FORMATION CONTROL OF MULTIPLE MOBILE ROBOTS VIA SWITCHING STRATEGY

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Abstract. In this paper we consider the formation control of multiple nonholonomic mobile robots within a desired target set. A switching control method is proposed for the group of robots based on local sensor-based information. At first, a finite-time control law is employed to make all robots move effectively into the target set. Once all the robots get in the target set, finite-time control combined with feedback linearization is used in order to quickly stabilize the relative distance and orientation of the follower robots with respect to the leader robot. Both stability analysis and simulations results are provided.

Key Words. Robotic network, switching strategy, target aggregation, finite-time formation.

1. Introduction

During the last few years, coordination in robotic networks became an important and promising research area. There are many coordination tasks of multiple mobile robots including formation, coverage, and target aggregation, but the basic idea for the collaborate control design is to make the relative position and orientation of the robots in a desired formation (maybe within a given target set) or all the robots move effectively as a whole. A broad range of applications can be found in practical systems from underwater vehicles [1, 2], ground vehicles [3, 4, 5], unmanned aerial vehicles [6] to micro-satellites [7]. Up to now, various control methods have been proposed and applied to the coordination design of robotic networks, such as behavior-based approach [8, 9], virtual structure approach [10], and the leader-follower approach [4, 5]. With the behavior-based approach [8, 9], desired behaviors are assigned to each robot. The resulting action of each robot is achieved by weighing the relative importance of each behavior. The virtual structure approach [10] treats the robot formation as a single virtual rigid structure so that the behavior of the networked robotic system is similar to that of a physical object and therefore, the formation of the whole group can be precisely maintained. By the leader-follower approach [4, 5], one robot of the considered multi-agent formation problem is designed as the leader and others are the followers. In this way, we only need to specify leader motion and the desired relative posture between leader and followers. Differently from the behavior-based and virtual structure approach, it has some advantages such as the simple structure, local sensor information, and easy implementation.

Recently, finite-time stabilizing control laws have been proposed for different systems including those with nonholonomic constraints [11, 12]. In fact, non-smooth
finite-time control synthesis can improve the system behaviors in some aspects like control accuracy and disturbance-rejection \cite{12,13}. Therefore, finite-time control ideas have been applied to multi-agent systems with help of explicitly constructed Lyapunov functions \cite{14}.

The objective of this paper is to deal with formation problems of multiple nonholonomic mobile robots within a target set. Most existing results did not discuss the formation together with the convergence to a desired region. Here our problem is formulated as follows: the robots are firstly required to flock to a target region \cite{15}, and then achieve the desired formation. The formation is controlled by keeping constant the distances between robots and an angle of the follower robots. The angle is taken differently from that of \cite{4}, which is referred to the leader reference frame. This guarantees a high degree of distribution. We adopt a switching finite-time feedback controller to move robots into the target set, then achieve the desired distance and angle. Compared with the asymptotical controller, like \cite{4} or \cite{16}, the finite-time controller can provide faster convergent rate.

This paper is organized as follows. First, mathematical preliminaries and the model of the mobile robot are given in Section 2. Section 3 introduces the switching formation controller, with subsection 3.1 and 3.2 dealing with the two cases when the distance between robots is far or near, respectively. Stability analysis is also presented in Section 3. Following that, simulations results are given to illustrate the effectiveness of the proposed method. Finally, this paper is ended by concluding remarks.

2. Preliminaries

In this section, we briefly discuss notions of switched systems, finite-time controllers and the dynamic model of the mobile robot.

Consider the following nonlinear equation:

\[ \dot{\xi} = f_{\sigma(t)}(\xi, u), \quad \xi \in \mathbb{R}^n, u \in \mathbb{R}^m \]

where \(\sigma : [0, + \infty) \to \varphi\) is a right continuous map, called the switching signal for this time-dependent switching system, and \(\varphi\) is a finite index set.

Finite-time controller are nonlinear controllers which can drive to the desired state in finite-time and stabilize around it. Finite-time stability of some equilibria of nonlinear systems has been studied in the literature \cite{17}. Consider a first-order control system of the form \(\dot{z} = u\), and the following control law

\[ u_1 = -\alpha \text{sig}(z)^c, \quad \alpha > 0, \quad 1 > c > 0 \]

where

\[ \text{sig}(z)^c = |y|^c \text{sgn}(y) \]

\(\text{sgn}(\cdot)\) is the signum function, can drive the system trajectory to zero in finite-time \(T\), and the settling-time function \(T : \mathbb{R} \to \mathbb{R}_{\geq 0}\) is given by \cite{18}

\[ T(z_0) = \frac{|z_0|^{1-c}}{\alpha(1-c)}. \]

Next we consider a wheeled mobile robot, as shown in Fig. 1 and its kinematic model can be written as:

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \]

where the triple \((x, y, \theta)\) denotes the position and the orientation of the the vehicle in the inertial frame, \(v, \omega\) are the linear and angular velocities of the mobile robot.
The mobile robots have velocity level nonholonomic constraint, which can be represented by
\[ \dot{x} \sin \theta - \dot{y} \cos \theta = 0. \]

3. Target Aggregation and Formation

In this section, we consider the formation formed by \( N \) labeled robots with a given target region. Without lose of generalization, let the robot labeled 1 act as the leader, the others are followers.

