CHAOs IDENTIFICATION OF MOBILE ROBOT NAVIGATION
BASED ON LYAPUNOV EXPONENTS

HONGYAN SHI AND WANLI LIU

Abstract. With developing of mobile robot technology and increasing of working environment complexity, chaos probably appears in the information data for mobile robot navigation, affects navigation and endangers mobile robot. So it has greatly theoretical and practical value to avoid chaos appearing in mobile robot navigation. In this paper, since the information data is time series of single variables, the improved C-C method is proposed to reconstruct phase space and the extended small data sets method is proposed to calculate the largest Lyapunov exponent for the information data. Through typical chaos system, the methods are proved effectual. And chaos in the information data for mobile robot navigation are respectively identified and analyzed.

Key Words. mobile robot navigation, chaos identification, phase space reconstruction, Lyapunov exponents, small data sets method.

1. Introduction

Mobile robot navigation technology is main research task in mobile robot technology, and key technology for intelligentizing and autonomy of mobile robot[1]. Mobile robot navigation is realized based on environment information time series by sensor. So the reliability and stability of the information data affect mobile robot navigation and control. With developing of mobile robot technology and increasing of working environment complexity, chaos probably appears in the information data for mobile robot navigation[2]. This makes the planed path based on environment information data time series be unoptimizable, even makes mobile robot become chaos. The result affects navigation and endangers mobile robot. So eliminating or suppressing chaos is necessary for mobile robot navigation when chaos appears in environment information data. And chaos identification is the first work for chaos control. This paper improves chaos identification methods based on the researched results, and identifies chaos in mobile robot navigation environment information time series.

2. Summary of chaos identification

Chaos is an analogously irregular and stochastic phenomena appearing in certainty systems, and special movement form for non-linear dynamic systems. Chaos exists widely in nature, such as physics, chemistry, biology, geology and technology, sociology etc [3,4]. Since American scientist Lorenz found chaos phenomena when
he researched weather [5], chaos theory developed very fast and many chaos identification methods were found. These methods are divided into qualitative methods and quantitative methods [4].

Qualitative methods mainly include phase space reconstruction method, frequency spectrum method and Poincare sections method. Generally, the incidence of system state variables is the attraction domain when $t \rightarrow \infty$. There are four attraction domains for autonomous system: the stable attraction domain, the periodic attraction domain, the quasi-periodic attraction domain and the strange attraction domain. And the stable attraction domain is for stable system, the periodic attraction domain is for periodic system, the quasi-periodic attraction domain is for quasi-periodic system, the strange attraction domain is for chaos system. Phase space reconstruction method is that reconstruct the phase space for reappearing the attraction domain of system using the real time series. The frequency spectrum is the result spectrum lines of Fast Fourier transformation (FFT) for the real time series. The frequency spectrum for periodic system and quasi-periodic system is independent and discrete. The frequency spectrum for non-periodic system is consecutive. The frequency spectrum for stochastic white noise is smooth and consecutive because they are produced by mass independent factors. But chaos movement is greatly complex. For example, in the process of period-doubling bifurcation, there are some wave crests corresponding new frequency-division and frequency-doubling for every bifurcation in the frequency spectrum, so the frequency spectrum is non-smooth, and there are noise background and width wave crests in the frequency spectrum. Poincare section is an appropriate section selected in multi-dimension phase space. On the section the conjugate variables are constant. We research the intercept points of system movement track on Poincare section. If the intercept points are a moving point or seldom discrete points, the system is periodic system. If the intercept points are a close line, the system is quasi-periodic system. If the intercept points are some serried points, the system is chaos system.

Quantitative methods mainly include largest Lyapunov exponent method, correlation dimension method and Kolmogorov entropy method. Lyapunov exponents are important characteristic measures, and measure neighborhood phase points divergence in system phase space. The Lyapunov exponents are less than zeros for the stable system. The Lyapunov exponents equal zeros for the periodic system. The Lyapunov exponents are larger than zeros for the chaos system, and plus Lyapunov exponents is sufficient condition for chaos systems. Moreover, the Lyapunov exponents will increase following chaos degree of system. Correlation dimension is an important characteristic measure, and measures the complexity of attraction domain for system phase space. For stochastic time series, the correlation dimension will increase following embedding dimension. For chaos time series, the correlation dimension is saturated. Kolmogorov entropy is defined with chaos. It denotes the average loss ratio of information, and is an important characteristic measure for chaos system. For regular movement, $K = 0$. For stochastic movement, $K = \infty$. For chaos movement, $0 < K < \infty$. Moreover, K will increase following chaos degree of system, and the loss ratio of information will increase.

