RECONSTRUCTION OF CUBIC CURVES FROM TWO OR MORE IMAGES USING GEOMETRIC INTERSECTION

SANJEEV KUMAR, BALASUBRAMANIAN RAMAN, AND NAGARAJAN SUKAVANAM

Abstract. Reconstruction of algebraic curves in 3-D space from its perspective views has received huge attention by many researchers. In this paper, reconstruction of a cubic curve in 3-D space from its two or more arbitrary perspective views is described. Two different methods are proposed for the reconstruction process based on the geometrical intersection approach. The advantages of proposed methods are to avoid the matching problem that occurs in pair of images of the curve and the reconstruction of non-planar curve. Parameters of a curve in 2-D digitized image planes are determined using least-square curve fitting. The generalized conical surfaces are obtained by these parameters and the center of focus of the respective cameras. Simulation studies have been conducted to observe the effect of noise on errors in the process of reconstruction. The proposed methods have been verified using synthetic as well as real world data.

Key Words. 3-D reconstruction, Cubic Curves, Generalized Conic Surfaces, Image Correspondence, Gaussian Noise, Least Square Curve fitting.

1. Introduction

In Computer Vision, cubic curves are playing important roles to recover 3-D information of an object from its various 2-D images. If the shape of a complex object is not defined with the help of straight lines or conics or both then cubic curves come next to accomplished this task. Generally, cubic curves are represented by algebraic polynomials of degree 3. Reconstruction of cubic curve from its 2-D images is more difficult when compare to the reconstruction of straight lines or conics due to the following reasons.

- Cubic curves are not always planar curve as in case of straight lines and conics.
- They have more complex shape.
- Existence of multiple correspondence in their different perspective images.

This work draws upon continuing progress on the application of projective geometry to computer vision. However, most of the work are limited in the reconstruction of lines or conics. This motivates us to extend our methods [2, 11] for the reconstruction of cubic curves.

Received by the editors June 11, 2008.
2000 Mathematics Subject Classification. 94A08; 68U10.
1.1. Relation to the Literature. In the past few years, most of the work in this field is limited to reconstruct 3-D lines and conics from arbitrary perspective views. The case of reconstruction of conics in 3-D space has been discussed by several researchers in their work. Xie et al. [12] have given the analytical formulation for the reconstruction of quadratic curves, but did not determine the unique solution from the roots of a quadratic equation. In case of planar curves they have assumed the existence of point to point correspondence between the sets of contour points on the pair of projected curve segments. Kanatani et al. [8] have developed computation procedures to interpret the 3D geometry of conics in the scene from their projections using real life examples. Quan [10], has solved correspondence in the case of conics from two views. He has also solved the ambiguity (non-uniqueness) in the solutions with the use of a non-transparency constraint of a conic section. Kahl et al. [4] have shown how corresponding conics in two images can be used to estimate the epipolar geometry in terms of fundamental or essential matrix. Balasubramanian et al., [2] have proposed a methodology to reconstruct quadratic curves in 3-D space together with error analysis in reconstruction process. Kumar et al. [11] have proposed a method to reconstruct a quadratic curve in 3-D space using intersection of conical surface spanned by the image of curve and center of focus of cameras. These methods was limited to reconstruct conics only and the reconstruction of cubic curve was not described by these methods.

