ADMISSIBLE CONDITIONS OF FUZZY DESCRIPTOR SYSTEMS
BASED ON FUZZY LYAPUNOV FUNCTION APPROACH

YUHAO YUAN\(^1\), QINGLING ZHANG\(^1\), DAQING ZHANG\(^1\), AND BING CHEN\(^2\)

Abstract. This paper discusses the admissible conditions of T-S fuzzy descriptor systems via the so-called fuzzy Lyapunov function which is a multiple Lyapunov function. The Lyapunov function is defined by fuzzily blending quadratic Lyapunov functions. Based on a Lyapunov function approach, we give admissible conditions for open-loop fuzzy systems. In addition, relaxed admissible conditions and fuzzy controller design scheme are also derived by considering the property of the time derivative of premise membership functions. All the conditions are formulated in the format of linear matrix inequality. At last, an example is given to illustrate that the new sufficient conditions is less conservative than the results given by previous literature.

Key Words. T-S fuzzy descriptor system, admissible conditions, fuzzy Lyapunov function, linear matrix inequality (LMI)

1. Introduction

In the last decade, the issue of stability analysis for nonlinear models described by Takagi-Sugeno (T-S) models has been considered actively. In this T-S fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model is obtained by 'blending' these linear models through membership functions. As a common belief, the control technique based on the T-S fuzzy model is conceptually simple and effective for the control of complex systems.

In 1999, Taniguchi and Tanaka et al extended the T-S fuzzy model to descriptor nonlinear systems [1, 2]. They brought the concept of T-S fuzzy descriptor systems forward, and the analysis of the stability problems have been done too. It has been proven that the T-S fuzzy model is a universal approximator of any smooth nonlinear systems having a first order that is differentiable [3]. Therefore, it is meaningful to consider applying the fuzzy descriptor model to approximate the nonlinear descriptor system, and designing the fuzzy controller.

For the nonlinear descriptor systems which described in T-S model, there some results concerning on the analysis and synthesis have been proposed [1-7]. In admissibility analysis of fuzzy descriptor control systems, main approaches have been based on a single Lyapunov function. These methods basically reduce to the problem of finding a common Lyapunov function for a set of admissible conditions. If the conditions are in terms of LMIs, the problem can be numerically solved by recent developed convex optimization techniques. Nevertheless, restriction to the class of quadratic Lyapunov function candidate may lead to significant conservativeness. Some researchers also have managed to reduce the conservatism too. Based on the theory of interval dynamics systems, the T-S fuzzy descriptor system is converted
into linear continuous-time descriptor system with norm-bounded perturbations [8]. Hence some analysis and design techniques in robust control theory for descriptor systems can be used to analyze and design T-S fuzzy descriptor systems. This approach reduces the difficulty in solving the problem, but the particular properties of the T-S fuzzy systems has not been taken into account. In [9] the robust admissibility of the uncertain discrete-time fuzzy descriptor systems is analyzed in terms of matrix norm. However, this approach can not be applied to the continuous T-S fuzzy systems.

In this paper, based on the fuzzy Lyapunov function, the admissible condition for the fuzzy descriptor system will be investigated. Fuzzy Lyapunov function is a multiple Lyapunov function, which is defined by fuzzy blending quadratic Lyapunov functions. The fuzzy Lyapunov function shares the same membership function with the T-S fuzzy descriptor model of a descriptor system. Hence the time derivative of the fuzzy Lyapunov function contains the time derivative of premise membership functions. An pivotal point is how we can deal with the time derivative of premise membership functions. All the conditions derived here are represented in terms of LMIs and contain bounds of the time derivative of membership functions.

The paper is organized as follows. Section 2 defines a fuzzy Lyapunov function and derives admissible conditions. Then a relaxed admissibility approach that utilizes a property of the time derivative of membership functions is given. Section 3 presents the fuzzy controller design scheme.

**Notations:** Matrix $X > 0$ ($X \geq 0$) denotes that $X$ is a positive (semi-positive) definite matrix, $A > (\geq) B$ denotes $A - B > (\geq) 0$. Symbol $I$ stands for the unit matrix with appropriate dimensions.

