THE PASSIVITY CONTROL FOR T-S FUZZY SYSTEMS

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Abstract. In this paper, we propose the notion of strict passivity to T-S fuzzy system and consider the problem of passivity control for a kind of uncertain T-S fuzzy system with time-delay. The sufficient conditions which make the closed-loop system be stable and strictly passive are obtained for the system. The conditions are expressed as linear matrix inequalities (LMIs). So checking the stability and passivity of the system can be finished by LMI Toolbox. The design scheme of state feedback controller which guarantees stability and strict passivity of the closed-loop system is obtained simultaneously. Finally, an example is given to show the validity and feasibility of the proposed approach.

Key Words. linear matrix inequality (LMI), passivity, time-delay, T-S fuzzy systems, uncertain

1. Introduction

The notion of passivity of energy across resistors in an electrical circuit has been widely used to analyze stability of a general class of interconnected nonlinear systems [1]. In 1972, J. C. Willems studied passivity for a nonlinear systems represented in state-space form. It allows for a more geometric interpretation of the notions such as stored and dissipated energy in terms of Lyapunov functions [2,3]. In 1990’s, C. I. Byrnes addressed the problem which finite-dimensional nonlinear system is rendered passive via state feedback and provided rather complete answer in terms of geometric nonlinear system theory [1]. W. Q. Sun proposed that if the nonlinearity or uncertainty can be characterized by a positive real system, then the classical results in stability theory can be used to guarantee robust stability provided an appropriate closed-loop system is strictly positive real (see for example [4]). In recent years, dissipation and passivity recognized as an important tool of studying nonlinear systems has played a major role in stability theory (see for example [5-8] and the references therein). It enriches and develops further the stability theory based on Lyapunov theory.

It is well known that the T-S fuzzy model is an effective method in approximating a complex nonlinear system(see for example [9] and the references therein). Fuzzy technique has been extensively used in nonlinear system control for more than two decades [9,10]. However, little attention has been paid to the dissipation and passivity control for the uncertain T-S fuzzy systems with time-delay. In this paper, we consider the problem of passivity control for a kind of uncertain T-S fuzzy system with time-delay. The notion of strict passivity is proposed to T-S fuzzy system. The sufficient conditions which make the closed-loop system be stable and strictly passive are obtained for the system. The conditions are expressed as linear
matrix inequalities (LMIs). So testing the stability and passivity of the system can be finished by LMI Toolbox. The design scheme of state feedback controller is obtained simultaneously. Finally, an example is given to show the validity and feasibility of the proposed approach.

This paper is organized as follows. In section 2, we present problem formulation and preliminaries. In section 3, passive analysis is done for the system without control input. In section 4, we consider the problem of passivity control for a kind of uncertain T-S fuzzy system with time-delay. In section 5, a numerical example is given to demonstrate the effectiveness and simplicity of the control method proposed. Finally, concluding remarks are made in section 6.

2. Problem Formulation and Preliminaries

In this section, we consider uncertain T-S fuzzy system with time-delay, its ith fuzzy rule is of the following form:

\[(1) \quad R_i: \text{ if } \xi_1(t) \text{ is } M_{1i}, \text{ and } \xi_2(t) \text{ is } M_{2i}, \ldots \text{ and } \xi_p(t) \text{ is } M_{pi},\]

then \[
\begin{align*}
\dot{x}(t) & = (A_{1i} + \Delta A_{1i})x(t) + A_{2i}x(t-d) + B_{1i}w(t) + (B_{2i} + \Delta B_{2i})u(t), \\
z(t) & = (C_{1i} + \Delta C_{1i})x(t) + D_{1i}w(t) + (D_{2i} + \Delta D_{2i})u(t), \quad i = 1, 2, \ldots, r, \\
x(t) & = 0, \quad \forall t \in [-d, 0],
\end{align*}
\]

