MODELING AND IDENTIFICATION OF NON-LINEAR SYSTEMS BY A MULTIMODEL APPROACH: APPLICATION TO A THROTTLE VALVE

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Abstract The control and the diagnosis of non-linear systems require modelling and identification techniques, adapted to numerical computation. This paper deals with such issues. Nonlinearities that are considered are dead zones, hysteresis and saturations. These nonlinearities are simplified according to a multimodel approach that has additive unknown inputs and that divides nonlinearities into several discrete-time linear models. These unknown inputs are used to define commutation conditions from one local model to the other. An automaton is proposed as global model. For identification purpose, we propose an algorithm that uses both classification and linear identification techniques. Finally, this algorithm is applied to the throttle valve of combustion engine.

Key Words: Nonlinear systems, Modeling, Identification, Throttle valve.

1. Introduction

In order to develop cleaner and more economical engines that satisfy the European antipollution standards, the field of embedded electronics has become increasingly present in the automotive industry. The principal challenges are the control of the combustion engines and the diagnosis of the various elements constituting this engine. The control and the diagnosis require modelling and identification of the components, adapted to numerical computation. One element, which plays a key role in the optimization of the fuel combustion, is the throttle valve. This component consists of a metal plate, which rotates around a fixed axis. In the literature, both control and diagnosis of throttle valve are based on continuous models resulting from physical equations with known parameters [10]. These models make it possible to have very useful information for the control engineers, but in many cases models are difficult to obtain with an acceptable precision. On the one hand, the throttle valve presents non-linear behaviors. On the other hand, specifications required for embedded systems are strong. Calculators use digital data and must satisfy real time constraints. For fast diagnosis and control, discrete time modelling and identification becomes necessary where instantaneous derivatives are replaced by delays [11].

Few studies have considered the valve identification in discrete time. Gagner [3] has proposed an identification method for the linear zones without including nonlinearities. Rossi [12] and Jankovic [6] have presented the valve by considering combustion performances and vehicle dynamics. Özgüner [10] and Kitahara [7] have studied control

In this paper, we will present a new approach to model non-linear systems by a set of discrete linear models which commutates between them with respect to switching conditions. The article is organized in two sections. In section 2, the multimodel approach is described. This modelling approach is based on dividing the operating space constituted by inputs, outputs and their derivatives in several operating modes. Thereafter, in order to have linear models, bounded additive inputs are introduced. The switching conditions are deduced from operating mode behaviours and new inputs limits. Then, application of our multi-model approach for throttle valve is provided. Section 3 deals with identification of linear multimodels. First, the identification approach is described. It is inspired from PWARX (PieceWise AutoRegressive eXogenous) method [14], applied to throttle valve [8]. Several improvements are proposed in comparison with usual PWARX method. Our method is based on linear multimodels with additive inputs instead of several linear models. Moreover, in our approach we use polyhedral multi-dimensional domain partition in order to classify data instead of local estimation of the parameters. The instantaneous derivatives of measurements are calculated by Legendre polynomials. Then, application of our approach on the real throttle valve is provided.

2 Multimodel approach

2.1 Modelling problem statement

Let $S(\dot{x}(t), x(t), u(t), y(t))=0$ be a non-linear process, where $u(t)$ is the control input vector, $x(t)$ the state vector and $y(t)$ the output vector.

Non-linear processes considered can be characterised by two properties. (1) Nonlinearities are caused by bounded unknown inputs such as friction forces. Such inputs induce nonlinearities like hysteresis and dead zones. (2) Operating space can be represented by multi-dimensional domain over outputs, inputs and their derivatives (for example outputs phases diagram). The operating space can be divided in $m$ modes that lead to polyhedral partition of multi-dimensional domain. Figure 1 illustrates a polyhedral partition of outputs phases diagram.

![Polyhedral partition of outputs phases diagram](image)

The problem is how to get all models corresponding to the operating modes and switching conditions between them in order to obtain a multimodel that represent accurately all non-linear behaviours. In the next paragraph, we propose a new modelling approach to solve this problem.

2.2 Approach Principle
The suggested model is based on the representation of the continuous system \( S(\dot{x}(t), x(t), u(t), y(t)) \) with several linear models that commutate thanks to switching conditions. This modelling approach requires the definition of two sets: possible linear models and commutation conditions. Figure 2 illustrates such a multimodel.