2. Fuzzy Lyapunov Function and Admissible Conditions

2.1. Definition and Admissible Conditions.

The $i$th fuzzy rule is of the following form:

(1) \[ R_i : \text{IF } z_1(t) \text{ is } N_{i1} \text{ and } \cdots \text{ z}_p(t) \text{ is } N_{ip}, \text{ THEN} \]

(2) \[ E\dot{x}(t) = A_ix(t) + B_iu(t), \quad i = 1, 2, \cdots, r. \]

where

$N_{ij}$: fuzzy sets;

$z_i(t)$: premise variables;

$x(t) \in R^n$: state vector;

$u(t) \in R^m$: control input;

$r$: number of rules;

$E, A_i \in R^{n \times n}, B_i \in R^{n \times m}$: constant matrix.

**notes:**

$E$ may be singular;

The premise variables vector $z_i(t)$ is a function of the state(or time).

By taking a standard fuzzy inference strategy, that is, using a singleton fuzzifier, procedure fuzzy inference and center average defuzzifier, the final fuzzy model of the systems is inferred as follows

(3) \[ E\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_ix(t) + B_iu(t)), \]

where

\[ h_i(z(t)) = \prod_{j=1}^{p} N_{ij}(z_j(t))/\sum_{i=1}^{r} \prod_{j=1}^{p} N_{ij}(z_j(t)), \quad \sum_{i=1}^{r} h_i(z(t)) = 1. \]
In this paper we also require \( h_i(z(t)) \) to be \( C^1 \) functions. It should be noted that the assumption is satisfied for fuzzy models constructed from smooth (at least \( C^1 \)) nonlinear systems by using a sector nonlinearity approach \([10]\). The sector nonlinearity approach can construct a global or semiglobal fuzzy model that exactly represent the dynamics of a nonlinear system \([12]\).

We employ the following candidate fuzzy Lyapunov function for the T-S fuzzy system (3):

\[
V(x(t)) = \sum_{i=1}^{r} h_i(z(t))x^T(t)E^TP_ix(t),
\]

where \( E^TP_i = P_i^TE \geq 0 \). \( P_i \)'s is nonsingular matrices.

This candidate Lyapunov function satisfies

1) \( V(x(t)) \) is \( C^1 \),
2) \( V(0) = 0 \) and \( V(x(t)) \geq 0 \) for \( x(t) \neq 0 \),
3) \( \| x(t) \| \to \infty \Rightarrow V(x(t)) \to \infty \).

The fuzzy Lyapunov function shares the same membership functions with the T-S fuzzy descriptor model of system. Hence the time derivative of the fuzzy Lyapunov functions contains the time derivative of premise membership functions. An important point is how we can deal with the time derivative of premise membership function. For simplicity, \( x, u, h_i \) will be used instead of \( x(t), u(t), h_i(z(t)) \), \( i = 1, 2, \cdots, r \), respectively.

Consider the open-loop system of (3)

\[
E\dot{x} = \sum_{i=1}^{r} h_iA_ix,
\]

we have the admissible definition as follows.

**Definition 1.** \([4]\) The fuzzy system (5) is admissible if it is continuous on \((0, \infty)\) and exponentially stable.

The following conditions for the admissibility can be derived.

**Theorem 1.** The system (5) is admissible if there exist nonsingular matrices \( P_i \)'s such that

\[
E^TP_i = P_i^TE \geq 0,
\]

\[
\mu_{\min}E^TP_\rho + \frac{1}{r}(P_i^TA_j + A_j^TP_i) < 0.
\]

\( 1 \leq i, j, \rho \leq r \). \( m = 1, 2 \).

In order to prove the Theorem 1, we need the following lemma.

**Lemma 1.** Suppose that for a given piecewise continuous matrix \( A(t) \in \mathbb{R}^{n \times n} \),
if there exists a bounded time-varying matrix $P(t) \in \mathbb{R}^{n \times n}$ and a scalar $\alpha > 0$ satisfying the following inequality

$$A^T(t)P(t) + P^T(t)A(t) \leq -\alpha I$$

for all $t$, then the following hold:

(i) $A(t)$ is invertible;

(ii) $A^{-1}(t)$ is bounded.