Fig. 2 shows the system of multiple nonholonomic mobile robots. Two robots are separated by a distance of \( l_{1k} \) between the center of the leader robot 1 and the front castor of the follower robot \( k \).

\[ l_{1k} \text{ is given by} \]
\[ l_{1k} = \sqrt{(x_1 - x_k)^2 + (y_1 - y_k)^2} \]
\[ \text{where} \ (x_1, y_1) \text{ is the Cartesian coordinates of the leader robot,} \ (x_k, y_k) \text{ is the position of robot} \ k \text{'s castor} \]
\[ x_k = x_k + d \cos \theta_k, \quad y_k = y_k + d \sin \theta_k. \]
The angle from the reference to the line of sight from follower robot \( k \) to leader robot 1 is given by \( \zeta_{1k} \)

\[
(5) \quad \zeta_{1k} = \arctan 2(y_1 - \bar{y}_k, x_1 - \bar{x}_k).
\]

Differentiating equations (4) and (5), we get the kinematic equations for the follower robot \( k \):

\[
(6) \quad \begin{cases}
i_{1k} = -v_k \cos \phi_{1k} + v_1 \cos(\zeta_{1k} - \theta_1) - d\omega_k \sin \phi_{1k} \\
\dot{\zeta}_{1k} = \frac{1}{i_{1k}} (-v_1 \sin(\zeta_{1k} - \theta_1) + v_k \sin \phi_{1k} - d\omega_k \cos(\zeta_{1k} - \theta_1)) \\
\dot{\theta}_k = \omega_k,
\end{cases}
\]

where \( \phi_{1k} \equiv \zeta_{1k} - \theta_k \).

We define the shape of the formation as \( r = [r_{12}, \ldots, r_{1N}]^T \), where \( r_{1k} \equiv [i_{1k}, \zeta_{1k}]^T, k = 2, \ldots, N \).

**Definition 3.1.** Set \( l_{1k}^d > d, k = 2, \ldots, N \), the group of robots make a \( r^d = [r_{12}^d, \ldots, r_{1N}^d]^T \) formation, if \( \exists T > 0, \forall t \geq T : l_{1k}(t) = l_{1k}^d, \quad \zeta_{1k} = \zeta_{1k}^d, \quad k = 2, \ldots, N \).

The formation changing of the \( N \) robots can be described by a state-dependent dynamical graph [19]. [5] invents control graph to describe the behaviors of the robots in the formation, whose nodes represent robots and edges represent control relations between robots. In a single leader setting, the control graph is a depth 1 tree, with the leader robot as the root and all follower robots as nodes, each with an edge directed to the root. In general, the control graph is a tree with depth more than 1, whose leaves are follower robots, root is a leader robot and all other nodes act as both follower and leader. Sometimes, virtual robots are added to the group to represent either obstacles or predefined trajectories that are along such features as walls.

We propose our switching controller for the formation with target aggregation. Let \( T_s \) be the switching time, which is given explicitly in [12]. When \( t \leq T_s \), a finite-time controller in leader-follower frame is used to guide all robots flock to the target with a virtual leader as selected reference point within the target set \( \mathcal{S} \). Once the robots enter the desired region \( \mathcal{S} \), a robot will be selected as the leader, while other robots act as followers. Then their coordination control based on finite-time design to achieve the predefined formation.

\[
(7) \quad \dot{\xi}_k(t) = \begin{cases}
f_1(\xi_k(t), u^1(t)), & \text{if } t \leq T_s \\
f_2(\xi_k(t), u^2(t)), & \text{if } t > T_s
\end{cases}
\]

where \( \xi_k(t) \equiv (i_{1k}, \zeta_{1k}, \theta_k) \in \mathbb{R}^3, u^1 = (v_k, \omega_k) \in \mathbb{R}^2 \) is given in [2], for robot \( k = 2, \ldots, N \), \( u^2 \) is given in [13]. for robot 1, \( u^2 \) is the leader’s running rule.