where \(M_{ji}(j = 1, 2, \ldots, p)\) is the fuzzy set, \(r\) is the number of if-then rules, \(x(t) \in \mathbb{R}^n\) is the state vector, \(u(t) \in \mathbb{R}^m\) is the control input, \(w(t) \in \mathbb{R}^l\) is the exogenous input, and \(z(t) \in \mathbb{R}^q\) is the controlled output, \(d \geq 0\) is time delay constant, \(A_{1i}, A_{2i}, B_{1i}, B_{2i}, C_{1i}, D_{1i}, D_{2i}\) are known constant matrices with appropriate dimensions. \(\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_p(t)]^T\) is the premise variables. The matrices \(\Delta A_{1i}, \Delta B_{2i}, \Delta C_{i}, \Delta D_{2i}\) represent the time-varying parametric uncertainties with the following structure:

\[
[\Delta A_{1i}, \Delta B_{2i}] = H_{1i}F_i[E_{1i}, E_{2i}],
\]

\[
[\Delta C_{i}, \Delta D_{2i}] = H_{2i}F_i[E_{1i}, E_{2i}], \quad i = 1, 2, \ldots, r,
\]

where \(H_{1i}, H_{2i}, E_{1i}\) and \(E_{2i}\) are known constant matrices with appropriate dimensions, and \(F_i\) are unknown real matrices satisfying \(F_i^TF_i \leq I\). By using the fuzzy inference methods with singleton fuzzifier and weighted average defuzzifier, the overall fuzzy model for the system can be inferred as follows:

\[(2) \quad \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[(A_{1i} + \Delta A_{1i})x(t) + A_{2i}x(t-d) + B_{1i}w(t) + (B_{2i} + \Delta B_{2i})u(t)],
\]

\[
z(t) = \sum_{i=1}^{r} h_i(\xi(t))[(C_{1i} + \Delta C_{1i})x(t) + D_{1i}w(t) + (D_{2i} + \Delta D_{2i})u(t)], \quad i = 1, 2, \ldots, r,
\]

\[
x(t) = 0, \quad \forall t \in [-d, 0],
\]
\[ \beta_i(\xi(t)) = \prod_{j=1}^{P} M_{ji}(\xi_j(t)) \geq 0, \]
\[ h_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{i=1}^{r} \beta_i(\xi(t))} \geq 0, \]
\[ \sum_{i=1}^{r} h_i(\xi(t)) = 1, \]

where \( M_{ji}(\cdot) \) is the grade of membership of \( \xi_j(t) \) in \( M_{ji} \) and \( h_i(\xi(t)) \) can be regarded as the normalized weight of each if-then rule.

The following lemma is used later.

**Lemma 1.** [see [11]] Given matrices \( Q, H \) and \( E \) with appropriate dimensions and \( Q \) is symmetric, then
\[ Q + HF E + E^T F^T H^T < 0 \]
for all \( F \) satisfying \( F^T F \leq I \), if and only if there exists some scalar \( \varepsilon > 0 \) such that
\[ Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0. \]

### 3. Passivity Analysis

The system of the form (2) is said to be strictly passive if there exists a positive constant \( \delta \), for each solution with initial condition \( x(0) = 0 \), such that the following matrix inequality
\[ V(x(\tau)) \leq \int_{0}^{\tau} z^T(t)w(t)dt - \delta \int_{0}^{\tau} w^T(t)w(t)dt \]
for all positive constant \( \tau \) and \( w(t) \in L_2[0, \tau] \), where \( V \) is a nonnegative continuous function satisfying \( V(0) = 0 \).