**Proof.**

(i) Suppose $A(t)$ is not invertible, then there must exist some $t_0$ and $x \in \mathbb{R}^n, x \neq 0$, such that $A(t_0)x = 0$.

Multiplying (8) on the left side by $x^T$ and on the right side by $x$, respectively, then the left-hand of the inequality in (8) can be rewritten as

$$x^T A^T(t_0)P(t_0)x + x^T P^T(t_0)A(t_0)x = 0,$$

However,

$$-\alpha x^T x = -\alpha \|x\|^2 < 0.$$

This is in contradiction with (10). Therefore, $A(t)$ is invertible.

(ii) Let $y \in \mathbb{R}^n$ be given and let $x = A^{-1}(t)y$. Then follows from (8) that

$$2x^T A^T(t)P(t)x \leq -\alpha x^T x = -\alpha \|x\|^2.$$

namely,

$$\alpha \|A^{-1}(t)y\|^2 \leq -2y^T P(t)A^{-1}(t)y.$$

From Cauchy-Schwarz inequality

$$|(\alpha, \beta)| \leq \|\alpha\|\|\beta\|,$$

we have

$$-2y^T P(t)A^{-1}(t)y \leq 2\|P(t)^T y\| \|A^{-1}(t)y\|.$$

Considering with (11) and (12), we have

$$\|A^{-1}(t)y\| \|A^{-1}(t)y\| - \frac{2}{\alpha} \|P(t)^T y\| \leq 0,$$

$$0 \leq \|A^{-1}(t)y\| \leq \frac{2}{\alpha} \|P(t)^T y\|.$$

Since matrix $P(t)$ is bounded, this proves (ii).

**Proof of Theorem 1.** Without loss of generality, let

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, E = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, A_i = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix},$$

where $\Sigma$ is invertible.

Suppose that there exist matrices $P_j$ such that (6) holds, then $P_j$ is constructed as

$$P_j = \begin{bmatrix} P_{ij}^1 \\ P_{ij}^2 \\ P_{ij}^3 \end{bmatrix}, j = 1, 2, \cdots, r.$$
Inferred from condition (7), we have

\[ P^T A_j + A_j^T P_i < 0. \quad 1 \leq i, j \leq r. \]

Furthermore, for the system (5), we have

\[
(\sum_{j=1}^{r} h_j A_j)^T (\sum_{i=1}^{r} h_i P_i) + (\sum_{i=1}^{r} h_i P_i)^T (\sum_{j=1}^{r} h_j A_j) = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_{22} \end{bmatrix} < 0.
\]

Let

\[
\sum_{j=1}^{r} h_j A_j = \begin{bmatrix} \sum_{j=1}^{r} h_j A_1^j & \sum_{j=1}^{r} h_j A_2^j \\ \sum_{j=1}^{r} h_j A_3^j & \sum_{j=1}^{r} h_j A_4^j \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix},
\]

\[
\sum_{i=1}^{r} h_i P_i = \begin{bmatrix} \sum_{i=1}^{r} h_i P_1^i & 0 \\ \sum_{i=1}^{r} h_i P_2^i & \sum_{i=1}^{r} h_i P_3^i \end{bmatrix} = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix},
\]

and

\[
\Gamma_{11} = A_1^T P_1 + P_1^T A_1 + A_3^T P_2 + P_2^T A_3,
\]

\[
\Gamma_{12} = A_1^T P_3 + P_1^T A_2 + P_2^T A_4, \quad \Gamma_{22} = A_3^T P_3 + P_3^T A_4.
\]

It follows that \( \Gamma_{22} < 0 \), and \( \|P_3\| \leq \sum_{i=1}^{r} \|P_i\| \), then \( A_4 \) is invertible and its invertible is bounded by lemma 1. Thus \( x_2 \) can be expressed

\[ x_2 = -A_4^{-1} A_3 x_1. \]