Qualitative methods analyze system externally, the analyzing results show only whether the chaos appear in system, but do not explain system chaos degree. Quantitative methods analyze system quantitatively, the analyzing results not only show whether the chaos appear in system, but also explain system chaos degree, and are necessary precondition for next research. So this paper will identify the chaos in the mobile robot navigation using quantitative method. The largest Lyapunov
exponent method is common method in quantitative methods, because the largest Lyapunov exponent will be calculated easily and the result will be accurate comparing with correlation dimension method and Kolmogorov entropy method. Since Wolf researched Lyapunov exponents track calculating method in 1985, it is the main research problem that how to calculate plus and the largest Lyapunov exponents fast and accurately. For this problem, researchers proposed some Lyapunov exponents calculating methods. The small data sets method [6,7,8] is a common method by Rosenstein M.T, Collins J.J and Deluca C.J. But too large computing amount and information data are in this method, and the calculated Lyapunov exponents would change following information data length, so the calculated result is unstable. For these problems, this paper proposes the extended small data sets method based on small data sets method. The method will bidirectionally search phase points when calculate Lyapunov exponent of system, and compare with the results of the small data sets method and Wolf algorithm. The calculated results indicate that the algorithm in this paper can calculate Lyapunov exponents fast and accurately and analyze dependably chaos in system.

3. Phase space reconstruction

Phase space reconstruction is necessary condition for calculating Lyapunov exponents of time series. Environment information data of mobile robot navigation is single variable time series. Takens embedding theorem considers that the evolution of any variable in system is decided by others associating with it, i.e. the whole system developing process is implicated in the evolution of any variable. Phase space reconstruction theorem considers that the reconstructed phase space with embedding dimension $m$ ($m \geq 2D + 1$, $D$ is correlation dimension of system) and delay time $\tau$ can present character of the whole system. Many phase space reconstruction methods decide independently embedding dimension $m$ and delay time $\tau$. But the method of selecting delay time $\tau$ is greatly complex. There are noise and estimated error. So if delay time $\tau$ is too small, the reconstructed phase space track will press to a same position, the information of system will be difficultly revealed, and the redundancy error will appear. If delay time $\tau$ is too larger, the dynamic character of system will change fast, the reconstructed phase space will become complex, the dynamic system will be distorted, and disrelated error will appear. Fig.1 is the two-dimension reconstructed phase space of Lorenz system for different delay time $\tau$. From the figure, we can get the conclusion explained in the above text. The research results indicate that the master causes affecting reconstructed phase space are not only embedding dimension $m$ and delay time $\tau$ that were independently decided, but also embedding window width $\tau_w$ decided by embedding dimension $m$ and delay time $\tau$. For this, Kim, Eykholt and Salas proposed C-C method that decided synchronously embedding window width $\tau_w$ and delay time $\tau$ using correlation integral of system [9,10].

3.1. C-C method. A real one-dimension time series is in (1).

$$\{x(1), x(2), \cdots, x(N)\}$$

Where $N$ is time series length. If embedding dimension is $m$ and delay time is $\tau$, then the reconstructed phase space for (1) is in (2).

$$X = \{X_i, i = 1, 2, \cdots, M\}$$

Where $X_i = [x(i), x(i+\tau), \cdots, x(i+(m-1)\tau)]$, $M = N - (m - 1)\tau$ For the reconstructed phase space, correlation integral is defined as (3).
\[ C(m, N, r, \tau) = \frac{2}{M(M-1)} \sum_{1 \leq i < j \leq M} H(r - d_{ij}) \]

\[ = \frac{2}{M(M-1)} \sum_{1 \leq i < j \leq M} H(r - \| X_i - X_j \|) \]

\( H(\cdot) \) is Heaviside unit function.

\[ H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \]

Correlation dimension of system is defined as (5).

\[ D(m, \tau) = \lim_{r \to 0} \frac{\ln C(m, r, \tau)}{\ln r} \]

Where \( C(m, r, \tau) = \lim_{N \to \infty} C(m, N, r, \tau) \).