Consider the system (2) with \( u(t) = 0 \) as follows
\[ \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[(A_{1i} + \Delta A_{1i})x(t) + A_{2i}x(t-d) + B_{1i}w(t)], \]
\[ z(t) = \sum_{i=1}^{r} h_i(\xi(t))[(C_i + \Delta C_i)x(t) + D_{1i}w(t)], \quad i = 1, 2, \ldots, r, \]
\[ x(t) = 0, \quad \forall t \in [-d, 0]. \]

**Theorem 1.** For system (3) with the zero initial condition, if there exist a common symmetric positive definite \( P \in \mathbb{R}^{n \times n} \) and a positive constant \( \delta \) such that the following LMIs:
\[ \sum_{i=1}^{r} h_i(\xi(t))(-D_{1i} - D_{1i}^T + \delta I) < 0, \]
\[ \sum_{i=1}^{r} h_i(\xi(t)) \begin{pmatrix} \Gamma_i & PA_{2i} & \Lambda_i \\ A_{2i}^T P & -Q & 0 \\ \Lambda_i^T & 0 & \Omega_i \end{pmatrix} < 0, \quad i = 1, 2, \ldots, r, \]
where
\[ \Gamma_i = (A_{1i} + \Delta A_{1i})^T P + P(A_{1i} + \Delta A_{1i}) + Q, \]
\[ \Lambda_i = PB_{1i} - (C_i + \Delta C_i)^T, \]
\[ \Omega_i = -D_{1i} - D_{1i}^T + \delta I, \]
then the system (3) is stable and strictly passive.

Remark 1. When the system (3) without time-delays and uncertainties and $h_i(\xi(t)) = 1$, $h_j(\xi(t)) = 0$, $\forall j \neq i$, the result of theorem 1 is the sufficient condition to test positive real for linear system (see [12]).

Theorem 2. For system (3) with the zero initial condition, if there exist a common symmetric positive definite $X \in \mathbb{R}^{n \times n}$ and positive constants $\delta$ and $\varepsilon_i$ such that the following LMIs:

$$-D_{1i} - D_{1i}^T + \delta I < 0,$$

(6)

$$
\begin{pmatrix}
\bar{\Gamma}_i & A_{2i} & \bar{\Lambda}_i & \varepsilon_i H_{1i} & X E_{1i}^T & X Q \\
A_{2i}^T & -Q & 0 & 0 & 0 & 0 \\
\bar{\Lambda}_i & 0 & \Omega_i & -\varepsilon_i H_{2i} & 0 & 0 \\
\bar{\varepsilon}_i H_{1i}^T & 0 & -\varepsilon_i H_{2i}^T & -\varepsilon_i I & 0 & 0 \\
E_{1i} X & 0 & 0 & 0 & -\varepsilon_i I & 0 \\
Q X & 0 & 0 & 0 & 0 & -Q \\
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
$$

(7)

where

$$\bar{\Gamma}_i = X A_{1i}^T + A_{1i} X, \quad \bar{\Lambda}_i = B_{1i} - X C_{1i}^T, \quad X = P^{-1}.$$

then the system (3) is stable and strictly passive.

We can check the stability and passivity for the system (3) by solving LMIs in Theorem 2.

4. Passivity Control

The parallel distributed compensation (PDC) gives a procedure to design a fuzzy controller from the given T-S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model [9]. The designed fuzzy controller shares the same fuzzy sets as the model in the premise parts. For the fuzzy model (1), we construct the following fuzzy controller via the PDC:

$$u(t) = \sum_{i=1}^r h_i(\xi(t)) K_i x(t),$$

(8)

where $K_i$ is the local feedback gain. By substituting (8) into system (2), we obtain the following closed-loop system:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\xi(t)) h_j(\xi(t)) \{[A_{1i} + \Delta A_{1i} + (B_{2i} + \Delta B_{2i}) K_j] x(t) + A_{2i} x(t - d) + B_{1i} w(t)\},$$

$$z(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\xi(t)) h_j(\xi(t)) \{[C_i + \Delta C_i + (D_{2i} + \Delta D_{2i}) K_j] x(t) + D_{1i} w(t)\}, \quad i = 1, 2, \ldots, r,$$

$$x(t) = 0, \quad \forall t \in [-d, 0],$$

Theorem 3. For system (9) with the zero initial condition, if there exist a common symmetric positive definite $P \in \mathbb{R}^{n \times n}$ and a positive constant $\delta$ such that the following LMIs:

$$\sum_{i=1}^r h_i(\xi(t)) (-D_{1i} - D_{1i}^T + \delta I) < 0,$$

(10)
express the above equation as

\[ \Lambda \]

where

\[ \Gamma_{ij} = [A_{i1} + \Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j]^TP + P[A_{i1} + \Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j] + Q, \]

\[ \Lambda_{ij} = PB_{1i} - [C_i + \Delta C_i + (D_{2i} + \Delta D_{2i})K_j]^T, \]

\[ \Omega_i = -D_{1i} - D_{1i}^T + \delta I, \]

then the system \( (9) \) is stable and strictly passive.

**Proof.** Consider the following nonnegative functional

\[ V(x(t)) = \frac{1}{2}x^T(t)Px(t) + \frac{1}{2} \int_{t-d}^t x^T(s)Qx(s)ds, \]

where \( Q \in \mathbb{R}^{n \times n} \) is a symmetric positive definite. Taking its time derivative along the solution of the closed-loop system \( (9) \) and arranging terms, we have

\[ \dot{V}(x(t)) = z^T(t)w(t) + \frac{1}{2} \delta w^T(t)w(t) \]

\[ = \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r h_i(\xi(t))h_j(\xi(t)) \times \]

\[ \left( x^T(t)[\Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j]^TP + P[A_{i1} + \Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j] + Q \right) x(t) + \]

\[ x^T(t)PB_{1i} - [C_i + \Delta C_i + (D_{2i} + \Delta D_{2i})K_j]^T \times \]

\[ w(t) + w^T(t)\Delta D_{2i}K_j]x(t) + x^T(t)PA_{2i}x(t - d) + \]

\[ x^T(t - d)A_{2i}^TPx(t) - x^T(t - d)Qx(t - d) + \]

\[ w^T(t)\left(-D_{1i} - D_{1i}^T + \delta I\right)w(t). \]

In terms of the augmented state vector \( Y(t) = [x^T(t), x^T(t - d), w^T(t)]^T, \) we can express the above equation as

\[ \dot{V}(x(t)) = z^T(t)w(t) + \frac{1}{2} \delta w^T(t)w(t) \]

\[ = \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r h_i(\xi(t))h_j(\xi(t))Y^T(t) \left( \begin{array}{ccc} \Gamma_{ij} & PA_{2i} & \Lambda_{ij} \\ A_{2i}^T & -Q & 0 \\ \Lambda_{ij}^T & 0 & \Omega_i \end{array} \right) Y(t), \]

where

\[ \Gamma_{ij} = [A_{i1} + \Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j]^TP + P[A_{i1} + \Delta A_{i1} + (B_{2i} + \Delta B_{2i})K_j] + Q, \]

\[ \Lambda_{ij} = PB_{1i} - [C_i + \Delta C_i + (D_{2i} + \Delta D_{2i})K_j]^T, \]

\[ \Omega_i = -D_{1i} - D_{1i}^T + \delta I. \]

From the inequality \( (11) \), we have

\[ \dot{V}(x(t)) = z^T(t)w(t) + \frac{1}{2} \delta w^T(t)w(t) < 0, \]

from which stability of system \( (9) \) is obtained when \( w(t) = 0 \). Note that \( V(x(0)) = 0 \). Therefore, by integrating the inequality \( (12) \) from zero to \( \tau \), we obtain

\[ V(x(\tau)) \leq \int_0^\tau z^T(t)w(t)dt - \frac{1}{2} \delta \int_0^\tau w^T(t)w(t)dt, \]
for all positive constant \( \tau \) and \( w(t) \in L_2[0, \tau] \). Thus, the system (9) is strictly passive.