Reconstruction of algebraic curves having degree more than two is very difficult task in computer vision. Ma [9] has given a closed form solution for the reconstruction of algebraic curves (conics and cubic curves) in 3-D space. However, Ma’s approach is applicable only for the reconstruction of planar curve (having degree \( n > 2 \)) and not for the non-planar curves. In the case of non-planar curves, he has used the concept of shape from motion instead of stereo. M-H An et al. have given an introduction to this problem in their paper [1], however, their method is also limited for the reconstruction of planar curve only. In the case of non-planar curves their method is not applicable. Yang et al. [13] have introduced a unified theoretical framework for extracting poses and structures of 2-D symmetric patterns (specially curves) in 3-D space from calibrated perspective images. They have shown that the key to consistent detection and segmentation of symmetric structures from their 2-D perspective images is to examine the relations among all types of symmetry as an algebraic group. Kaminski et al. [6, 7] have introduced a number of new results in the context of multi-view geometry from general algebraic curves. They have addressed three different representations of curves: (i) the regular point representation for which we show reconstruction from two views of a curve of degree \( n \) admits two solutions, one of degree \( n \) and the other of degree \( n(n - 1) \), (ii) the dual space representation for which derived a lower bound for the number of views necessary for reconstruction as a function of the curve degree and genus, and (iii) a new representation based on the set of lines meeting the curve which does not require any curve fitting in image space. Kahl et al. [5] have presented a completely automatic approach for matching and reconstructing space curves in multiple views. Their method solves an inverse problem for a generative model, therefore the resulting reconstructions are consequences of the model.

1.2. Overview of our Approach. The idea of present methods are motivated from the research papers [2, 11]. We deal with the methodology of reconstruction of a curve in 3-D space from two or three arbitrary perspective views. Projection of the original curve into two image planes results two discrete conic in which the curves are obtained by using least square technique. From these two conic equations
combined with the collinearity equations results two different conical surfaces in 3D space. In the first method, we deal with the intersection of line obtained from the projected points in first image plane and the center of focus of the first camera with the conical surface which obtained using curve in second image plane and center of focus of the second camera. In second approach the intersection of these two surfaces (spanned from images of curve and center of focus of cameras) contains the reconstructed curve in 3-D space. The uniqueness in reconstruction is obtained by using the additional constraint called third perspective view. The reconstructed cubic curves with the detailed derivations and with the simulated examples with synthetic and real data are shown in this paper. In this work, we are dealing with the digitized and normalized image planes and we are considering various noisy cases. Further we also analyze the error in reconstruction process. The SSD (sum of square of differences) between the reconstructed and original cubic curves in 3-D space has been used as the criterion for the measurement of the error.

1.3. Organization of the Paper. The rest of this paper is organized as follows. The mathematical model for imaging setup is shown in section 2. Section 3 provides the detail methodologies. Section 4 gives the simulation results of the reconstructed curves for both the methodologies together with error analysis and the comparison studies. The paper ends in section 5 with the concluding remark.

2. Formulation of the problem

The imaging set-up is shown in Figures 1 and 2 for line-surface intersection and two surface intersection methods respectively. Let $I_1$ and $I_2$ be the first and second image planes of the pair of cameras $C_1$ and $C_2$ respectively. Let the position and the orientation of one camera be known with respect to another and both have a common field of view. Let $O_1xyz$ be the rectangular cartesian frame of reference with its origin $O_1$ at the center of projection of camera $C_1$. A point $W$ in 3-D space, with co-ordinates $(x_w, y_w, z_w)$ with respect to the frame of reference at $C_1$, is viewed by both the cameras $C_1$ and $C_2$. Let $O_2x'y'z'$ be the second rectangular cartesian co-ordinate system, not necessarily parallel to $O_1xyz$ system, with its origin $O_2$ at the center of projection of the second camera $C_2$. Let the co-ordinates of the second camera $C_2$ with respect to $O_1$ be $(x_d, y_d, z_d)$. Let $P_1(X_1, Y_1, f_1)$ and $P_2(X_2, Y_2, f_2)$ be the corresponding pair of projections of point $W$ on the pair of image planes $I_1$ and $I_2$ respectively. Let $f_1$ and $f_2$ be the focal lengths of the first and the second cameras respectively. Let the perspective projections of a cubic curve $\Gamma$ in 3-D space be a pair of cubic curves $\Gamma_1$ and $\Gamma_2$, on the first and second image planes respectively as shown in Figure 1. The problem is to reconstruct the 3-D cubic curve $\Gamma$ from the pair of its images $\Gamma_1$ and $\Gamma_2$. The relation between the coordinates of the point $W(x_w, y_w, z_w)$ and that of the image point $P(X_1, Y_1, f_1)$ is given by the perspective equation:

\begin{equation}
X_1 = f_1 \frac{x_w}{z_w}, \quad Y_1 = f_1 \frac{y_w}{z_w}
\end{equation}