The length \( N \) of real time series is finite, and \( r \) is not infinite small data. So correlation dimension is placed of by the gradient of linear region in general.

\[ D(m, \tau) = \frac{\ln C(m, N, r, \tau)}{\ln r} \]

The theory of C-C method is that time series is divided into \( t \) non-overlapping subsequence time series as (7). The length \( L \) of subsequence time series is integer.
less than $N/t$.

\[
\{x(1), x(1 + t), x(1 + 2t), \cdots \}
\]
\[
\{x(2), x(2 + t), x(2 + 2t), \cdots \}
\]
\[
\vdots
\]
\[
\{x(t), x(t + t), x(t + 2t), \cdots \}
\]

(7)

Then we calculated BDS statistical value $S(m, N, r, t)$ using (8).

\[
S(m, N, r, t) = \frac{1}{t} \sum_{l=1}^{t} \left[ C_l(m, L, r, t) - C_l^m(1, L, r, t) \right]
\]

(8)

We can calculate a series of $S(m, N, r, t)$ value with different $m$, $r$ and $t$. And define differential values as (9).

\[
\Delta S(m, t) = \max[S(m, N, r_i, t)] - \min[S(m, N, r_j, t)], (i \neq j)
\]

(9)

We can receive equations as (10) with $S(m, N, r, t)$ and $\Delta S(m, t)$.

\[
\begin{align*}
\Delta S(t) &= \frac{1}{n_m} \sum_{i=1}^{n_m} \Delta S(m_i, t) \\
S(t) &= \frac{1}{n_m n_r} \sum_{i=1}^{n_m} \sum_{j=1}^{n_r} S(m_i, N, r_j, t) \\
S_{cor}(t) &= \Delta S(t) + |S(t)|
\end{align*}
\]

(10)

The optimization delay time $\tau$ of the reconstructed phase space is $t$ corresponding with the zero point of $S(m, N, r, t)$ or the first minimum value point of $\Delta S(t)$. And the optimization embedding window width $\tau_w$ of the reconstructed phase space is $t$ corresponding with global minimum value point of $S_{cor}(t)$. Then embedding dimension $m$ of the reconstructed phase space is $m = (\tau_w + \tau)/\tau$.

In fact, we get delay time $\tau$ and embedding window width $\tau_w$ using $S(m, N, r, t)$ using (11) statistical theory. Moreover it would make the computing amount diminishes and the computing precision depresses that divided time series into $t$ non-overlapping subsequence time series. So there are two problems in the computing process. First, the zero point of $S(m, N, r, t)$ is not the first minimum value point of $\Delta S(t)$ in computing delay time $\tau$ process. Second, the optimization embedding window width $\tau_w$ of the reconstructed phase space is $t$ corresponding with global minimum value point of $S_{cor}(t)$ in theory. But in fact, there are some local minimum value points being near of the global minimum value point. They affect the deciding of the optimization embedding window width $\tau_w$ of the reconstructed phase space. For the two problems, this paper proposes improved C-C method.

### 3.2. Improved C-C method.

In improved C-C method, we do not divide time series into $t$ non-overlapping subsequence time series again when compute statistical value $S(m, N, r, t)$. And we calculate statistical value $S(m, N, r, t)$ using (11).

\[
S(m, N, r, t) = C(m, N, r, t) - C^m(1, N, r, t)
\]

(11)

The calculating method in (11) solves these two problems and improves the computing precision. But the time series is not divided into $t$ non-overlapping subsequence time series, it makes the computing amount increase greatly. For depressing the computing amount, we can first divide time series into $k$ non-overlapping subsequence time series and calculate statistical value $S(m, N, r, t)$ using (12).

\[
S(m, N, r, t) = \frac{1}{k} \sum_{i=1}^{k} [C_l(m, L, r, t) - C^m_l(1, L, r, t)]
\]

(12)
Where $k$ is a parameter to balance computing precision and computing speed. The time complexity of computing statistical value $S(m,N,r,t)$ is $O(1/k^2)$. We can use (12) to depress computing amount to $1/k^2$ of the original. There is an error in computing correlation integral $C(m,N,r,\tau)$ when $k \neq 1$, but the error is a constant. So the computing precision will not depress following increasing of $t$.