**Theorem 4.** For system (9) with the zero initial condition, if there exist a common symmetric positive definite \( X \in \mathbb{R}^{n \times n} \) and positive constants \( \delta, \varepsilon_i \) and \( \varepsilon_{ij} \) such that the following LMIs:

\[
-D_{1i} - D_{1i}^T + \delta I < 0, \quad i = 1, 2, \ldots, r,
\]

\[
-\begin{pmatrix}
\tilde{\Gamma}_{ii} & A_{2i} & \tilde{\Lambda}_{ii} & \varepsilon_i H_{1i} & \tilde{\Xi}_{ii} & X Q
\end{pmatrix}
\begin{pmatrix}
\varepsilon_i H_{1i}^T & 0 & 0 & 0 & 0
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\]

\[
-\begin{pmatrix}
\tilde{\Gamma}_{ij} + \tilde{\Gamma}_{ji} & A_{2i} & \tilde{\Lambda}_{ij} + \tilde{\Lambda}_{ji} & \varepsilon_{ij} H_{1i}^T & \tilde{\Xi}_{ij} & \tilde{\Xi}_{ji} & X Q
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{ij} H_{1j} & 0 & 0 & 0 & 0
\end{pmatrix} < 0, \quad i < j \leq r,
\]

where

\[
\begin{align*}
\tilde{\Gamma}_{ii} &= X A_{1i}^T + A_{1i} X + N_i^T B_{2i}^T + B_{2i} N_i, \\
\tilde{\Phi}_{ij} &= A_{2i} + A_{2j}, \\
\tilde{\Lambda}_{ij} &= B_{1i} - X C_{1i}^T - N_j^T D_{2i}, \\
\tilde{\Xi}_{ij} &= X E_{1i}^T + N_i^T E_{2i}^T, \\
X &= P^{-1}, \quad N_i = K_i X,
\end{align*}
\]

then the system (9) is stable and strictly passive. The state feedback controller is

\[
u(t) = \sum_{i=1}^{r} h_i(\xi(t)) N_i X^{-1} x(t).
\]

**Proof.** The matrix inequality (10) holds if

\[
-D_{1i} - D_{1i}^T + \delta I < 0, \quad i = 1, 2, \ldots, r,
\]

that is, the desired inequality (13) holds.
From the inequality (11), it follows that

$$
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \begin{pmatrix}
\Gamma_{ij} & PA_{2i} & \Lambda_{ij} \\
A_{ij}^TP & -Q & 0 \\
\Lambda_{ij}^T & 0 & \Omega_i
\end{pmatrix}
= \sum_{i=1}^{r} h_i^2(\xi(t))G_{ii} + \sum_{i<j} h_i(\xi(t))h_j(\xi(t))(G_{ij} + G_{ji}) < 0,
$$

where,

$$
G_{ij} = \begin{pmatrix}
\Gamma_{ij} & PA_{2i} & \Lambda_{ij} \\
A_{ij}^TP & -Q & 0 \\
\Lambda_{ij}^T & 0 & \Omega_i
\end{pmatrix},
$$

$$
\Gamma_{ij} = [A_{11} + \Delta A_{11} + (B_{2i} + \Delta B_{2i})K_{j}]^TP + P[A_{11} + \Delta A_{11} + (B_{2i} + \Delta B_{2i})K_{j}] + Q,
$$

$$
\Lambda_{ij} = PB_{1i} - [C_i + \Delta C_i + (D_{2i} + \Delta D_{2i})K_{j}],
$$

$$
\Omega_i = -D_{1i} - D_{1i}^T + \delta I.
$$

Therefore, the inequality (11) holds only if there exists some positive constant \(\varepsilon_i\) such that

$$
\begin{pmatrix}
\Gamma_{ij} & PA_{2i} & \Lambda_{ij} \\
A_{ij}^TP & -Q & 0 \\
\Lambda_{ij}^T & 0 & \Omega_i
\end{pmatrix} + \varepsilon_i \begin{pmatrix}
PH_{1i} & 0 & 0 \\
0 & -H_{2i} & 0 \\
0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
E_{1i}^+ & K_{i}^T & E_{2i}^T \\
0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
F_i^T & H_{1i}^TP & 0 & -H_{2i}^T
\end{pmatrix} + \varepsilon_i^{-1} \begin{pmatrix}
E_{1i}^+ & K_{i}^T & E_{2i}^T \\
0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
E_{1i} & E_{2i}K_i & 0 & 0
\end{pmatrix} < 0.
$$

