QUADRATIC STABILIZATION FOR A CLASS OF SWITCHED NONLINEAR SINGULAR SYSTEMS

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Abstract. This paper is concerned with the quadratic stabilization for a class of switched nonlinear singular systems. For this class of switched systems, when each individual subsystem fails to be minimum-phase, the quadratic stabilization problem is solved via designing switching law. This extends available results for non-switched nonlinear singular systems.

Key Words. Switched nonlinear singular systems, single Lyapunov functions, zero dynamics, quadratic stabilization.

1. Introduction

A switched system is a dynamic system described by a family of continuous-time subsystems and a rule that orchestrates the switching between them. There has been increasing interest in the stability analysis and designing methodology recently in the literature about switched systems [1-6]. In general, a switched system does not inherit properties of the individual subsystems. A well-known example is that switching among globally exponentially stable subsystems can lead to instability [1]. Stability of switched systems under arbitrary switching is a desirable property because this property enables one to seek for other system performances by switching without changing stability. A common Lyapunov function for all subsystems ensures asymptotic stability under arbitrary switching laws [1]. Once switched systems do not possess a common Lyapunov function, they still may be asymptotically stable under some properly chosen switching law. The single Lyapunov function method is a powerful and effective tool for finding a switching law.

Switched singular systems constitute an important class of switched systems, which arise in electrical networks [11,12], economic systems [13]. The analysis of switched singular systems is more sophisticated, since discrete dynamic, regularity, impulse elimination and state consistence of such systems should be considered at the same time. Recently some preliminary results on switched linear singular systems have been given [11-14]. Reachability was analyzed by using geometric approach [14]; asymptotic properties, including complexity reduction and limit behavior, of large-scale hybrid singular systems were analyzed [15]. However, most of the results above mentioned are suitable for switched linear singular systems, little work deals with switched nonlinear singular systems.

On the other hand, for a (non-switched) nonlinear singular system, by using geometric method, the concept and the algorithm of zero dynamics has been recently obtained and the stabilization and decoupling problems are also studied by its normal form and minimum-phase property in [8-10].
In this paper, we will generalize the result in [8,10], which make known that the stability of zero dynamics implies the stabilization of a (non-switched) nonlinear singular system, to switched singular systems. We deal with stabilization of a class of switched nonlinear singular systems (SNSS) under some switching laws. When zero dynamics of such SNSS does not possess a common Lyapunov function, we will show that if a convex combination system of zero dynamics is stable, then the SNSS is still stabilized under some constructed switching law. In this case, we do not need any subsystem of the SNSS is stable.

2. Problem statement

Consider the switched nonlinear singular system

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
& \vdots \\
\dot{\xi}_{r-1} &= \xi_r \\
\dot{\xi}_r &= a_\sigma(t)(\xi) + b_\sigma(t)(\xi)z + c_\sigma(t)(\xi)u_\sigma(t) \\
\dot{\eta} &= q_\sigma(t)(\xi) + p_\sigma(t)(\xi)z + R_\sigma(t)(\xi)u_\sigma(t)
\end{align*}
\]

where \( \xi = (\xi_1, \xi_2, \ldots, \xi_n)^T \in \mathbb{R}^n \) is the vector of differential variables, \( z \in \mathbb{R}^s \) is the vector of algebraic variables, \( \sigma : [0, +\infty) \to \{1, \ldots, m\} \) is the switching law, \( u \) is control input, \( a_i(\xi), f_i(\xi), h_i(\xi), c_i(\xi), q_i(\xi), p_i(\xi), R_i(\xi), b_i(\xi), g_i(\xi), i = 1, \ldots, m, \) are the matrix-valued smooth functions having appropriate sizes, and \( a_i(0) = 0, f_i(0) = 0, q_i(0) = 0, i = 1, \ldots, m. \)

**Definition 1:** System (1),( 2) is said to be quadratically stabilizable under some switching if there exist positive quadratic function \( w(\xi) \), smooth nonlinear controllers \( u_i(\xi), u_i(0) = 0, \) \( i = 1, \ldots, m, \) and a switching law \( \sigma_0(t) \), such that along the solution of the corresponding closed-loop system with switching law \( \sigma_0(t), \)

\[
\dot{w} \leq -\beta \|\xi\|^2
\]

holds, where \( \beta > 0. \)

Our goal is to find conditions under which system (1),( 2) is quadratically stabilizable under some switching.