Substitution of (14), we obtain

\[ \Sigma \dot{x}_1 = (A_1 - A_2 A_4^{-1} A_3)x_1. \]

Consider the fuzzy Lyapunov function (4) and differentiate it along the solution of (5), we have

\[
\dot{V}(x) = \sum_{\rho=1}^{r} \dot{h}_\rho x^T E^T P_\rho x + \sum_{i=1}^{r} h_i (\dot{x}^T E^T P_i x + x^T P_i^T E \dot{x})
\]

\[ = \sum_{\rho=1}^{r} \dot{h}_\rho x^T E^T P_\rho x + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j x^T (P_i^T A_j + A_j^T P_i) x. \]

The time derivative of membership functions can be written as

\[ \dot{h}_\rho = \sum_{m=1}^{2} \omega_{\rho m} \mu_{\rho m}, \quad \sum_{m=1}^{2} \omega_{\rho m} = 1, \quad \omega_{\rho m} \geq 0. \quad \rho = 1, 2, \ldots, r. \]

We will show how to select these parameters in the Appendix. Furthermore, (15) can express as follows

\[
\dot{V}(x) = \sum_{\rho=1}^{r} \sum_{m=1}^{2} \omega_{\rho m} \mu_{\rho m} x^T E^T P_\rho x + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j x^T (P_i^T A_j + A_j^T P_i) x
\]

\[ = x^T \left\{ \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{\rho=1}^{r} \omega_{\rho m} \mu_{\rho m} \left[ \mu_{\rho m} E^T P_\rho + \frac{1}{r} (P_i^T A_j + A_j^T P_i) \right] \right\} x. \]

From (7) in Theorem 1, let

\[ \mu_{\rho m} E^T P_\rho + \frac{1}{r} (P_i^T A_j + A_j^T P_i) = Q_{\rho m ij}, \]
then $Q_{\rho m i j} < 0$. Define $Q = \max_{\rho, m, i, j} Q_{\rho m i j}$, we have $Q < 0$ and

$$
\mu_{\rho m} E^T P_\rho + \frac{1}{r} (A^T_i A_j + A^T_j A_i) \leq Q, \quad 1 \leq i, j, \rho \leq m = 1, 2.
$$

Then for (16), we have

$$
\dot{V}(x) \leq x^T rQx.
$$

Since $rQ < 0$, there must exists some positive scalar $\alpha$, such that $rQ + \alpha I < 0$, i.e., $rQ < -\alpha I$. Hence, we have

$$
\dot{V}(x) < -\alpha x^T x.
$$

By taking notice of that the structural properties matrices $E$ and $P_i$, we set $V_1(x_1) = x_1^T \Sigma^T P_1 x_1$, then

$$
(17) \quad \dot{V}_1(x_1) = \dot{V}(x) < -\alpha x^T x < -\alpha x_1^T x_1.
$$

This proves the square integrability of $x_1$ and hence $x_2$ because of (14). The exponential stability follows [11].

2.2. Relaxed Admissible Conditions.

This section presents a relaxed admissible conditions by considering an important property of the time derivative of membership functions [12]. The property is

$$
\sum_{\rho=1}^{r} \dot{h}_\rho = 0.
$$

From this property, we have

$$
\dot{h}_r = -\sum_{\rho=1}^{r-1} \dot{h}_\rho.
$$

By using (19), the admissible conditions given in Theorem 1 can be relaxed as follows in Theorem 2.

**Theorem 2.** System (5) is admissible if exist a nonsingular matrices $P_i$s satisfy

$$
E^T P_i = P_i^T E \geq 0,
$$

$$
\mu_{\rho m} (E^T P_\rho - E^T P_r) + \frac{1}{2(r-1)} (P_i^T A_j + A^T_j P_i + P_j^T A_i + A^T_i P_j) < 0.
$$

$$
1 \leq i \leq j \leq r, \quad \rho = 1, 2, \cdots, r - 1, \quad m = 1, 2.
$$

**Proof.** Consider the time derivative of the fuzzy Lyapunov function (4), differentiate it along the solution of (5), we have

$$
\dot{V}(x) = \sum_{\rho=1}^{r} \dot{h}_\rho x^T E^T P_\rho x + \sum_{i=1}^{r} \sum_{j=1}^{r} \dot{h}_i h_j x^T (P_i^T A_j + A^T_j P_i) x
$$

$$
= \sum_{\rho=1}^{r-1} \dot{h}_\rho x^T E^T P_\rho x + \dot{h}_r x^T E^T P_r x + \sum_{i=1}^{r} \sum_{j=1}^{r} \dot{h}_i h_j x^T (P_i^T A_j + A^T_j P_i) x.
$$