By the Schur complements, the above matrix inequality holds if and only if

$$
\begin{pmatrix}
\Gamma_{ij} - Q & PA_{2i} & \Lambda_{ij} & PH_{1i} & \Xi_{ii} & I \\
A_{ij}^TP & -Q & 0 & 0 & 0 & 0 \\
\Lambda_{ij}^T & 0 & \Omega_i & -H_{2i} & 0 & 0 \\
H_{1i}^TP & 0 & -H_{2i} & -\varepsilon_i^{-1}I & 0 & 0 \\
\Xi_{ii}^T & 0 & 0 & -\varepsilon_iI & 0 & 0 \\
I & 0 & 0 & 0 & -Q^{-1}
\end{pmatrix} < 0,
$$

(16)
where

\[ \tilde{\Xi}_{ii} = E_{1i}^T + K_{1i}^T E_{2i}^T, \]

other symbols are the same meaning as the above. Pre-and Post-multiplying both sides of the above inequality (16) by \( \text{diag}(P^{-1}, I, I, \varepsilon_i I, I, Q) \), respectively, we can obtain that the inequality (16) is equivalent to

\[
\begin{pmatrix}
\hat{\Gamma}_{ii} & A_{2i} & \check{\Lambda}_{ii} & \varepsilon_i H_{1i} & \tilde{\Xi}_{ii} & XQ \\
A_{2i}^T & -Q & 0 & 0 & 0 & 0 \\
\check{\Lambda}_{ii}^T & 0 & \Omega_i & -\varepsilon_i H_{2i} & 0 & 0 \\
\varepsilon_i H_{1i}^T & 0 & -\varepsilon_i H_{2i}^T & -\varepsilon_i I & 0 & 0 \\
\tilde{\Xi}_{ii}^T & 0 & 0 & 0 & -\varepsilon_i I & 0 \\
QX & 0 & 0 & 0 & 0 & -Q
\end{pmatrix} < 0, \quad i = 1, 2, \ldots, r,
\]

where

\[
\hat{\Gamma}_{ii} = XA_{1i}^T + A_{1i}X + N_i^T D_{2i}^T + B_{2i} N_i,
\]

\[
\check{\Lambda}_{ii} = B_{1i}X - X C_i^T - N_i^T D_{2i}^T,
\]

\[
\tilde{\Xi}_{ii} = X E_{1i}^T + N_i^T E_{2i},
\]

\[
X = P^{-1}, \quad N_i = K_i X.
\]

That is, the desired inequality (14) holds.

Note that

\[
G_{ij} + G_{ji} = \begin{pmatrix}
\hat{\Gamma}_{ij} + \hat{\Gamma}_{ji} & \hat{\Phi}_{ij} & \check{\Lambda}_{ij} + \check{\Lambda}_{ji} \\
\hat{\Phi}_{ij}^T & -2Q & 0 \\
\check{\Lambda}_{ij}^T + \check{\Lambda}_{ji}^T & 0 & \Omega_i + \Omega_j
\end{pmatrix} + \\
\left( \begin{pmatrix}
PH_{1i} \\
0 \\
-H_{2i}
\end{pmatrix} \right) \left( \begin{pmatrix}
PH_{1j} \\
0 \\
-H_{2j}
\end{pmatrix} \right) \left( \begin{pmatrix}
F_i \\
0 \\
F_j
\end{pmatrix} \right) \times \\
\left( \begin{pmatrix}
E_{1i} + E_{2i} K_i \\
0 \\
E_{1j} + E_{2j} K_i
\end{pmatrix} \right) + \\
\left( \begin{pmatrix}
E_{1i}^T + K_{1i}^T E_{2i} \\
0 \\
0
\end{pmatrix} \right) \left( \begin{pmatrix}
E_{1j}^T + K_{1j}^T E_{2j} \\
0 \\
0
\end{pmatrix} \right) \times \\
\begin{pmatrix}
F_i^T \\
0 \\
F_j^T
\end{pmatrix} \left( \begin{pmatrix}
H_{1i}^T P \\
0 \\
-H_{2i}^T
\end{pmatrix} \right),
\]