We need following assumptions.

**Assumption A:** each subsystem of SNSS (1),(2) is regular.

**Assumption B:** matrix \( \begin{pmatrix} g_i \\ b_i \\ c_i \end{pmatrix} \) is nonsingular at \( \xi_0 = 0, \) \( i = 1, \ldots, m. \)

If Assumption A holds, there will exist \( r_i(\xi), \) such that \( g_i + h_ir_i \) is nonsingular, \( i = 1, \ldots, m. \) Let

\[
(3) \quad u_i = r_i z + \tilde{u}_i,
\]

from (3) and (2), we have

\[
(4) \quad z = -(g_i + h_ir_i)^{-1}(f_i + h_i\tilde{u}_i).
\]
Substituting (4) into (1), then the equation (1) becomes

\[\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
& \vdots \\
\dot{\xi}_{r-1} &= \xi_r \\
\dot{\xi}_r &= \tilde{a}_{\sigma(t)}(\xi) + \tilde{c}_{\sigma(t)}(\xi)\tilde{u}_{\sigma(t)} \\
\dot{\eta} &= P_{\sigma(t)}(\xi) + Q_{\sigma(t)}(\xi)\tilde{u}_{\sigma(t)}
\end{align*}\]

where

\[\begin{align*}
\tilde{a}_i &= a_i - (b_i + c_i)(g_i + h_i r_i)^{-1} f_i \\
\tilde{c}_i &= c_i - (b_i + c_i)(g_i + h_i r_i)^{-1} h_i \\
P_i &= q_i - (b_i + c_i)(g_i + h_i r_i)^{-1} f_i \\
Q_i &= R_i - (b_i + c_i)(g_i + h_i r_i)^{-1} h_i
\end{align*}\]

From lemma 1 of [10], it is known that \(\tilde{c}_i\) is nonsingular, \(i = 1, \ldots, m\). The zero dynamics of (5) is defined by the system

\[\dot{\eta} = \tilde{Q}_{\sigma(t)}(0, \eta),\]

where \(\zeta = (\xi_1, \ldots, \xi_r)^T, \eta = (\xi_{r+1}, \ldots, \xi_n)^T\) and

\[\tilde{Q}_i(\zeta, \eta) = P_i(\zeta, \eta) - Q_i(\zeta, \eta)\frac{\tilde{a}_i(\zeta, \eta)}{\tilde{c}_i(\zeta, \eta)}\]

Zero dynamics (6) is again a switched nonlinear system, whose stability play a key role in the stabilization of system (1), (2).

3. Quadratic stabilization under some switching

When the zero dynamics of system (1), (2) fails to be minimum-phase, it is possible to realize the quadratic stabilization under some switching law by single Lyapunov function technique.

Theorem 1: Suppose Assumption A and Assumption B hold, and a convex combination system of the zero dynamics (6) is quadratically stable, that is, there exists a positive quadratic function \(U(\eta)\), such that

\[\frac{\partial U}{\partial \eta} \sum_{i=1}^{m} \delta_i \tilde{Q}_i(0, \eta) \leq -\beta \|\eta\|^2,\]

where \(\delta_i \geq 0, i = 1, \ldots, m, \sum_{i=1}^{m} \delta_i = 1\), then SNSS (1), (2) is quadratically stabilizable under some switching.

Proof: Assumption A implies that system (1), (2) can be translated into system (5) by using (3) and (4). Choose

\[\tilde{u}_i = \frac{1}{\tilde{c}_i}(-\tilde{a}_i - d_0 \xi_1 - d_1 \xi_2 - \ldots - d_{r-1} \xi_r), i = 1, \ldots, m,\]

where positive coefficients \(d_0, d_1, \ldots, d_{r-1}\) are selected to ensure
is Hurwitz. Let $P$ satisfy $PA + A^TP \leq -I$. The closed-loop system (5), (8) takes the form

\begin{align}
\dot{\zeta} &= A\zeta \\
\dot{\eta} &= H_{\sigma(t)}(\zeta, \eta)
\end{align}

where $H_i(\zeta, \eta) = P_i + Q_i \cdot \frac{1}{\alpha_i}(-\bar{a}_i - d_0 \xi_1 - d_1 \xi_2 - \ldots - d_{r-1} \xi_r)$. It is easily verified that

