SEMI-BLIND MULTIUSER DETECTION BASED ON IMPROVED PASTD SUBSPACE TRACKING FOR MC-CDMA UPLINK

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Abstract In this paper, an adaptive semi-blind multiuser detection (MUD) algorithm based on improved projecting approximation subspace tracking with deflation (PASTd) subspace tracking is proposed for multicarrier code division multiple access (MC-CDMA) uplink where the base station receiver has the knowledge of the spreading sequences of all the users within the cell, but not that of the users from other cells. It is known that the PASTd algorithm has the drawback of slow convergence rate. Based on this, we develop an improved PASTd algorithm and apply it to the adaptive linear hybrid semi-blind multiuser detection. The improved PASTd algorithm guarantees the orthonormality between the estimated eigenvectors such that a fast convergence rate can be achieved. Simulation results show the proposed semi-blind MUD has a fast convergence rate and provides the similar output signal-to-interference-plus-noise-ratio (SINR) and bit error rate (BER) as the Singular Value Decomposition (SVD) semi-blind MUD.

Key Words Semi-blind multiuser detection, Subspace tracking, PASTd algorithm, MC-CDMA

1. Introduction

MC-CDMA has received considerable attention for future high-speed wireless systems [1]. The uplink transmission of MC-CDMA suffers from distortion of code-orthogonality among users by the instantaneous frequency response of channels, which causes an increase in the effective multiple access interference (MAI). Thus, multiuser detection is required for reducing its effects.

Recently, various blind multiuser detection techniques have been developed, which can suppress MAI by only exploiting the spreading codes of the desired user [2]. However, in uplinks, the spreading sequences of all the users within the cell are known at the base station receiver, which should assist in further suppressing the MAI. This idea was proposed in [3]-[5], and after that various semi-blind multiuser detectors [6]-[10] have been proposed for CDMA uplink, which can cancel interferers from both known and unknown users, while utilizing the information about known users.

A linear hybrid semi-blind multi-user detector has been proposed in [4]. This detector adopted SVD to obtain the signal subspace, which requires high computational complexity. Many efficient adaptive subspace tracking algorithms with low complexity such as projection approximation subspace tracking (PAST) in [11], PASTd in [11] and orthonormal projection approximation subspace tracking (OPAST) in [12] have been proposed. We found that the convergence rate of the PASTd algorithm is fairly slow. In this paper, we develop an improved PASTd subspace tracking algorithm and use it, along
with the closed-form expression for the hybrid semi-blind MUD, to propose an adaptive semi-blind MUD algorithm for MC-CDMA uplink. The improved PASTd subspace tracking algorithm has a faster convergence rate than the PASTd algorithm [11]. The proposed semi-blind MUD offers better performance than PASTd semi-blind MUD and OPAST semi-blind MUD in terms of the convergence rate, SINR and BER. The simulation results are provided to demonstrate the performance of the proposed algorithm.

2. Signal model

We consider a synchronous MC-CDMA communication system in the uplink [8], where \( K \) users with known codes are in the cell, and \( \bar{K} \) users with unknown codes are in the other cells. The modulating symbols \( b_k \in \{-1,1\} \) are spread and mapped to \( N \) different subcarriers by the Inverse Fast Fourier Transform (IFFT), where \( N \) is the length of spreading sequence. This signal is transmitted through a multipath fading channel, which is assumed to have \( L \) paths, hence the channel impulse response (CIR) can be expressed as

\[
h_k(t) = \sum_{l=0}^{L-1} h_{k,l} \cdot \delta(t - mT_c)
\]

where \( h_{k,l} \) is the complex channel gain experienced by the signal of the \( k \)th user in the \( l \)th path, which obeys Rayleigh fading, and \( T_c \) is the chip-duration. At the receiver, \( N \) point FFT is invoked for demodulating samples of the received signal. Hence the received signal can be expressed in vectorial form as

\[
r = \sum_{k=1}^{K} A_k b_k \mathbf{g}_k + \sum_{k=1}^{\bar{K}} \bar{A}_k \bar{b}_k \bar{\mathbf{g}}_k + \sigma \mathbf{n}
\]

where \( A_k \) and \( b_k \) denote the received amplitude and the transmitted symbols of the \( k \)th user within the cell, respectively. \( \mathbf{g}_k = \mathbf{C}_k \mathbf{H}_k \) is the effective signature waveform of the \( k \)th user. \( \mathbf{C}_k \) is a diagonal matrix which is composed of the spreading sequence of the \( k \)th user within the cell and \( \mathbf{H}_k \) denotes the frequency domain channel transfer function which can be expressed as the \( N \) point FFT of \( \mathbf{h}_k = [h_{k,0}, \ldots, h_{k,L-1}]^T \). \( \bar{\mathbf{g}}_k \) and \( \bar{A}_k \) and \( \bar{b}_k \) are the corresponding information of the \( k \)th user in the other cells. \( \mathbf{n} \) denotes a white Gaussian noise vector with zero mean and covariance matrix \( \mathbf{I}_N \).