Determination of the operating modes is obtained according to the following steps:

1. Determine a map representing \( m \) modes that lead to polyhedral multi-dimensional partition domain using inputs, outputs and their derivatives.

2. Consider nonlinearities as unknown bounded inputs vector \( v(t) \). Afterwards, build \( m \) continuous time linear models \( S_i(\dot{x}(t), x(t), u(t), v(t), y(t))=0 \) \( i \in \{1,\ldots,m\} \), and then \( m \) discrete time linear models \( S_{di}(u(k), x(k), x(k+1), y(k), v(k))/i \in \{1,\ldots,m\} \).

3. Deduce switching conditions from properties of the process in each operating mode. Use also these properties to estimate the unknown inputs \( v \), and compare the estimated values to unknown inputs limits.

4. Represent the multimodel with an automaton, where places are for models and transition for switching conditions.

We will apply our approach in the next paragraph to a throttle valve. This component, used in combustion engine of vehicles, has dead zone and hysteresis nonlinearities caused by frictions.

2.3 Apply multimodel approach to a throttle valve

Throttle valve (fig 3) is used to control airflow in combustion engine. The objective is to increase engine torque and decrease exhaust pollution. The airflow increases proportionally with the throttle desired angle \( \theta_d \), and in that case the engine produces more torque. This component consists of a metal plate, which rotates around a fixed axis. Plate position is controlled by a voltage excitation (input signal) of type PWM (Pulse Width Modulated) applied to electrical DC current motor (Fig.4).

Fig 3. Throttle valve
The D.C motor torque $Ce$ (Fig.4) is used to control angular position $\theta_d$ of throttle valve. Other torques due to the return spring $Cr$ and frictions $Cf$ influence the behaviour of this plate.

![Fig 4. Electro – mechanical model of throttle valve](image)

**a) Modelling of the electrical device**

The electrical device is modeled (Fig.4) by an inductance $L$, a resistance $R$ and an electromotive force $E = k\omega$ induced by rotation of the rotor angle. $k$ is a constant and $\omega$ is the angular velocity of the motor rotor. The equation of the electrical part is as follows:

$$u = L\frac{di}{dt} + Ri + k\omega$$

(1)

**b) Modelling of the mechanical part**

The mechanical part using is modeled according to (2), such that:

$$j\frac{d\Omega}{dt} = Ce - Cf - Cr$$

(2)

with:

- $Ce = Ki$: Electrical torque (N-m),
- $K$: Constant
- $Cf$: Torques caused by mechanical frictions (Nm)
- $Cr$: Spring resistive torque (Nm)

$$\Omega = \frac{d\theta}{dt} = n\omega$$: Throttle plate velocity

- $n$: Gears ratio.
- $\theta$: Throttle plate position.

**c) Nonlinearities of the valve**

Fig.5 gives response $y(t)$ to sinusoidal inputs $u(t)$ for a real throttle. This figure suggests that the device is a non-linear process. The typical input/output feature (fig.6) of the electronic throttle valve has 2 main nonlinearities. The first is a dead zone where the throttle plate remains motionless for position $\theta_0 \approx 10$ degrees even if the input signal varies in a given interval. The second is a hysteresis combined with saturation. These nonlinearities are caused by the resistive torques (spring and frictions).
The resistive torque applied by the spring is represented in Fig. 7, and his mathematical equation is given by (3):

$$C_r = k_r(\theta - \theta_0) + CR1$$

Fig 5. Static response of the valve

Fig 6. Input - output signature

With:

- $k_r$: Spring gain,
- $D$: Spring offset = Constant,
- $\theta_0$: Default position,
- $CR1$: Function representing nonlinearity.
The valve model takes into account a static friction model [13]. Fig.8 gives the friction torque according to the plate. Such a torque is given by equation (4).

\[ C_f = f_v \Omega + CF1 \]  

where \( f_v \) and \( f_c \) are constant, and \( CF1 \) is a function representing nonlinearities. These nonlinearities induce saturation zones and hysteresis.