where

\[
\hat{\Gamma}_{ij} = A_{1i}^T P + PA_{1i} + K_{1i}^T B_{2i} + PB_{2i} K_j + Q,
\]

\[
\check{\Lambda}_{ij} = PB_{1i} - C_i^T - K_{1i}^T D_{2i},
\]

\[
\hat{\Phi}_{ij} = PA_{2i} + PA_{2j}.
\]
Applying Lemma 1, the inequality $G_{ij} + G_{ji} < 0$ holds if and only if there exists some positive constant $\varepsilon_{ij}$ such that

$$\begin{pmatrix}
\Gamma_{ij} + \tilde{\Gamma}_{ij} & \hat{\Phi}_{ij} & \tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & \varepsilon_{ij} P H_{1i} \\
\hat{\Phi}_{ij}^T & -2Q & 0 & 0 \\
\tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & 0 & \Omega_i + \Omega_j & 0 \\
\varepsilon_{ij} H_{1i} & 0 & -H_{2i} & 0
\end{pmatrix} \times
\begin{pmatrix}
\tilde{\Xi}_{ij} \\
0 \\
0 \\
0
\end{pmatrix}
< 0. $$

By the Schur complement formula, the above matrix inequality holds if and only if

$$\begin{pmatrix}
\Gamma_{ij} + \tilde{\Gamma}_{ij} - 2Q & \hat{\Phi}_{ij} & \tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & P H_{1i} \\
\hat{\Phi}_{ij}^T & -2Q & 0 & 0 \\
\tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & 0 & \Omega_i + \Omega_j & 0 \\
\varepsilon_{ij} H_{1i} & 0 & -H_{2i} & 0 \\
\varepsilon_{ij}^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
< 0, $$

where

$$\tilde{\Xi}_{ij} = E_{1i}^T + K_{ij}^T E_{2i}^T,$$

other symbols are the same meaning as the above. Pre-and Post-multiplying both sides of above inequality by $\text{diag}(P^{-1}, I, I, \varepsilon_{ij} I, \varepsilon_{ij} I, I, I, Q)$, respectively, we can obtain that the inequality (17) is equivalent to

$$\begin{pmatrix}
\tilde{\Gamma}_{ij} + \tilde{\Gamma}_{ji} & \hat{\Phi}_{ij} & \tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & \varepsilon_{ij} H_{1i} \\
\hat{\Phi}_{ij}^T & -2Q & 0 & 0 \\
\tilde{\Lambda}_{ij} + \hat{\Lambda}_{ji} & 0 & \Omega_i + \Omega_j & 0 \\
\varepsilon_{ij} H_{1i} & 0 & -\varepsilon_{ij} H_{2i} & 0 \\
\varepsilon_{ij}^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
< 0.$$
\[\begin{pmatrix}
\varepsilon_{ij} H_{1j} & \bar{\Xi}_{ij} & \bar{\Xi}_{ji} & XQ \\
0 & 0 & 0 & 0 \\
-\varepsilon_{ij} H_{2j} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\varepsilon_{ij} I & 0 & 0 & 0 \\
0 & -\varepsilon_{ij} I & 0 & 0 \\
0 & 0 & -\varepsilon_{ij} I & -\frac{1}{2}Q \\
\end{pmatrix} < 0, \quad i < j \leq r,\]

where

\[
\begin{align*}
\tilde{\Gamma}_{ij} &= X A_{1i}^T + A_{1i}X + N_j^T B_{2i}^T + B_{2i}N_j, \\
\tilde{\Phi}_{ij} &= A_{2i} + A_{2j}, \\
\tilde{\Lambda}_{ij} &= B_{1i} - X C_i^T - N_j^T D_{2i}^T, \\
\bar{\Xi}_{ij} &= X E_{1i}^T + N_j^T E_{2i}, \\
X &= P^{-1}, \quad N_i = K_iX.
\end{align*}
\]

That is, the desired inequality (15) holds. This completes the proof.