For convenience and without loss of generality, we assume that the signature waveforms of all users are linearly independent. Denote \( \mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_K] \), \( \bar{\mathbf{G}} = [\bar{\mathbf{g}}_1, \ldots, \bar{\mathbf{g}}_{\bar{K}}] \), \( \mathbf{A} = \text{diag}(A_1^2, \ldots, A_K^2) \) and \( \bar{\mathbf{A}} = \text{diag}(\bar{A}_1^2, \ldots, \bar{A}_{\bar{K}}^2) \). The autocorrelation matrix of the received signal \( \mathbf{r} \) is then given by

\[
\mathbf{R} = \mathbb{E} \{ \mathbf{r} \mathbf{r}^H \} = \sum_{k=1}^{K} A_k^2 \mathbf{G}_k \mathbf{G}_k^H + \sum_{k=1}^{\bar{K}} \bar{A}_k^2 \bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H + \sigma^2 \mathbf{I}_N
\]

\[
= \mathbf{G} \mathbf{G}^H + \bar{\mathbf{G}} \bar{\mathbf{G}}^H + \sigma^2 \mathbf{I}_N
\]

By performing an eigen-decomposition of the matrix \( \mathbf{R} \), we get

\[
\mathbf{R} = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \Lambda_n \end{bmatrix} [\mathbf{U}_s^H \ \mathbf{U}_n^H]
\]

where \( \Lambda_s = \text{diag}(\lambda_1, \ldots, \lambda_{K+\bar{K}}) \) contains the \( K+\bar{K} \) largest eigenvalues of \( \mathbf{R} \) in descending order and \( \mathbf{U}_s = [\mathbf{u}_1, \ldots, \mathbf{u}_{K+\bar{K}}] \) contains the corresponding orthonormal
eigenvectors; \( \Lambda_s = \sigma^2 \mathbf{I}_{N-K-K} \), and \( \mathbf{U}_s = [\mathbf{u}_{k+1}, \ldots, \mathbf{u}_N] \) contains the \( N-K-K \) orthonormal eigenvectors that correspond to the eigenvalue \( \sigma^2 \). The range space of \( \mathbf{U}_s \) is called the signal subspace and its orthogonal complement, the noise subspace, is spanned by \( \mathbf{U}_n \).

3. Semi-blind multiuser detection

Consider an uplink transmission, where the base station receiver has the knowledge of the spreading sequences of users within the cell, while that of users from other cells are unknown. Without loss of generality, we assume user 1 is the desired user. The semi-blind linear hybrid detector for user 1 can be formulated as the following multiple constrained optimization problem

\[
\min E \{ (A_1 b_1 - m_1^H r)^2 \} \quad \text{s.t.} \quad m_1^H \mathbf{G} = \mathbf{f}
\]

where \( \mathbf{G} \) is composed of the effective spreading sequences of all known users and \( \mathbf{f} = [1, 0, \ldots, 0]^T \), \( f \in \mathbb{R}^{K \times 1} \). By the method of Lagrange multipliers, the solution of (5) in terms of the signal subspace parameters can be written as [4]

\[
\mathbf{m}_1 = \mathbf{U}_s \mathbf{A}_s^{-1} \mathbf{U}^H \mathbf{G} (\mathbf{G}^H \mathbf{U}_s \mathbf{A}_s^{-1} \mathbf{U}^H \mathbf{G})^{-1} \mathbf{f}
\]

A linear hybrid semi-blind multiuser detector for demodulating the desired user’s data in (2) is in the form of a correlator followed by a hard limiter

\[
\hat{b}_1 = \text{sgn}(m_1^H r)
\]

It can be seen from (7) that the proposed semi-blind multiuser detection can be expressed by signal subspace parameters. So the subspace methods make the importance of solving a detector turn to be the subspace tracking algorithm. We get the detector when we obtain the signal subspace parameters. In this paper, an improved PASTd subspace tracking algorithm is proposed to obtain the signal subspace parameters adaptively.

4. Improved PASTd subspace tracking algorithm

There is a lack of orthonormality between the eigenvectors in the PASTd algorithm [11], which induces the slow convergence rate of the PASTd algorithm. To alleviate this drawback, an improved PASTd subspace tracking algorithm is proposed. The basic idea of the improved PASTd algorithm is to orthonormalize all of the estimated eigenvectors at each iteration. After the \( k \)th eigenvector has been extracted, we orthonormalize the \( k \)th eigenvector and the \( k-1 \) eigenvectors that have been estimated before the \( k \)th eigenvector. The same orthonormalization processing is used after we extracted the next eigenvector. Applying this orthonormalization procedure repeatedly, all of the estimated eigenvectors have been orthonormalized at each iteration. We assume \( \mathbf{w} = \mathbf{U}_i(:,1:k-1) \) is the \( k-1 \) eigenvectors at the \( i \)th iteration and \( \mathbf{U}_i(:,k) \) is the \( k \)th eigenvector of the \( i \)th iteration. An orthonormalization step can be made as follows after the \( k \)th eigenvector has been extracted at the \( i \)th iteration

\[
\mathbf{U}_i(:,k) = \mathbf{U}_i(:,k) - \mathbf{w} \mathbf{w}^H \mathbf{U}_i(:,k)
\]

\[
dd = \| \mathbf{U}_i(:,k) \|
\]

\[
\mathbf{U}_i(:,k) = \mathbf{U}_i(:,k) / dd
\]