We can get the conclusion from (5) that correlation integral $C(m,N,r,\tau)$ is relative to $r$ in (13) when $r \rightarrow 0$.

\begin{equation}
\lim_{r \rightarrow 0} C(m,N,r,\tau) \propto r^D
\end{equation}

Where $D$ is the correlation dimension of system. But from analyzing (3) we can get that the result of calculating the correlation integral $C(m,N,r,\tau)$ is relative to the selection of $r$. The conclusion is that $C(m,N,r,\tau) = 1$ when $r$ is too large and $C(m,N,r,\tau) = 0$ when $r$ is too small. These two states will not present the inbeing character of system, and have not practical value. It is the non-scale range.

Fig.2 (a) is the ln$r$ − ln$C(r)$ diagram of Lorenz system with non-scale range. From the diagram we can get that the value of ln$C(r)$ will nonlinearly evolve following ln$r$ when $r$ is too large or too small and enter to the non-scale range. If we calculate the correlation dimension $D(m,\tau)$ using (6) under the non-scale range, the computing precision of result is low, even the result is wrong.

For avoiding the non-scale range to affect the calculated result, $r$ value should be selected between $\sigma/2$ and $2\sigma$ ($\sigma$ is the mean-square error of time series). Fig.2 (b) is the ln$r$ − ln$C(r)$ diagram of Lorenz system when $\sigma/2 \leq r \leq 2\sigma$. From the diagram we can get that the value of ln$C(r)$ would sub-linearly evolve following ln$r$ without the non-scale range.

![FIGURE 2. The ln$r$ − ln$C(r)$ diagram. (a) with non-scale range; (b) without non-scale range](image)

We can get the conclusion from the above C-C method computing process that selection of $m$ and $r$ are very important. The selections not only affect the computing precision but also decide the computing amount of the whole computing process. For depressing the computing amount, $m$ is integer between 2 and 5 generally, and $r$ is commonly selected 3 or 4 values between $\sigma/2$ and $2\sigma$.

The distance in computing correlation integral $C(m,N,r,\tau)$ is commonly calculated using 2-Norm in (14).

\begin{equation}
d_{ij} = \| X_i + X_j \|_2 = \sqrt{\sum_{k=1}^{m} (X_{ik} - X_{jk})^2}
\end{equation}
For depressing the computing amount, $2$-Norm can be place of $\infty$-Norm in (15), because they were topological equivalent in Euclidean Space.

\[(15) \quad d_{ij} = \|X_i + X_j\|_\infty = \max_{1 \leq k \leq m} \|X_{ik} - X_{jk}\|]\]

4. Lyapunov exponent calculation

4.1. Wolf method. Wolf method can calculate the largest Lyapunov exponents based on phase track, phase plane and phase cubage of system.

If the embedding dimension is $m$ and the delay time is $\tau$, then the reconstructed phase space $X$ for chaos time series $\{x(1), x(2), \cdots, x(N)\}$ is in (16).

\[(16) \quad X(t) = \{x(t), x(t + \tau), \cdots, x(t + (m - 1)\tau)\}\]

Where $t = 1, 2, \cdots, M, M = N - (m - 1)\tau$. If the initial time is $t_0$ and the initial phase point is $X(t_0)$, and the minimum distance between $X(t_0)$ and its neighboring phase points $X_0(t_0)$ is $L_0$. Pursuing the evolvement of the two phase points, the distance $L'_0 > \varepsilon (\varepsilon > 0, \varepsilon$ is threshold value, $L_0 = \|X(t_1) - X(t_0)\|$) when time is $t_1$. Then the phase point $X(t_1)$ is saved, and we find the other phase point $X_1(t_1)$ with $L_1 = \|X(t_1) - X_1(t_1)\| < \varepsilon$. We will continue the above calculating process until $x(t)$ arrives the end of time series $x(N)$. The iterative times of the above pursuing process is $t_M - t_0$. The largest Lyapunov exponent can be calculated with (17).