5. Numerical Example

Consider the following uncertain T-S fuzzy system with time-delays:

\[
R_i : \begin{align*}
& \text{if } x_1(t) \text{ is } M_i, \\
& \text{then } \dot{x}(t) = (A_{1i} + \Delta A_{1i})x(t) + A_{2i}x(t - d) + B_{1i}w(t) + (B_{2i} + \Delta B_{2i})u(t), \\
& z(t) = (C_i + \Delta C_i)x(t) + D_{1i}w(t) + (D_{2i} + \Delta D_{2i})u(t), \quad i = 1, 2, \\
& x(t) = 0, \quad \forall t \in [-d, 0].
\end{align*}
\]

The membership functions are chosen as follows

\[h_1(x_1(t)) = \frac{x_1^2(t)}{2}, h_2(x_1(t)) = 1 - \frac{x_1^2(t)}{2},\]
where state vector $x(t) = [x_1^T(t), x_2^T(t), x_3^T(t)]^T$, $x_i(t) \in \mathbb{R}^1, i = 1, 2, 3$. The control parameters are defined as follows:

$$
A_{11} = \begin{pmatrix} 5 & 0 & 9 \\ 8 & 0 & 9 \\ 9 & 1 & 0 \end{pmatrix}, A_{12} = \begin{pmatrix} 1 & -4 & 3 \\ 6 & 0 & 7 \\ 7 & 8 & 2 \end{pmatrix}, A_{21} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix},
$$

$$
A_{22} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}, B_{11} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, B_{12} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, B_{21} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, B_{22} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},
$$

$$
C_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -5 & -5 \end{pmatrix}, C_2 = \begin{pmatrix} 1 & 4 & -4 \\ 0 & -1 & 1 \end{pmatrix},
$$

$$
D_{11} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, D_{12} = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}, D_{21} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, D_{22} = \begin{pmatrix} 1 & 0 \\ 4 & 0 \end{pmatrix},
$$

$$
H_{11} = \begin{pmatrix} 1 & 3 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}, H_{12} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}, H_{21} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 2 & 1 \end{pmatrix}, H_{22} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},
$$

$$
E_{11} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix}, E_{12} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -4 \end{pmatrix}, E_{21} = \begin{pmatrix} 0 & 0.4 \\ 0 & 0.2 \end{pmatrix},
$$

$$
E_{22} = \begin{pmatrix} 0.4 & 0 \\ 0 & 1 \end{pmatrix}, Q = I.
$$

Solving the matrix inequalities in the Theorem 4 with the LMI toolbox in Matlab, we can obtain its solutions as

$$
P = \begin{pmatrix} 2.8357 & 3.5439 & 6.9144 \\ 3.5439 & 6.2824 & 10.0083 \\ 6.9144 & 10.0083 & 21.5518 \end{pmatrix} > 0,
$$

$$
K_1 = \begin{pmatrix} 0.6053 & 3.7205 & 4.5610 \\ -11.3740 & -15.2382 & -30.5896 \end{pmatrix},
$$

$$
K_2 = \begin{pmatrix} 0.1867 & 1.7199 & 0.5317 \\ -20.8500 & -29.0288 & -57.2956 \end{pmatrix}.
$$

The state feedback gain matrices are obtained simultaneously.

6. Conclusion

In this paper, we have addressed the problem of passivity control for a kind of uncertain T-S fuzzy system with time-delay. The notion of strict passivity has been proposed to T-S fuzzy system. The sufficient conditions which make the closed-loop system be stable and strictly passive have been obtained for the system. An example has been given to show the validity and feasibility of the proposed approach.

References


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