The 3-D co-ordinates of point $W(x_w, y_w, z_w)$ with respect to second camera $C_2$, is given by

\begin{equation}
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} = \lambda \begin{pmatrix}
\cos \alpha_1 & \cos \beta_1 & \cos \gamma_1 \\
\cos \alpha_2 & \cos \beta_2 & \cos \gamma_2 \\
\cos \alpha_3 & \cos \beta_3 & \cos \gamma_3
\end{pmatrix} \begin{pmatrix}
(x_w - x_d) \\
(y_w - y_d) \\
(z_w - z_d)
\end{pmatrix}
\end{equation}
In the above equations $\alpha_i$, $\beta_i$, $\gamma_i$ ($i = 1, 2, 3$) are the respective direction cosines of the axes of $O_2x' y' z'$ with respect to $O_1xyz$ system and can be expressed in $\theta$, $\phi$ and $\psi$ (Eulerian angles) [2]. $\lambda$ is a scale factor between the two reference frames and without loss of generality this is considered to be 1, in the present work. Using Equation (2), the relation between the object space point $W(x_w, y_w, z_w)$ and the image point $P_2(X_2, Y_2, f_2)$ is given by the perspective equations [1, 2]:

\[
X_2 = f_2 \frac{(x_w - x_d)\cos\alpha_1 + (y_w - y_d)\cos\beta_1 + (z_w - z_d)\cos\gamma_1}{(x_w - x_d)\cos\alpha_3 + (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3}
\]

\[
Y_2 = f_2 \frac{(x_w - x_d)\cos\alpha_2 + (y_w - y_d)\cos\beta_2 + (z_w - z_d)\cos\gamma_2}{(x_w - x_d)\cos\alpha_3 + (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3}
\]

Equations (1), (3) and (4) are the collinearity equations for a pair of arbitrary perspective views.

3. Projective Reconstruction Methodologies

In this paper, we use least-square curve fitting to obtain the parameters of the projected cubic curves in various 2D image planes. In present section, the detailed process of two different methodologies for the reconstruction of cubic curves in 3D space are explained.

In the proposed approach, a fully calibrated imaging setup has been used for the reconstruction of cubic curves in 3D space. The knowledge of the following input parameters is necessary for the reconstruction process.
The set of pixel coordinates for the curve $\Gamma_1$ on the first image plane $I_1$: $(X_{1i}, Y_{1i}), i = 1, 2, \ldots, m$.

The set of pixel coordinates for the curve $\Gamma_2$ on the second image plane $I_2$: $(X_{2i}, Y_{2i}), i = 1, 2, \ldots, n$.

Intrinsic and extrinsic parameters for all cameras i.e. focal lengths, center of focus etc. We assume that $f_1 = f_2 = 1$ for the computational simplicity.

Let the equations of the cubic curves $\Gamma_1$ and $\Gamma_2$ in the respective first and second image planes $I_1$ and $I_2$ which obtained from least-square curve fitting is represented by the polynomials

\begin{align} 
\Gamma_1 : \sum_{i+j+k=3} a_{ijk} x_i^r y_j^r z_k^r &= 0 \\
\Gamma_2 : \sum_{i+j+k=3} b_{ijk} x_i'^r y_j'^r z_k'^r &= 0 
\end{align}