The combination of equations 1 to 4 leads to (5):

\[
\begin{align*}
\frac{d\theta}{dt} &= n \\
J \frac{d\omega}{dt} &= -k_r(\theta - \theta_0) - f_v \omega + Ki - CR1 - CF1 \\
L \frac{di}{dt} &= -k_0 - Ri + u
\end{align*}
\]  

Let us define the following state variables \( x_1 = (\theta - \theta_0) \), \( x_2 = \omega \) and \( x_3 = i \), and \( y = x_1 \) stands for the output. The model of throttle valve is given by (6):

\[
\begin{align*}
\dot{x}_1 &= a_{12}x_2 \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 - CR - CF \\
\dot{x}_3 &= a_{32}x_2 + a_{33}x_3 + b_3u
\end{align*}
\]  

with: \( a_{12} = n \), \( a_{21} = \frac{-k_r}{nJ} \), \( a_{22} = \frac{-f_v}{J} \), \( a_{23} = \frac{K}{nJ} \), \( a_{32} = \frac{-k}{L} \), \( a_{33} = \frac{-R}{L} \), \( b_3 = \frac{L}{L} \), \( CF = \frac{CF1}{nJ} \), \( CR = \frac{CR1}{nJ} \).

In order to rewrite non-linear model (6) as a multimodel, we apply our approach in 4 steps.

**Step 1.** From fig.7, fig.8 and equation (6), we can observe that nonlinearities of throttle valve are caused by output \( y = x_1 \) and output derivative \( \dot{y} = \dot{x}_1 = a_{12}x_2 \). As a consequence, we divide the throttle valve behavior in 5 regions according to a polyhedral partition. Each region \( i \) \((1 \leq i \leq 5)\) corresponds to an operating mode and is represented by a model \( M_i \), defined as follows:
Mode 1: \((x_1=0, x_2=0\) and \(\dot{x}_2 = 0\)) (Dead zone \(M_1\))
Mode 2: \((x_1>0\) and \(x_2 \geq 0\)) (\(M_2\))
Mode 3: \((x_1>0\) and \(x_2 \leq 0\)) (\(M_3\))
Mode 4: \((x_1>0, x_2=0\) and \(\dot{x}_2 = 0\)) (Saturation mode \(M_4\))
Mode 5: \((x_1<0\) and \(x_2 \geq 0\)) (\(M_5\))
Mode 6: \((x_1<0\) and \(x_2 \leq 0\)) (\(M_6\))
Mode 7: \((x_1<0, x_2=0\) and \(\dot{x}_2 = 0\)) (Saturation mode \(M_7\))

\[ \dot{y} = a_{12} x_2 \]

Fig 9. Operating modes

**Step 2.** Non-linear behaviors are removed in order to obtain linear multimodel. In equations (3) (resp. (4)), we can observe that nonlinearity is caused by \(CR1\) (resp. \(CF1\)). These nonlinearities are represented by \(CR\) and \(CF\) in equation (6). Several models have been discussed for these nonlinearities. Özguner [10] proposes to define:

\[
CR = \alpha \cdot \text{sign}(x_1)\]
\[
CF = \mu \cdot \text{sign}(x_2)\]

with \(\mu = \frac{f_r}{nJ}, \alpha = \frac{D}{nJ}\).

Canudas [2] proposes to use the current as input signal, and represents friction with LuGre dynamical model [1].

One can observe that values of nonlinearities \(CR\) and \(CF\) are bounded respectively by \(\pm \alpha\) and \(\pm \mu\). In the following, we propose another presentation of the nonlinearities. In modes \(M_1, M_4\) or \(M_7\) the torque induced by the control input \(u\) is equal to resistor torque. The position of the throttle plate remains constant as long as the torque induced by input is higher or lower than a given value. There is an input value able to move the throttle plate from its static position (\(M_1, M_4\) or \(M_7\)) to other modes \(M_2, M_3, M_5\) or \(M_6\). Then we can write \(CR\) and \(CF\) nonlinearities as:

\[
CR = \begin{cases} 
-\alpha \leq CR \leq \alpha & \text{if } y=x_1=0 \\
-\alpha & \text{if } y=x_1<0 \\
\alpha & \text{if } y=x_1>0 
\end{cases} 
\]

\[
CF = \begin{cases} 
-\mu \leq CF \leq \mu & \text{if } \dot{y}=a_{12}x_2=0 \\
-\mu & \text{if } \dot{y}=a_{12}x_2<0 \\
\mu & \text{if } \dot{y}=a_{12}x_2>0 
\end{cases} 
\]

In order to obtain continuous time models \(S_i(\dot{x}(t), x(t), u(t), v(t), y(t)) = 0\) and discrete time models \(S_i(u(k), x(k), x(k+1), y(k), v(k))/i \in \{1,...,7\}\), we will study the cases above
the dead zone where \( y \geq 0 \) (i.e. \( M_1, M_2, M_3 \) and \( M_4 \)). The cases where \( y < 0 \) (i.e. \( M_5, M_6 \) and \( M_7 \)) are similar.