The improved PASTd subspace tracking algorithm is summarized as follows

For \( i = 1, 2, \cdots, M \)
\( \mathbf{x}_i(:,1) = \mathbf{r}_i \)
For \( k = 1, 2, \cdots, K + \bar{K} \)
\( y_i = \mathbf{U}_{i-1}(:,k)^H \mathbf{x}_i(:,k) \)
\( \Lambda_i(k,k) = \beta \Lambda_{i-1}(k,k) + |y_i|^2 \)
\( \mathbf{U}_i(:,k) = \mathbf{U}_{i-1}(:,k) + [\mathbf{x}_i(:,k) - \mathbf{U}_{i-1}(:,k)y_i]y_i^H / \Lambda_i(k,k) \)
\( \mathbf{x}_i(:,k+1) = \mathbf{x}_i(:,k) - \mathbf{U}_i(:,k)y_i \)
If \( k \geq 2 \)
\( \mathbf{w} = \mathbf{U}_i(:,1:k-1) \)
\( \mathbf{U}_i(:,k) = \mathbf{U}_i(:,k) - \mathbf{w} \mathbf{w}^H \mathbf{U}_i(:,k) \)
End
\( dd = \| \mathbf{U}_i(:,k) \| \)
\( \mathbf{U}_i(:,k) = \mathbf{U}_i(:,k) / dd \)
End
End

where \( \beta \) is the forgetting factor, \( M \) is the number of iterations and \( K + \bar{K} \) is the number of users. The improved PASTd algorithm guarantees the orthonormality of the weight matrix spanned by the signal subspace at each iteration.

5. Simulation results

We consider a synchronous MC-CDMA uplink system, where five users with known codes are in the cell, while two users with unknown codes are in the other cells. A multipath channel with three paths is considered. The channel coefficients are randomly generated according to a complex Gaussian distribution. The channel coefficients of users in the cell are available at the receiver. The user 1 is specified as the desired user. All of the interference users have the same power. There are six 10dB MAI’s in the channel, all relative to the desired user’s signal. The signature sequence of desired user is Wash codes of length \( N = 32 \). The forgetting factor \( \beta \) is set to be 0.995. The performance of the proposed semi-blind MUD based on the improved PASTd (newPASTd) is compared with that of semi-blind MUD based on SVD, OPAST in [12], and PASTd in [11]. The performance measure is the output SINR given in [13] and BER.

The data plotted is the averages over 100 Monte Carlo simulations and 3000 iterations in each simulation.

![Figure 1. Output SINR versus number of iterations](image-url)
Fig. 1 illustrates the SINR performance of the detectors versus number of iterations where the SNR of user 1 is 20 dB. It shows that SVD MUD, newPASTd MUD and PASTd MUD can track subspace efficiently; with a fast convergence rate, OPAST MUD reaches the steady state with low values of the SINR; the convergence rate of PASTd MUD is very slow so it is unsuitable for real-time system. It is obvious that newPASTd MUD has a high convergence rate and high stable SINR, and almost has identical performance with SVD MUD.

![Figure 2. Output SINR versus SNR](image)

The SINR performance versus the SNR of user 1 is plotted in Fig. 2. In Fig. 2, it can be seen the SINR of OPAST MUD and PASTd MUD are much lower than that of SVD MUD and newPASTd MUD; the newPASTd MUD presents the same performance as SVD MUD in each SNR. It shows that the newPASTd MUD has high steady SINR and has strong ability to eliminate MAI.

![Figure 3. Output BER versus number of iterations](image)

Fig. 3 displays the BER performance of four detectors where the SNR of user 1 is 20 dB. It shows the BER performance of PASTd MUD and OPAST MUD are very poor; the
performance of newPASTd MUD is close to that of the SVD MUD.

Fig.4 presents the BER performance versus the SNR of user 1. 3000 symbols are used to calculate the steady-state BER with various inputs SNR. It can be seen that the BER performance of OPAST MUD and PASTd MUD are poor; newPASTd MUD performs like SVD MUD in low SNR, for moderate and high SNR, the BER of newPASTd MUD is little higher than that of SVD MUD.

![Figure 4. Output BER versus SNR](image)

6. Conclusion

In this paper, an adaptive hybrid semi-blind multiuser detector based on the improved PASTd subspace tracking is proposed for MC-CDMA uplink. We have developed an improved PASTd subspace tracking algorithm and applied it to the hybrid semi-blind multiuser detection. The improved PASTd algorithm has a fast convergence rate. The simulation results have shown that the proposed semi-blind detector outperforms the semi-blind MUD based on OPAST and PASTd, and is close to the SVD semi-blind MUD in terms of the convergence rate, SINR and BER.

References


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