Substituting the collinearity equations given by Equations (1),(3) and (4) in the Equations (5) and (6), we obtained the representation of the surface $S_1$ and $S_2$ (assume $f_1 = f_2 = 1$):

\begin{align} 
S_1 : \sum_{i+j+k=3} a_{ijk} x_i^w y_j^w z_k^w &= 0 \\
S_2 : \sum_{i+j+k=3} b_{ijk} x_i'^w y_j'^w z_k'^w &= 0 
\end{align}
By transforming \((x', y', z')\) in Equation (8) to \((x, y, z)\) using the camera extrinsic parameter data (see equation (2)), \(S_2\) can be represented using the same coordinate system of \(S_1\):

\[
S_2 : \sum_{i+j+k\leq 3} b'_{ijk}x^iy^jz^k = 0
\]

Notice that the equations (7) and (8) are homogenous polynomials, where as the equation (9) is nonhomogeneous because the origin of the coordinate system \(O_1\) is not located on the surface \(S_2\). The following two methodologies are proposed for the 3-D reconstruction of cubic curves.

3.1. Line-surface intersection method (LSI method). Select any point \(P_1(X_1, Y_1)\) on the curve \(\Gamma_1\) in the first image plane. The equation of the line \(O_1P_1\) shown in Figure 1, joining the center of projection of the first camera \(O_1\) and \(P_1\), is given by

\[
\frac{x}{X_1} = \frac{y}{Y_1} = \frac{z}{f_1} = r
\]

as we assume without lose of generality, \(f_1\) is to be unity. The intersection of line \(O_1P_1\) with surface \(S_2\) is obtained as follows:

From the equation (10), one can obtain

\[
x = rX_1 \\
y = rY_1 \\
z = rf_1
\]

On substituting the above values of \(x, y, z\), equation (9) can be rewritten as:

\[
ar^3 + br^2 + cr + d = 0
\]

where

\[
a = \sum_{i+j+k=3} b'_{ijk}X_1^iY_1^j \\
b = \sum_{i+j+k=2} b'_{ijk}X_1^iY_1^j \\
c = \sum_{i+j+k=1} b'_{ijk}X_1^iY_1^j \\
d = \sum_{i+j+k=0} b'_{ijk}X_1^iY_1^j
\]

It can be seen that \(a, b, c\) and \(d\) are functions of

1. Coefficients of various powers of \(X_2\) and \(Y_2\) in the cubic curve represented by equation (6).
2. Extrinsic parameters of second camera (i.e. \(\theta, \phi, \psi, x_d, y_d\) and \(z_d\)).

The coordinates of the points of intersection of the line \(O_1P_1\) and surface \(S_2\) in 3D space in terms of \(r\) are the roots of cubic polynomial given by the equation (11). Choose a set of \(m\) contour points \(P_{i1}(X_{i1}, Y_{i1})\), \(i = 1, 2, ..., m\), on the curve \(\gamma_1\) in the first image plane. We have obtained three sets of coordinates values \((x_{i1}, y_{i1}, z_{i1})\), \((x_{i2}, y_{i2}, z_{i2})\) and \((x_{i3}, y_{i3}, z_{i3})\), \(i = 1, 2, ..., m\). Hence this gives three sets of depth values of the points as candidates for the solution. The method for obtaining accurate unique solution out of these three solutions is described in the next section.
3.2. Two surfaces intersection method (SSI method). It is clear from the model of imaging setup explained in section 2, that the required curve in space is the intersection curve of two surfaces represented by $S_1$ and $S_2$. The intersection of these two generalized conic surfaces may have some extra points other than the required curve $\Gamma$. Hence, we use the third perspective view $\Gamma_3$ of $\Gamma$. We project all the points which we have obtained from the intersection in the third perspective image plane and from the following steps the required curve in 3D space have been obtained. We have used the following steps for the reconstruction and uniqueness of cubic curve in 3D space.