Let us assume that input \( v = CR + CF = \alpha v_1 + \mu v_2 \), with \( v_1 \) and \( v_2 \) stand for elementary nonlinearities, bounded by \( \pm 1 \). Input \( v \) has its maximum (resp. minimal) value in the modes \( M_2 \) and \( M_5 \) (resp. \( M_6 \) and \( M_7 \)). Indeed, \( v \) depends on the values of \( v_1 \) and \( v_2 \):

- \( v_1 = 1 \), if the metal plate is above the dead zone (\( M_2 \), \( M_3 \) and \( M_4 \)),
- \( v_1 = -1 \), if it is below the dead zone (\( M_5 \), \( M_6 \) and \( M_7 \)),
- \(-1 < v_1 < 1 \), in dead zone mode \( M_f \).

In the same way,

- \( v_2 = 1 \), if the plate has a positive velocity (\( M_2 \) and \( M_3 \)),
- \( v_2 = -1 \), for negative velocity (\( M_5 \) and \( M_6 \))
- \(-1 < v_2 < 1 \), if velocity is zero.

Equation (6) can be rewritten as a continuous or discrete time linear system with two inputs \( u \) and \( v \) according to equations (9) and (10) or with three inputs \( u, v_1 \) and \( v_2 \) as (11):

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & a_{12} & 0 \\
a_{21} & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 & -1 \\
b_3 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\tag{9}
\]

\[
\begin{bmatrix}
x_j(k+1) \\
x_j(k+1) \\
x_j(k+1)
\end{bmatrix} =
\begin{bmatrix}
ad_{11} & ad_{12} & ad_{13} \\
ad_{21} & ad_{22} & ad_{23} \\
ad_{31} & ad_{32} & ad_{33}
\end{bmatrix}
\begin{bmatrix}
x_j(k) \\
x_j(k) \\
x_j(k)
\end{bmatrix} +
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22} \\
b_{31} & b_{32}
\end{bmatrix}
\begin{bmatrix}
u(k) \\
v(k)
\end{bmatrix}
\tag{10}
\]

\[
\begin{bmatrix}
x_j(k+1) \\
x_j(k+1) \\
x_j(k+1)
\end{bmatrix} =
\begin{bmatrix}
ad_{11} & ad_{12} & ad_{13} \\
ad_{21} & ad_{22} & ad_{23} \\
ad_{31} & ad_{32} & ad_{33}
\end{bmatrix}
\begin{bmatrix}
x_j(k) \\
x_j(k) \\
x_j(k)
\end{bmatrix} +
\begin{bmatrix}
b_{11} & vb_{11} & vb_{12} \\
b_{21} & vb_{21} & vb_{22} \\
b_{31} & vb_{31} & vb_{32}
\end{bmatrix}
\begin{bmatrix}
v_1(k) \\
v_2(k)
\end{bmatrix}
\tag{11}
\]

Using equation (9) as well as properties in each mode (fig. 9), discrete time local models \( S_d(u(k), x(k), x(k+1), y(k), v(k)) \) are obtained. In fact, by writing \( v = \alpha v_1 + \mu v_2 \) as a function of the control input \( u \) and state \( x \) and comparing it to a bounded value, we can deduce the operating mode and switching conditions as follows.

**In mode 1.** The system is stationary. The throttle plate position is zero as well as velocity and acceleration (\( x_1 = \dot{x}_1 = x_2 = \dot{x}_2 = 0 \)). The plate remains in this mode as long as the torque induced by the voltage \( u(t) \) is lower than resistor torque maximum. The current in circuit increases so that it moves outside the dead zone. The model \( S_1 \) is given by (12):

\[
S_1(\dot{x}, x, u, y, v) = \begin{cases} 
\dot{x}_1 = x_1 = \dot{x}_2 = \dot{x}_3 = 0 \\
\dot{x}_3 = a_{32}x_2 + a_{33}x_3 + b_3u \\
y = \dot{y} = 0
\end{cases}
\tag{12}
\]
and the discrete time model $S_{d2}(u(k), x(k), x(k+1), y(k), v(k))$ is given by (13):

$$
\begin{align*}
\begin{cases}
    x_1(k+1) &= x_1(k) = 0 \\
    x_2(k+1) &= 0 \\
    x_3(k+1) &= a_{d3} x_3(k) + b_3 u(k) \\
    y(k) &= x_1(k) = 0
\end{cases}
\end{align*}
$$

