarrier of the \( m \)th transmit antenna.

The PAPR of the transmitted OFDM signal of (1) is defined as

\[
PAPR = \max_{0 \leq t \leq NT} \left| x_m(t) \right|^2 = \max_{0 \leq t \leq NT} \left\{ \frac{1}{NT} \int_0^{NT} \left| x_m(t) \right|^2 dt \right\}
\]

where \( E[\cdot] \) denotes the expected value. Then, the complementary cumulative distribution function (CCDF), which is the probability that the PAPR of an OFDM symbol exceeds the given threshold \( PAPR_0 \), can be expressed as

\[
CCDF = Pr(PAPR > PAPR_0)
\]

An ordinary method for PAPR reduction in MIMO-OFDM is to apply SLM scheme for each transmit antenna individually, as shown in Figure 1. In the individual SLM (ISLM), \( V \) statistically independent sequences are generated for each antenna by multiplying the fixed \( N \)-dimensional phase vectors \( \{P^{(v)}\} \) to the input sequences, where \( P^{(v)}=[P_0^{(v)}, P_1^{(v)}, \ldots, P_{N-1}^{(v)}] \), \( 1 \leq v \leq V \) with each component \( P_n^{(v)} \in \{\pm 1, \pm j\} \). The sequence with the lowest PAPR is individually transmitted from each transmit antenna and results in the SI of \( \log_2 V \) bits at each transmit antenna. Consequently, the CCDF of the PAPR of the OFDM signals at each transmit antenna is written as

\[
Prob[PAPR > PAPR_0] = [1 - (1 - e^{-PAPR_0})^V]^V
\]

In the concurrent SLM-based MIMO-OFDM system, the \( N \) subcarrier sequences from \( M \) transmit antennas are multiplied subcarrierwise with the same sequence, which is one of the \( V \) sequences \( P^{(v)} \), as shown in Figure 2. The sequence with the lowest average
PAPR over all $N$ transmit antennas is chosen. Then, the probability that the PAPR of a randomly generated $MN$-OFDM symbol over all $M$ transmit antennas exceeds $\text{PAPR}_0$ is given by

\begin{equation}
\text{Prob}[\text{PAPR} > \text{PAPR}_0] = [1 - (1 - e^{\text{PAPR}_0})^{MN}]^v
\end{equation}

3. **Decomposed SLM algorithms**

In this paper, decomposed SLM algorithms are proposed to further improve the PAPR performance without increasing the computational complexity for MIMO-OFDM systems, which is shown in Figure 3 and Figure 4.

3.1 **Decomposed individual SLM (D-ISLM) algorithm**

Firstly, real and imaginary part of the complex baseband signal $A_m$ for the $m$th transmit antenna are separated as

\begin{equation}
A^m = A_m^r + jA_m^i \quad (1 \leq m \leq M)
\end{equation}
where $A_r^m$ is the real part and $A_i^m$ is the imaginary part of the frequency-domain vectors, and for both, real ($A_r^m$) and imaginary part ($A_i^m$), $V$ candidates are generated. Since the signals $A_r^m$ and $A_i^m$ are real-valued, the phase vectors $p_r^m$ and $p_i^m$ (not necessarily the same for both quadrature components) have to be real, too, i.e., $p_r^m, p_i^m \in \{\pm 1\}$. These candidates are transformed into time-domain using $2V$ “real-to-complex” IDFTs. Then each combination of one real and one imaginary candidate is formed, leading to $V^2$ candidate time-domain signals. From these, the best candidate $a^m = a_r^m + ja_i^m$ with minimum PAPR is selected. Consequently, performance of conventional SLM with $V^2$ candidates is achieved. Noteworthy, the required side information is $M \log_2 V^2$.

**D-ISLM algorithm** is described as follows.

For $m = 1, 2, \ldots, M$
for \( u, v = 1, 2, \ldots, V \)
\[
\alpha_R^m(u) = \text{IDFT}\{A_R^m(u) \cdot p_R^m(u)\}, \quad \alpha_I^m(v) = \text{IDFT}\{A_I^m(v) \cdot p_I^m(v)\}
\]
end

\[
[a^m, p_R^m(u'), p_I^m(v')] = \arg\min_{u, v = 1, 2, \ldots, V}\{\text{PAPR}[\alpha_R^m(u) + j\alpha_I^m(v)]\}
\]
end

### 3.2 Decomposed concurrent SLM (D-CSLM) algorithm

To reduce the SI, we investigate a decomposed concurrent SLM (D-CSLM) algorithm. The real and imaginary part of signals from \( M \) transmit antennas are multiplied subcarrierwise with the same phase factor sequences \( p_R \) and \( p_I \) respectively, then the sequence with the lowest average PAPR over all \( M \) transmit antennas is chosen. Noteworthy, since the subcarrier sequences with the lowest average PAPR for transmission are constructed by using the same phase vector to all of transmitters, transmit antennas can bear the same SI of \( \log_2 V^2 \).