Furthermore, from the property (19)

$$
\dot{V}(x) = \sum_{\rho=1}^{r-1} \dot{h}_\rho x^T (E^T P_\rho - E^T P_r) x + \sum_{i=1}^{r} \sum_{j=1}^{r} \dot{h}_i h_j x^T (P_i^T A_j + A^T_j P_i) x
$$
Assume that

\[ \dot{h}_\rho = \sum_{m=1}^{2} \omega_{\rho m} h_{\rho m}, \sum_{m=1}^{2} \omega_{\rho m} = 1, \omega_{\rho m} \geq 0, \rho = 1, 2, \ldots, r - 1, \]

we have

\[
\dot{V}(x) = \sum_{\rho=1}^{r-1} \sum_{m=1}^{2} \omega_{\rho m} h_{\rho} x^T (E^T P_\rho - E^T P_\rho) x + \sum_{i=1}^{r} \sum_{j=1}^{r-1} \sum_{\rho=1}^{2} h_i h_j \omega_{\rho m} x^T \left\{ \mu_{\rho m} (E^T P_\rho - E^T P_\rho) \right\} x \\
+ \sum_{i=1}^{r} \sum_{j=1}^{r-1} \sum_{\rho=1}^{2} h_i \omega_{\rho m} x^T \left\{ \frac{1}{2(r-1)} (P_i^T A_j + A_j^T P_i) \right\} x \\
+ \sum_{i=1}^{r} \sum_{j=1}^{r-1} \sum_{\rho=1}^{2} \sum_{m=1}^{2} h_i h_j \omega_{\rho m} x^T \left\{ \mu_{\rho m} (E^T P_\rho - E^T P_\rho) + \frac{1}{r-1} (P_i^T A_j + A_j^T P_i) \right\} x \\
+ \sum_{i=1}^{r} \sum_{j=1}^{r-1} \sum_{\rho=1}^{2} h_i h_j \omega_{\rho m} x^T \left\{ \frac{1}{2(r-1)} (P_i^T A_j + A_j^T P_i) \right\} x.
\]

The admissibility follows from the proof of Theorem 1 with lemma 1.

**Remark 1.** By using the property (19), the number of LMIs in (21) is smaller than the one in (7). There \((r+1)\) LMIs constrains have been taken out of the problems. The differences of the computational complexity between these two conditions will be distinct when the given system is with high dimensions. In fact, the conditions in Theorem 2 coincides with the result of \([1, \text{Th.1}],[3, \text{Th.1}],[4, \text{Th.2.1}]\) and \([6, \text{Th.1}]\) when \(P_i\)s are same.

### 3. Fuzzy Controller Design

In this section we design fuzzy controller for the fuzzy system (3) based on the PDC. Consider the following fuzzy control law

\[
(22) \quad u = \sum_{i=1}^{r} h_i F_i x.
\]

The aim is to determine the local feedback gain \(F_i\) such that the closed-loop system (23) is regular, impulse-free and stable.

\[
(23) \quad E \dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j (A_i + B_i F_j) x.
\]

With the aid of the admissible analysis method presented in the last section, the result for the controller design is as follows