- Calculate $S_1^2$ and $S_2^2$.
- Construct a function $F$ which is defined as $F(x, y, z) = S_1^2(x, y, z) + S_2^1(x, y, z)$.
- Assume initial values of points $(x_0, y_0, z_0)$.
- Minimize the function $F$ using suitable unconstrained optimization technique.
- Find the sets of values $(x, y, z)$ for which the function $F$ is zero or very close to zero.
- Project all these points in the third view.
- Select the points which coincide with $\Gamma_3$. These are the points of reconstructed curve $\Gamma$ in 3D space.

4. Results and Discussions

Simulation studies have been performed for the reconstruction of cubic curves having different properties together with various sets of camera parameters in stereo imaging system. We have tested our algorithms in perfect stereo imaging system and general stereo imaging system. In perfect stereo system, we have considered that the second camera is situated at a distance $x_d$ along $x$-axis of the first camera and there is no translation along $y$ and $z$ directions. We consider that the rotation matrix of the coordinate frame of second camera is an identity matrix with respect to first camera coordinate system. In general stereo case, we consider the rotation as well as translation parameters in all directions.

4.1. Simulation Results using SSI method. In order to reconstruct cubic curves using SSI method uniquely, we employ an additional constraint called third view. This consists of the projection of the original curve $\Gamma$, i.e. set of pixel coordinates for the corresponding curve, $\Gamma_3 : (X_{3i}, Y_{3i}), i = 1, 2, \ldots, n$. Again least square curve fitting is used to construct the curve in third view. The equation of this curve is $f_3(X, Y) = 0$. Now project the reconstructed 3D points onto third image plane and select the points, whose projection in the third view, $(X, Y)$, satisfies the equation $f_3(X, Y) = 0$. For computational purpose, suitable threshold values ($\epsilon$) are considered.

Figure 3(a) represents the reconstruction of a cubic curve in 3D space when imaging setup is considered as prefect stereo case. The parametric representation of curve is given by $x = t; y = t^3 + 2t^2 + 5t + 1; z = 10$. The focal lengths of both the cameras are unity. The extrinsic parameters of the imaging setup is given as $x_d = 12, y_d = 0, z_d = 0, \theta = \phi = \psi = 0$. The value of threshold $\epsilon$ is considered as $1 \times 10^{-5}$. In this figure, the original and the reconstructed cubic curves are shown in 3D space. The 2D projection of reconstructed and original curves in third view is represented by figure 3(b). It can be seen that the proposed SSI method is able to work in the case of noisy images also. Figures 4(a)and 4(b) represent the
reconstruction of cubic curve in the presence of Gaussian noise of variance 0.001 (low noise case) and 1.0 (large noise case) respectively.

The proposed SSI method is also sufficient to reconstruct the cubic curves which have more complex shape and in case of general stereo. Figure 5(a) and 5(b) represent the original and the reconstructed curves in case of general imaging setup. The parametric representation of simulated curve is given by \( x = t^3 + t + 4; \ y = t^3 + 2t^2 + 5t + 1; \ z = 3t^3 + 2t^2 + 3t + 25 \). The extrinsic parameters of the imaging setup is given as \( x_d = 15, \ y_d = 10, \ z_d = 20, \ \theta = 5, \ \phi = 20, \ \psi = 15 \). Again the value of threshold \( \epsilon \), is considered as \( 1 \times 10^{-3} \). The 2D projection of reconstructed and original curves in third view is represented by figure 5(b). It can be seen that the proposed SSI method is able to work in case of noisy 2D images also. Figure 6(a) and 6(b) represent the reconstruction of cubic curve in presence of Gaussian noise of variance 0.0001 (low noise case) and 1.0 (large noise case) respectively.