\[(17) \quad \lambda_{max} = \frac{1}{t_M - t_0} \sum_{i=0}^{M} \ln \frac{L'_i}{L_i}\]

From the calculating process we can get that the calculated result will change with threshold value $\varepsilon$. For accurate result, the calculator needs abundant experience in the calculating process.

4.2. Extended small data sets method. For solving the shortcoming of Wolf method, the small data sets method is proposed by Rosenstein M. T, Collins J. J. and Deluca C. J. in [6]. This method is used to calculate largest Lyapunov exponent based on the real time series. But this method searches unidirectionally neighboring phase points. The calculated result using small data sets method will change following the length of time series, and it is unstable. For this problem, this paper proposed the extended small data sets method based on the small data sets method.

The reconstructed phase space $X$ for chaos time series is (16). For $\forall X_i \in X, \exists X_j \in X$, and their distance is minimum. i.e.

\[(18) \quad d_i(0) = \min_{X_j \in X} \|X_i - X_j\|, |i - j| > p\]

Where $p$ is the average period of the time series calculated with FFT [11]. Then $X_i$ and $X_j$ go forward $k$ steps along the time orbit and arrive to $X_{i+k}$ and $X_{j+k}$, moreover $X_{i+k}, X_{j+k} \in X$.

\[(19) \quad d_i(k) = \|X_{i+k} - X_{j+k}\|\]

The relation between $d_i(0)$ and $d_i(k)$ is approximate as in (20).

\[(20) \quad d_i(k) \approx d_i(0)e^{\lambda_i \Delta t}\]

Where $\Delta t$ is the sampling time interval of the time series, and $\lambda_i$ is Lyapunov exponent. The largest Lyapunov exponent can be calculated by average radiation
of every neighboring phase points in the basic orbit. The largest Lyapunov exponent can be calculated with (21).

\[ \lambda_{\text{max}} = \frac{1}{k \Delta t_n} \sum_{i=1}^{n_i} \ln \frac{d_i(k)}{d_i(0)} \]  

The theory of the extended small data sets method is that the largest Lyapunov exponent is calculated based on bidirectional searching neighboring phase points. \( X_i \) and \( X_j \) not only go forward \( k \) steps along time orbit, but also go backward \( k \) steps along time orbit and arrive to \( X_{i-k} \) and \( X_{j-k} \), moreover \( X_{i-k}, X_{j-k} \in X \).

\[ d_i(-k) = \| X_{i-k} - X_{j-k} \| \]  

The relation between \( d_i(0) \) and \( d_i(k) \) is approximate as in (23).

\[ d_i(-k) \approx d_i(0) e^{\lambda \Delta t} \]  

Then the calculating equation of the largest Lyapunov exponent will become to (24).

\[ \lambda_{\text{max}} = \frac{1}{2k \Delta t_n} \sum_{i=1}^{n_i} \ln \frac{d_i(k) d_i(-k)}{d_i^2(0)} \]  

5. Simulation and analysis

Simulation experiment is divided into two steps. First we prove the methods in this paper are effectual using the typical chaos system. Then based on the methods in this paper are practicable and ascendant, they are used to identify chaos in the sensor information data of mobile robot navigation.