**D-CSLM algorithm** is described as follows.

for \( m = 1, 2, \ldots, M \)

for \( u, v = 1, 2, \ldots, V \)
\[
\alpha_R^m(u) = \text{IDFT}\{A_R^m(u) \cdot p_R(u)\}, \quad \alpha_I^m(v) = \text{IDFT}\{A_I^m(v) \cdot p_I(v)\}
\]
end

\[
[a^m, p_R(u'), p_I(v')] = \arg\min_{u, v = 1, 2, \ldots, V}\{\text{average} \sum_{m=1}^{M} \text{PAPR}[\alpha_R^m(u) + j\alpha_I^m(v)]\}
\]

### 3.3 Performance analysis

Using the approximation \((1 - e^{-x})^V = 1 - ye^{-x}\), for \( x \neq 1 \), the CCDF of PAPR can be approximated by

\[
\text{Prob}[\text{PAPR}_{\text{ISLM}} > \text{PAPR}_0] \approx MN^V \cdot e^{\text{PAPR}_0 V}
\]

\[
\text{Prob}[\text{PAPR}_{\text{D-ISLM}} > \text{PAPR}_0] \approx MN^{V^2} \cdot e^{\text{PAPR}_0 V^2}
\]

\[
\text{Prob}[\text{PAPR}_{\text{CSLM}} > \text{PAPR}_0] \approx (MNe^{\text{PAPR}_0})^V
\]

\[
\text{Prob}[\text{PAPR}_{\text{D-CSLM}} > \text{PAPR}_0] \approx (MNe^{\text{PAPR}_0})^{V^2}
\]

In equation (7), it is clearly that compared with ISLM, D-ISLM is better by the factor \( V^2 \) of candidate signals, and in equation (8), D-CSLM is also better than CSLM. Moreover, the D-CSLM algorithm selects the sequence with the lowest average PAPR over all \( M \) transmit antennas, thus has certain PAPR performance degradation compared with D-ISLM.

The IDFTs and SI required are compared in Table.1.

<table>
<thead>
<tr>
<th></th>
<th>ISLM</th>
<th>D-ISLM</th>
<th>D-CSLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDFT ((M = 4, V = 8))</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>SI ((M = 4, V = 16))</td>
<td>16</td>
<td>32</td>
<td>8</td>
</tr>
<tr>
<td>IDFT ((M = 4, V = 16))</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>SI ((M = 4, V = 16))</td>
<td>16</td>
<td>32</td>
<td>8</td>
</tr>
</tbody>
</table>
4. Simulation results

Simulations results have been conducted to assess the performance of the proposed algorithms. We assume that $10^4$ random OFDM sequences were generated to obtain the CCDF. We use $M = 4$ transmit antennas and $N = 128$ subcarriers with QAM data symbols. The transmitted signal is oversampled by a factor of $L = 4$, and the phase vectors $p_R^m, p_I^m \in \{\pm 1\}$.

In Fig. 5, the CCDF is plotted for the ISLM algorithm and D-ISLM algorithm, respectively. For reference, the CCDF of original OFDM signals without PAPR reduction is included. It is clearly that D-ISLM shows much better performance than conventional ISLM employing the same complexity. With $V$ candidates (for real and imaginary part of each signal) performs almost the same as ordinary ISLM with $V^2$ candidates. The D-SLM algorithm gains over 1dB PAPR reduction for clipping levels in the order of $10^{-3}$ compared with ISLM. In Figure 6, the CCDF is plotted for CSLM and D-CSLM algorithm, respectively. It is clearly that D-CSLM shows much better performance than conventional CSLM with 0.7dB PAPR reduction for clipping levels $\Pr(\text{PAPR} > \text{PAPR}_0) = 10^{-3}$.

![Fig.5 PAPR reduction performance of ISLM and D-ISLM](image)

![Fig.6 PAPR reduction performance of CSLM and D-CSLM](image)
The CCDF plotted for D-ISLM and D-CSLM algorithms is shown in Figure 7. Obviously, D-CSLM shows worse performance than D-ISLM with nearly 2dB PAPR degradation for clipping levels $Pr(\text{PAPR} > \text{PAPR}_0) = 10^{-3}$.

In order to illustrate the SI of ISLM and D-ISLM, the bits of SI to transmit is plotted for different number of candidate signals $V = 2, 4, 8, \ldots, 1024$, and the number of antenna is $M = 2, 4, 8$ respectively. It is shown in Figure 8 that the D-ISLM requires more SI compared with the ISLM.

The SI of CSLM and D-CSLM is plotted when the number of antenna is $M = 2$. It is shown in Figure 9 that the D-CSLM requires only a slightly more SI compared with the CSLM. It is shown in Fig 10 that the D-CSLM requires less SI compared with D-ISLM. Therefore, the D-CSLM algorithm can achieve a moderate tradeoff between the PAPR performance and SI transmitted.

For comparison, the CCDF of D-ISLM and D-CSLM with $V = 2, 4, 8$ is compiled in Figure 11. It is clear that the PAPR performance becomes better as the number of candidate signals $V$. 

---

**Fig.7 PAPR reduction performance of D-ISLM and D-CSLM**

**Fig.8 SI for ISLM and D-ISLM**

**Fig.10 SI for ISLM and D-ISLM**
Fig. 9 SI for CSLM and D-CSLM

Fig. 10 SI for D-ISLM and D-CSLM

Fig 11. PAPR reduction performance for different $V$
5. Conclusions

Decomposed concurrent SLM algorithms are considered for PAPR reduction in MIMO-OFDM systems. PAPR reduction in MIMO-OFDM systems by decomposed SLM algorithm is considered. Its Main idea is to choose the combinations of pairs of candidate time-domain signals to transmit. As a result, the significant PAPR reduction performance can be achieved without increasing computational complexity. Furthermore, a D-CSLM algorithm is proposed to reduce the side information at the expense of a slight degradation of the PAPR performance. In summary, decomposed SLM algorithms are efficient to PAPR reduction in wireless MIMO-OFDM systems.

REFERENCES


Northeastern University at Qinhuangdao, Qinhuangdao, Hebei Province, 110004, China.