\begin{align}
H_i(0, \eta) &= \tilde{Q}_i(0, \eta).
\end{align}

Let $K_0 = \frac{\tilde{\alpha}^2 L^2}{2\theta}$, $\theta_0 = \sqrt{\frac{1}{\tilde{\alpha} \theta} + \frac{\tilde{\alpha} L}{4\theta_0}}$, where $\tilde{\alpha}$ satisfies $\|\frac{\partial U}{\partial \eta}\| \leq \tilde{\alpha} \|\eta\|$, $L$ is the local Lipschitz coefficient of $H_i(\zeta, \eta)$, i.e., $\|H_{\sigma_i}(\zeta, \eta) - H_i(0, \eta)\| \leq L \|\zeta\|$, $i = 1, \ldots, m$. It is easily tested that $K_0 - \frac{\tilde{\alpha} L}{2\theta_0} > 0, \beta - \frac{\tilde{\alpha} L \Delta^2}{2} > 0$. Let $W(\zeta, \eta) = K_0 \zeta^T \tilde{P} \zeta + U(\eta)$ and

\begin{align}
\Omega_i = \{\eta : \frac{\partial U}{\partial \eta} \tilde{Q}_i(0, \eta) + \beta \|\eta\|^2 \leq 0\}, \quad D_i = \{(\zeta^T, \eta^T)^T \eta \in \Omega_i\}.
\end{align}

From (7) it is easily shown that $\bigcup_{i=1}^m D_i = \mathbb{R}^n$. Let $\tilde{D}_1 = D_1, \tilde{D}_i = D_i - \bigcup_{j=1}^{i-1} \tilde{D}_j$, $i = 2, \ldots, m$. Then $\bigcap_{i=1}^m \tilde{D}_i = \emptyset$, $\bigcup_{i=1}^m \tilde{D}_i = \mathbb{R}^n$, where $\emptyset$ stands for the empty set. The switching law is determined by

\begin{align}
\sigma_0(t) = i, (\zeta^T, \eta^T)^T \in \tilde{D}_i, i = 1, \ldots, m.
\end{align}

So, when $i$-th subsystem is active, we have $\frac{\partial U}{\partial \eta} \tilde{Q}_i(0, \eta) \leq -\beta \|\eta\|^2$. Along the solution of the closed-loop system (5),(8) with switching law (11), we have

\begin{align}
\dot{W} \leq -K_n \|\zeta\|^2 + L_{H_i(0, \eta)} U(\eta) + L_{H_i(\zeta, \eta) - H_i(0, \eta)} U(\eta) \\
\leq -K_n \|\zeta\|^2 + L_{\tilde{Q}_i(0, \eta)} U(\eta) + L_{H_i(\zeta, \eta) - H_i(0, \eta)} U(\eta) \\
\leq -K_0 \|\zeta\|^2 - \beta \|\eta\|^2 + \tilde{\alpha} L \|\eta\| \|\zeta\| \\
\leq -K_0 \|\zeta\|^2 - \beta \|\eta\|^2 + \tilde{\alpha} L \frac{\theta_0^2}{2} \|\eta\|^2 + \frac{1}{2\theta_0^2} \|\zeta\|^2 \\
\leq -\tilde{K} (\|\zeta\|^2 + \|\eta\|^2),
\end{align}

where $\tilde{K} = \min\{K_0 - \frac{\tilde{\alpha} L}{2\theta_0}, \beta - \frac{\tilde{\alpha} L \Delta^2}{2}\} > 0$. So $W(\zeta, \eta)$ is a single Lyapunov function of system (5),(8) with (11), and the quadratic stabilization of SNSS (1),(2) under some switching follows.

\textbf{Remark 1:} As we known, the minimum-phase property is useful for the stability of stabilization of non-switched nonlinear systems and non-switched nonlinear singular systems. Theorem 1 indicates that the minimum-phase property also plays a key role for the stabilization problem of the SNSS. The proposed method depends on the stability of a convex combination system of zero dynamics of the SNSS.
4. Conclusion

We have addressed the problem of quadratic stabilization for a class of switched nonlinear singular systems. Sufficient conditions, which guarantee the SNSS is quadratically stabilizable under some switching, have been derived. We generalize the result that the stability of the zero dynamics implies the stabilization of a nonlinear singular systems to the case of switched nonlinear singular systems.

Acknowledgments

The author thanks the anonymous reviewers’ valuable comments. This research was supported by the NSF of China under Grant 60574011; 60574013.

References


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