### 3.1. Target aggregation

In this subsection, we consider how to control the robots automatically move in a predefined target set \( \mathcal{S} \). After all robots arrive in the target set, they will form the desired formation there, which is presented in the next subsection.

We achieve the target aggregation task within the same framework as formation control, described by [6]. This is done as follows. Arbitrarily choose a vector at the inner of \( \mathcal{S} \) with direction \( \theta_0, (x_0, y_0) \in \mathcal{S}^o \), which is then deemed as a virtual robot 0, whose velocity always equals to 0. From [6], we have the following dynamics for
all robots

\[
\begin{align*}
\dot{l}_{0k} &= -v_k \cos \phi_{0k} - d\omega_k \sin \phi_{0k} \\
\dot{\zeta}_{0k} &= \frac{1}{l_{0k}} \{v_k \sin \phi_{0k} - d\omega_k \cos(\zeta_{0k} - \theta_k)\} \\
\dot{\theta}_k &= \omega_k,
\end{align*}
\]

(8)

A location in the target set is then assigned for each robot to arrive at, which can be described by \(l_{0k}^d\) and \(\zeta_{0k}^d\), that is the distance between it and \((x_0, y_0)\) and the bearing from horizontal line. See Fig. 3 for illustration.

![Figure 3. Robots aggregate to the target set \(S\).](image)

Under the following control law, \(l_{0k}, \zeta_{0k}\) will converge in finite time to \(l_{0k}^d\) and \(\zeta_{0k}^d\), respectively,

\[
\begin{align*}
\omega_k &= \frac{\cos(\zeta_{0k} - \theta_k)}{d} \left\{ \alpha_2 l_{0k} \sin(\zeta_{0k} - \zeta_{0k}^d)^{c_2} + \rho_{0k} \sin(\zeta_{0k} - \theta_k) \right\} \\
v_k &= \rho_{0k} - d\omega_k \tan(\zeta_{0k} - \theta_k)
\end{align*}
\]

(9)

where

\[
\rho_{0k} = \frac{\alpha_1 \sin(l_{0k} - l_{0k}^d)^{c_1}}{\cos(\zeta_{0k} - \theta_k)},
\]

since the above control law leads to robots' dynamics in the \(l_{0k} - \zeta_{0k}\) variables of the form

\[
\begin{align*}
\dot{l}_{0k} &= -\alpha_1 \sin(l_{0k} - l_{0k}^d)^{c_1} \\
\dot{\zeta}_{0k} &= -\alpha_2 \sin(\zeta_{0k} - \zeta_{0k}^d)^{c_2}.
\end{align*}
\]

(10)

Define \(T_k\) to be the time when robot \(k\) achieves the target aggregation task, that is

\[
T_k = \max\{l_{0k} - l_{0k}^d, \zeta_{0k} - \zeta_{0k}^d\}.\]

which follows from the result of finite-time convergence \(\Box\).

Let \(T_*\) be the time when all robots get in the target set, that is,

\[
T_* = \max\{T_1, \ldots, T_N\}.\]

(12)

After that, the robots will focus on the desired formation using the technique proposed in the next subsection.
3.2. Finite-time Feedback Control. This subsection deals with the formation control of follower robots \( k = 2, \ldots, N \) when all robots are within the target set.

We use standard techniques of I/O linearization (referring to [20]) and finite-time control technique to build the following control law that gives finite-time convergent solutions of variables \( l_{1k} \) and \( \zeta_{1k} \)

\[
\left\{ \begin{array}{l}
\dot{\omega}_k = \frac{\cos(\zeta_{1k} - \theta_k)}{d} \{-\alpha_2 l_{1k} \text{sgn}(\zeta_{1k}^d - \zeta_{1k}) \} - v_1 \sin(\zeta_{1k} - \theta_1) \\
+ \rho_{1k} \sin(\zeta_{1k} - \theta_k) \\
v_k = \rho_{1k} - d\omega_k \tan(\zeta_{1k} - \theta_k)
\end{array} \right.
\]  \hspace{1cm} (13)

where \( \alpha_1, \alpha_2 > 0 \) are positive constants and \( \rho_{1k} = \frac{-\alpha_1 \text{sgn}(l_{1k}^d - l_{1k})c_1 + v_1 \cos(\zeta_{1k} - \theta_1)}{\cos(\zeta_{1k} - \theta_k)} \).