4.2. Simulation Results using LSI method. Simulation studies have been performed for the reconstruction of cubic curves using LSI method. Assuming the roots obtained from equation (11) to be positive (which is a constraint from the geometry of the imaging setup), only positive values of \( r_1, r_2 \) and \( r_3 \) are acceptable. Two cases arise obtained from the values of \( r_1, r_2 \) and \( r_3 \), given as follow:
4.2.1. Case 1: One root is real and rest are complex conjugate. Figure 7(a) and (b) show the reconstruction of cubic curve given by parametric representation as \((x = t, y = t^3 + 2t^2 + 5t + 1, z = 5t^2 - 3t + 20)\) and the parameters of imaging setup is given as \((x_d = -10, y_d = 0, z_d = 0)\) together with the condition that the coordinate frames of both cameras are parallel i.e. rotation part is trivial for the purpose of computational simplicity. Select only the solution in which one root has positive value and rest two are complex conjugate for the purpose of uniquely reconstruction of curve \(\Gamma\). Figure 7(b) illustrates the 2D projection in third view of the data given in figure 7(a), where the original and reconstructed curve is shown in 3D view.

4.2.2. Case 2: More than one roots are positive and real. To eliminate the ambiguity and provide a unique solution we need an additional constraint. For using the planarity constraint of Xie et. al. [12], point to point correspondence is necessary. Transparency constraint could be used to eliminate the ambiguity for solid objects, [1]. For more general case, if the curve is wired or transparent, unique solution is possible only with the help of a third view like as in SSI method, which identifies the unique solution. Figure 8(b) represents this additional constraint (third view) in which the required positive solution shown by \(\ast\) is superimposed to original curve. The points which we obtained from the second positive solution
is represented by + are also shown. Figure 8(a) represents the original and reconstructed cubic curve in 3D view. Using the constraint of continuity or smoothness of the curve, the two possible solutions are classified. If the surface enclosed by the curve is opaque, only one of the solution will be visible. In the case of transparent structure or wired curves, a third view is mandatory.

4.3. Experiment with real data. In this section, we show experimental results based on real data. In this experiment, we use the real image of a CD stand from which we obtain a cubic curve by multiplying the base circle and the vertical bar which is considered as a straight line in 3D space. The pair of stereo images together with additional third image are shown in figure 9.

We consider the cubic curve $\Gamma$ consisting of a conic $c$ and a line $l$, that is, $\Gamma = lc$. We follow the following process.

1. Take the stereo images using digital camera with the known focal length.
2. Read these images in MATLAB and select the points of interest using MATLAB preprocessing techniques (pixels on vertical bar and the boundary of the base circle).
3. Use the least square curve fitting in both images and obtain the best fit circles and lines.
(4) Follow SSI and LSI methods for the reconstruction of cubic in 3D space. The equation of circle and line in first image plane is given below

\[ l_1 : 0.67X_1 - Y_1 = 0 \]
\[ c_1 : -0.99X_1^2 - 1.01Y_1^2 - 0.01X_1Y_1 + 1.50X_1 + 1.02Y_1 - 0.78 = 0 \]

and the equations of circle and line in second image plane is given as

\[ l_2 : 0.40X_2 - Y_2 = 0 \]
\[ c_2 : -0.99X_2^2 - 0.99Y_2^2 + 0.03X_2Y_2 + 2.49X_2 + 0.95Y_2 - 1.75 = 0 \]

and the equations of circle and line in third image plane is given as

\[ l_3 : 0.91X_3 - Y_3 = 0 \]
\[ c_3 : -1.02X_3^2 - 0.99Y_3^2 + 0.06X_3Y_3 + 1.16X_3 + 1.97Y_3 - 1.81 = 0 \]