**Theorem 3.** The closed-loop system (23) is admissible via the fuzzy controller (22) if there exist \(\varepsilon > 0, s_1, s_2, \ldots, s_r\) nonsingular matrices \(P_i\)s and matrices \(F_i\)s such that

\[
(24) \quad E^T P_i = P_i^T E \geq 0,
\]
Proof. Consider the fuzzy Lyapunov function

$$V(x(t)) = \sum_{i=1}^{r} h_i(z(t))x^T(t)E^TP_i x(t),$$

then $\dot{V}(x)$ can be obtained as

$$\dot{V}(x) = \sum_{\rho=1}^{r} \dot{h}_\rho x^T E^TP_\rho x + \sum_{i=1}^{r} h_i(\dot{z}^T E^TP_i x + x^T P_i^T E\dot{z})$$

$$= \sum_{i=1}^{r} \dot{h}_i x^T P_i^T \left[ \sum_{j=1}^{r} \sum_{k=1}^{r} h_j k(A_j + B_j F_k) + \sum_{j=1}^{r} h_j k(A_j + B_j F_k)^T P_i \right] x$$

$$+ \sum_{\rho=1}^{r} \dot{h}_\rho x^T E^TP_\rho x$$

$$= \sum_{i=1}^{r} \dot{h}_i x^T E^TP_i x + \sum_{\rho=1}^{r} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j k x^T W_{ijk} x$$

$$= \sum_{\rho=1}^{r} \dot{h}_\rho x^T (E^TP_\rho - E^TP_r) x + \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} h_i h_j k x^T W_{ijk} x$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{\rho=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} h_i h_j k \omega_{pm} x^T \left[ \mu_{pm}(E^TP_\rho - E^TP_r) + \frac{1}{(r-1)} \dot{W}_{ijk} \right] x,$$

where

$$\dot{W}_{ijk} = W_{ijk} + W_{ikj} + W_{jik} + W_{kji} + W_{kij} + W_{kji},$$

$$W_{ijk} = P_i^T G_{jk} + G_{jk}^T P_i, G_{jk} = A_j + B_j F_k.$$
Note that for all real number $s_i$

$$(s_i I + P_i)^T (s_i I + P_i) \geq 0.$$  

Hence, we have

$$P_i^T P_i \leq s_i^2 I + s_i P_i + s_i^T.$$  

Then the following relation holds:

$$\dot{V}(x) < 0 \text{ at } x \neq 0 \text{ if the right-hand of inequality in (29) is negative-definite matrix.}$$

By Schur complement (25) is equivalent to (29).

The admissibility of system (23) follows from the proof of Theorem 2 with lemma 1. This completes the proof.

Remark. The condition in Theorem 3 are LMIs if $\varepsilon$ and $s_1, s_2, \ldots, s_r$ are given. Furthermore, we can replace $\varepsilon$ with $\varepsilon_{ijk}$, such that $\varepsilon$ takes on deferent values accordingly to $i, j, k$ and hence the dimension of the solution space can be enlarged.

4. Numerical Example

In this section, we compare the fuzzy Lyapunov approach (Theorem 3) with the Theorem 3 in [1], Theorem 6 in [1], Theorem 2.2 in [4] and Theorem 3 in [6], respectively.

Consider the following fuzzy model:

$$R_1 : \text{IF } x_1 \text{ is P, THEN}$$

$$E \dot{x} = A_1 x + B_1 u,$$

$$R_2 : \text{IF } x_1 \text{ is N, THEN}$$

$$E \dot{x} = A_2 x + B_2 u,$$

where the membership function of 'P', 'N' are given as following the effectiveness of the method

$$h_1(x_1) = 1 - \frac{1}{1 + e^{-2x_1}}, \quad h_2(x_1) = 1 - h_1(x_1).$$

And

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 5 & -4 \\ 10 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & a_{22} \\ 3 & 5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ b_{12} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. $$

Let $a_{22}, b_{12}$ take values in intervals $[-5, 10]$ and $[-10, 10]$, respectively. Let " * " denotes the points of $(a_{22}, b_{12})$ at which the system can be determined to be stabilized.