Note that we use the \text{sgn} function of the error, not a linear function of it as in the control input, fortunately, the induced \( l - \zeta \) dynamics has the following simple form with finite-time convergence rate

\[
\left\{ \begin{array}{l}
\dot{l}_{1k} = -\alpha_1 \text{sgn}(l_{1k} - l_{1k}^d)c_1 \\
\dot{\zeta}_{1k} = -\alpha_2 \text{sgn}(\zeta_{1k} - \zeta_{1k}^d)c_2
\end{array} \right.
\]  \hspace{1cm} (14)

Lemma 3.1. The follower robot \( k \) will approach the desired length \( l_{1k}^d \) and the desired relative angle \( \zeta_{1k}^d \) with the leader robot 1 in finite time under the control law (13).

Proof: Since under the control law (13), we have \( l - \zeta \) dynamics of the form (14), which is finite-time stable from (14).

This lemma and the results of the preceding subsection proved the effectiveness of our proposed switching control algorithm (13).

Since the optimal point-to-point paths of mobile manipulators are generally composed of straight lines and circular arcs [21], we now focus on these two leader robot motions.

Proposition 3.1. When the leader robot 1 follows a circular path, \( v_1 = K_1, \omega_1 = K_2 \), if and only if \( K_2 d \leq K_1 \), \( \theta_k \) will locally converge to \( \theta_1(0) + K_2 t - \arcsin(\frac{K_2 d}{K_1}) \).

Proof: Since \( l_{1k}, \zeta_{1k} \) are finite-time stabilized from (14) and (13), the zero dynamics of the system is then given by in terms of \( \theta_k \).

When \( v_1 = K_1, \omega_1 = K_2, \theta_1(t) = \theta_1(0) + K_2 t, \) and from (13)

\[
\dot{\theta}_k = \frac{\cos(\zeta_{1k}^d - \theta_k)}{d} \{-K_1 \sin(\zeta_{1k}^d - \theta_1) \\
+ \frac{K_1 \cos(\zeta_{1k}^d - \theta_1)}{\cos(\zeta_{1k}^d - \theta_k)} \sin(\zeta_{1k}^d - \theta_k)\}
\]  \hspace{1cm} (15)

Let \( e_{1k}^\theta = \theta_1 - \theta_k \), and then

\[
e_{1k}^\theta = K_2 - \frac{K_1}{d} \sin(e_{1k}^\theta)
\]  \hspace{1cm} (16)

To make equation (16) stable, it is required that \( K_2 d \leq K_1 \). Then the equilibrium point \( e_{1k}^\theta = \arcsin(\frac{K_2 d}{K_1}) \) is locally asymptotically stable. Thus, the follower robot \( k \) will locally converge to \( \theta_k = \theta_1 - e_{1k}^\theta = \theta_1(0) + K_2 t - \arcsin(\frac{K_2 d}{K_1}) \).

Proposition 3.1 shows that when leader robot 1 does not rotate too faster than it translates, the orientation difference \( e_{1k}^\theta \) keeps constant, though the linear and angular velocities of the follower robot are time-varying.
Proposition 3.2. When the leader robot follows a straight line, \( v_1 = K_1 \), \( \theta_k \) exponentially converges to \( \theta_1(0) \).

Proof: When \( \theta_1 = \theta_1(0) \), from (15),
\[
\dot{\theta}_k(t) = \frac{K_1}{d} \sin(\theta_1(0) - \theta_k(t))
\]
the result follows directly from integrating the above differential equation. \( \square \)

Proposition 3.2 tells us that, the follower robot will also follow a straight line with the same orientation as the leader robot.

4. Simulations

In this section, the proposed formation control algorithm is verified by the simulations. The simulations are done in MoibleSim, a software for simulating mobile robots and their environments developed by MobileRobots®.
The codes are written with the help of ARIA, an object-oriented, robot control applications-programming interface for intelligent mobile robots. The control laws are encapsulated in a action class of the follower robots. Each robot also has actions class that make them avoid collisions with other robots or the environment. The action classes have various levels of priority. Higher levels of priority are assigned to collision avoidance actions.

We take 5 robots in MobileSim in a two-dimensional space for the test. The target region is a disk with its center at the origin and its radius as 4 meters. The desired formation is a square with the leader robot 1 as the center and the 4 follower robots as vertexes of the square, whose length of sides are $2\sqrt{2}$ meters. Tracking errors are given in Fig. 4. Fig. 5 shows snaps of the robots’ trace. The top-left subfigure shows their initial states, while the down-right subfigure shows the final formation. The simulations show the effectiveness of the proposed method.

5. Conclusions

In this paper we investigated cooperative control of multiple nonholonomic mobile robots with concerning target aggregation and formation. We proposed a switching finite-time method, to drive the robots firstly to enter a given target set and achieve the desired formation within the target region. Both theoretical analysis and simulations were carried out.

References


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