Simulation experiment proves the methods are practicable and ascendant through two data series: Lorenz system and Henon system. First we can get numerical solutions of the Lorenz system in (25) and the Henon system in (26) using four-order Runge-Kutta method with the initial value of \([0, 0.01, 0]\) for Lorenz system and \([0, 0.1]\) for Henon system, and got rid of the initial 1000 values. Simulation experiment used the next 2000 values.

\[ \begin{align*} 
    \dot{x} &= -10x + 10y \\
    \dot{y} &= 28x - xz - y \\
    \dot{z} &= xy - \frac{8}{3} z 
\end{align*} \]

\[ \begin{align*} 
    x_{n+1} &= 1 - 1.4x_n^2 + y_n \\
    y_{n+1} &= 0.3x_n 
\end{align*} \]

We can reconstruct phase space using improved C-C method. The calculated parameters are that the embedding dimension is \( m = 5 \) and the delay time is \( \tau = 6 \) for Lorenz system and the embedding dimension is \( m = 3 \) and the delay time is \( \tau = 3 \) for Henon system. Fig.3(a) is the phase plane diagram of the reconstructed phase space for Lorenz system and Fig.4(a) is the phase plane diagram of the reconstructed phase space for Henon system. Comparing with the phase planes of the initial system in Fig.3(b) for Lorenz system and Fig.4(b) for Henon system, we can get that the reconstructed phase spaces present the character of the initial system, and illuminate that the improved C-C method is practicable.

For the reconstructed phase spaces, we calculate the largest Lyapunov exponents of different length time series using Wolf method, the small data sets method and the extended small data sets method, and the results are in Tab.1 for Lorenz system and in Tab.2 for Henon system. Through analyzing the calculated results, we can get that the results of extended small data sets method are more comparatively
concentrative and stable than the results of Wolf method and the small data sets method, i.e. they do not change following the length of time series, and are greatly near the real values. So the results of the extended small data sets method are effectual.

\begin{table}[h]
\begin{tabular}{lccc}
\hline
\textbf{N} & \textbf{method1($\lambda_{max}$)} & \textbf{method2($\lambda_{max}$)} & \textbf{method3($\lambda_{max}$)} \\
\hline
600 & 1.859 & 1.637 & 1.522 \\
800 & 1.641 & 1.583 & 1.516 \\
1200 & 1.461 & 1.413 & 1.498 \\
1600 & 1.487 & 1.491 & 1.511 \\
2000 & 1.509 & 1.511 & 1.507 \\
\hline
\end{tabular}
\end{table}

In the following simulation experiment, we will identify chaos in information data of mobile robot experiment. The experiment environment is a square area. The initial position of mobile robot is (0, 0), and the goal position is (100, 100). There are two static obstacles in the centre of the work space, and there is a moving obstacle moving from (0, 50) to (100, 50) with sine track. The distance time series of mobile robot and obstacles are \{x_i, i = 1,2,\cdots,N\}. We will analyze these time series with the methods in this paper.
First reconstruct phase space for the distance time series using the improved C-C method. The calculated parameters are that the embedding dimension is $m = 6$ and the delay time is $\tau = 7$ for the moving obstacle and the embedding dimension is $m = 3$ and the delay time is $\tau = 25$ for static obstacle. Then calculate the largest Lyapunov exponents of the reconstructed phase spaces. The results are in Tab.3.

### TABLE 3. The largest Lyapunov exponents of distance time series

<table>
<thead>
<tr>
<th>N</th>
<th>Moving obstacle($\lambda_{max}$)</th>
<th>Static obstacle($\lambda_{max}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.0323</td>
<td>0.0005</td>
</tr>
<tr>
<td>800</td>
<td>0.0373</td>
<td>-0.0003</td>
</tr>
<tr>
<td>1000</td>
<td>0.0333</td>
<td>-0.0007</td>
</tr>
</tbody>
</table>

Through analyzing the calculated results in Tab.3, we can get that chaos appears in the distance time series of mobile robot and moving obstacle, and chaos disappears the distance time series of mobile robot and static obstacle.

### 6. Conclusion

Based on the character of time series in mobile robot navigation, the character of chaos system and the character of traditional chaos values algorithms, this paper calculates the embedding dimension $m$ and the delay time $\tau$ for the reconstructed phase space using the improved C-C method, and calculates the largest Lyapunov exponent of the reconstructed phase space using the extended small data sets method. And proves that the methods are effective through the typical chaos system-Lorenz system and Henon system. Simulation results indicate that the extended small data sets method can calculate accurately the largest Lyapunov exponents of system, and the results are greatly stable. At last this paper dependably analyzes chaos appearing mobile robot navigation.

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