The resulting cubic \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) are given by the following three equations

\[ 0.67X_1^3 - 1.01Y_1^3 - 0.99X_1^2Y_1 + 0.66X_1Y_1^2 + 0.82X_1Y_1 - 1.00X_1^2 + 1.02Y_1^2 + 0.62X_1 - 0.78Y_1 = 0 \]
\[ 0.40X_2^3 - 0.99Y_2^3 - 1.01X_2^2Y_2 + 0.42X_2Y_2^2 + 2.11X_2Y_2 - 0.99X_2^2 + 0.95Y_2^2 - 0.70X_2 - 1.75Y_2 = 0 \]
\[ 0.09X_3^3 - 0.99Y_3^3 - 1.08X_3^2Y_3 + 0.97X_3Y_3^2 + 0.61X_3Y_3 - 1.06X_3^2 + 1.97Y_3^2 + 1.01X_3 - 1.81Y_3 = 0 \]
It can be seen that $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are homogeneous polynomials but when we transform the second and third cameras coordinate system into first camera coordinate system (i.e. world coordinate system) the polynomial corresponding to second and third images will be nonhomogeneous. After computing the intersection of surfaces $S_1$ and $S_2$ which are obtained from $\Gamma_1$ and $\Gamma_2$, we reconstruct the required cubic curve after projecting the intersection data on third view. The points obtained from the intersection of $S_1$ and $S_2$ and satisfied the constraint (i.e. $\Gamma_3$) are shown in figure 10.

4.4. Comparison Study. A qualitative comparison is given between proposed approaches and the methodology given by M-H An et al. [1] for the reconstruction of a planar cubic curve $\Gamma = x^3 - y^2 = 0, 2x - 3y - z + 10 = 0$ in the presence of noise (table 1, table 2 and table 3). Here we have considered only planar curve because M-H An methodology works only in the case of planar curve. Noise with the Gaussian distributions is added to the pixel coordinate values of the projection of cubic curve on the image planes. This perturbs the location of the pixels in both image planes. The three factors have considered for comparison criterion between the different approaches. First factor is angle between the planes having original and the reconstructed curves. The second factor is minimum distance between these planes and the third is sum of the square of the differences between the points of original and the reconstructed curves used in simulation studies along all three directions.

Effect of adding Gaussian noise in image planes is shown in tables 1, 2 and 3. Table 2 represents the error analysis when we use M-H An method [1] for the reconstruction process. When we add Gaussian noise of variance 1.0 in both image planes, this method fails to establish the correspondence between image plane because one can not find the characteristic number from the equations of the obtained curves in both image planes. Tables 3 and 4 represent the error analysis for the SSI and LSI methods respectively. It can be seen from the above three tables that the SSI method is better from the rest two for the reconstruction of cubic curves in noisy environment.

5. Conclusions

In this paper, we have given two methodologies for obtaining the reconstruction of a cubic curve in 3D space without the requirement of point to point correspondence, as well as the constraint of transparency on the curve. We do not consider
Table 1. Error in reconstruction corresponding to presence of Gaussian noise using M-H An [1]

<table>
<thead>
<tr>
<th>Noise (Variance)</th>
<th>Angle (Degree)</th>
<th>Distance</th>
<th>Sum of square distances</th>
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Table 2. Error in reconstruction corresponding to presence of Gaussian noise using SSI method

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<th>Sum of square distances</th>
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Table 3. Error in reconstruction corresponding to presence of Gaussian noise using LSI method

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<th>Sum of square distances</th>
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</tbody>
</table>

correspondence problem between image pair, so the given methodologies are simple to apply and save computational time compare to other classical reconstruction approaches. We have shown experimental study for both the methods using synthetic as well as real 2D images of a 3D cubic curve. The cubic reconstruction methods shown are simpler and stable as shown by simulation studies. The beauty of presented approaches is that these can be used in more general case (other than planar curves) as well as for the images which are noisy.

Acknowledgments

Sanjeev Kumar acknowledges the financial support of the Council of Scientific and Industrial Research, New Delhi, India through a Senior Research Fellowship (SRF)scheme (CSIR award no. 9/143(503)/2004-EMR). R. Balasubramanian expresses his sincere gratitude to Indian Space Research Organization (ISRO) and Ministry of Human Resources and Development (MHRD) for their financial support through grant numbers ISR-295-MTD and MHR03-05-(802,418,807,412).

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