From Figures 1-4 we can see that if $a_{22} \geq 0$, the conditions proposed in Theorem 3 and Theorem 6 of [1], Theorem 2.2 of [4] and Theorem 3 of [6] will be invalid for solving the stability problem of the system. However, Figure 5 illustrates that there are a mass of points of $(a_{22}, b_{12})$ above the line $a_{22} = 0$ at which the system can be stabilized, and, we can verify it through Theorem 3 proposed in this paper. Hence, the fuzzy Lyapunov approach has provided less conservative results.
Figure 1. Feasible area of Theorem 3 in [1]

Figure 2. Feasible area of Theorem 6 in [1]
Figure 3. Feasible area of Theorem 2.2 in [4]

Figure 4. Feasible area of Theorem 3 in [6]
5. Conclusion

This paper considers the admissibility problem for T-S fuzzy descriptor systems via the so-called fuzzy Lyapunov function which is a multiple Lyapunov function. Based on the fuzzy Lyapunov approach, we have obtained admissible conditions for open-loop fuzzy descriptor systems. Furthermore, utilizing the property of the time derivative of membership functions, we have proposed a new admissible conditions and fuzzy controller design scheme with less conservative. A design example has illustrated the utility of the fuzzy Lyapunov function scheme.

References

In the appendix, we present how to calculate the design parameters $\omega_{\rho m}$ and $\mu_{\rho m}$, such that

$$\dot{h}_\rho = \sum_{i=1}^{2} \omega_{\rho m} \mu_{\rho m}, \quad \sum_{i=1}^{2} \omega_{\rho m} = 1, \omega_{\rho m} \geq 0.$$ 

Consider the following fuzzy model

$$E \dot{x} = \sum_{i=1}^{r} h_i(A_i x + B_i u),$$

where

$$h_1(x_1) = \frac{1}{2}(1 + \sin x_1), \quad h_2(x_1) = 1 - h_1(x_1) = \frac{1}{2}(1 - \sin x_1).$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 5 & -4 \\ 10 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$ 

Consider the time derivative of $h_1(x_1)$

$$\dot{h}_1(x_1) = \frac{\partial h_1(x_1)}{\partial x_1} \dot{x}_1$$

$$= \frac{1}{2} \cos x_1 \left\{ \frac{1}{2}(1 + \sin x_1)[5 - 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2}(1 - \sin x_1)[2 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\}$$

$$= \cos x_1 \left[ \frac{7}{4} x_1 - \frac{3}{4} x_2 + \sin x_1 \left( \frac{3}{4} x_1 - \frac{5}{4} x_2 \right) \right].$$

We replace $\dot{h}_1(x_1)$ with the model representation

$$\sum_{m=1}^{2} \omega_{1m} \mu_{1m}, \quad \sum_{m=1}^{2} \omega_{1m} = 1, \omega_{1m} \geq 0.$$ 

Where

$$\mu_{11} = \max_{|x_1| \leq 1, \ |x_2| \leq 1} \dot{h}_1(x_1) = 2.4225,$$

$$\mu_{12} = \min_{|x_1| \leq 1, \ |x_2| \leq 1} \dot{h}_1(x_1) = -0.8064.$$
Then, from
\[ \omega_{11}\mu_{11} + \omega_{12}\mu_{12} = \cos x_1 \left[ \frac{7}{4} x_1 - \frac{3}{4} x_2 + \sin x_1 \left( \frac{3}{4} x_1 - \frac{5}{4} x_2 \right) \right] \]
and \[ \sum_{m=1}^{2} \omega_{1m} = 1, \]
we have
\[ \omega_{11} = \frac{1}{3.2289} (f(x_1, x_2) + 0.8064), \]
\[ \omega_{12} = \frac{1}{3.2289} (2.4225 - f(x_1, x_2)), \]
where
\[ f(x_1, x_2) = \cos x_1 \left[ \frac{7}{4} x_1 - \frac{3}{4} x_2 + \sin x_1 \left( \frac{3}{4} x_1 - \frac{5}{4} x_2 \right) \right]. \]

In the same way, \( \mu_{21}, \mu_{22}, \omega_{21}, \omega_{22} \) are obtained.

1Institute of Systems Science, Northeastern University, Shenyang, Liaoning Province, 110004, China
E-mail: yyhmds@sohu.com and qlzhang@mail.neu.edu.cn

2Institute of Complexity Science, Qingdao University, Qingdao, Shandong Province, 266071, China
E-mail: dongshuoch@sina.com