In mode 2. The electrical couple is larger than the maximum of the resistive torque. In this case the resistive torque takes the maximum of its value, then we will have $v = CR + CF = \alpha + \mu$ and $v_1 = v_2 = +1$. The plate velocity in this mode remains positive until the sum of couples becomes zero (i.e. dead zone or saturation). $S_2$ is given by (14):

$$
S_2(\ddot{x}, x, u, y, v = \alpha + \mu) \equiv \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} \begin{bmatrix} \alpha + \mu \end{bmatrix}
$$

and $S_{d2}(u(k), x(k), x(k+1), y(k), v(k) = \alpha + \mu)$ is given by (15):

$$
\begin{align*}
\begin{cases}
    x_1(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    x_2(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    x_3(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    y(k) &= x_1(k)
\end{cases}
\end{align*}
$$

In mode 3. The electrical torque is lower than the resistive torque that leads to a negative velocity. The plate velocity remains negative until the torques sum becomes zero (i.e. dead zone or saturation). $S_3$ is given by (16):

$$
S_3(\ddot{x}, x, u, y, \alpha - \mu) \equiv \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} \begin{bmatrix} \alpha - \mu \end{bmatrix}
$$

and $S_{d3}(u(k), x(k), x(k+1), y(k), v(k) = \alpha - \mu)$ is given by (17):

$$
\begin{align*}
\begin{cases}
    x_1(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    x_2(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    x_3(k+1) &= a_{d3} x_3(k) + b_{31} u(k) \\
    y(k) &= x_1(k)
\end{cases}
\end{align*}
$$

In mode 4. The torques induce a zero acceleration, a zero velocity ($\dot{x}_1=\dot{x}_2=0$) and $x_0>0$. $S_4$ is given by (18):
\[ S_d(\dot{x}, x, u, y, v) = \begin{cases} \dot{x}_1 = x_2 = \dot{x}_3 = y = 0 \\ \dot{x}_3 = a_{33} x_2 + a_{33} x_3 + b_3 u \\ y = x_1 \neq 0 \end{cases} \] (18)

\[
\begin{align*}
&x_1(k+1) = x_1(k) \neq 0 \\
&x_2(k+1) = 0 \\
&x_3(k+1) = ad_{31} x_1(k) + ad_{33} x_3(k) + b_{31} u(k) \\
&y(k) = x_1(k) \neq 0
\end{align*}
\] (19)

**Step 3.** In order to deduce switching conditions, we use equation (11) and velocity properties in each mode. \(v(k)\) value that lead to zero velocity is calculated and compared to bounded values. The conditions in each mode are as follows:

**Switching conditions in mode 1.** From (7), (8) and (13), the system is stationary if \(x_1 = 0\) and the behaviors in the mode \(M_1\) satisfies the following conditions:

\[
\begin{align*}
&x_1(k+1) = x_1(k) = 0 \\
x_2(k+1) = 0 \\
-\alpha - \mu < CR + CF < \alpha + \mu
\end{align*}
\] (20)

From these conditions, one can deduce \(v(k)\) input value associated to the resistive torque by using (11) and (20a):

\[
v(k) = \frac{(ad_{21} - 1)x_1(k) + ad_{12} x_2(k) + ad_{13} x_3(k) + b_{11} u(k)}{-b_{12}}
\] (21)

or (20b):

\[
v(k) = \frac{ad_{21} x_1(k) + ad_{22} x_2(k) + ad_{23} x_3(k) + b_{21} u(k)}{-b_{22}}
\] (22)

As long as the estimated value of \(v(k) = CR + CF\) satisfies condition (20c), the system remains in dead zone. Then, we replace this estimated value in expression of \(x_1(k+1)\) in (10). It must be zero while satisfying (20a). Let us define \(x_{\text{test}}\) by (21).

\[
x_{\text{test}} = ad_{11} x_1(k) + ad_{12} x_2(k) + ad_{13} x_3(k) + b_{11} u(k) + b_{12} (CR + CF)
\] (23)

Switching conditions in mode 1 are given by (24):

\[
\begin{align*}
&|x_{\text{test}}| < \zeta \\
&-\alpha - \mu < v(k) < \alpha + \mu
\end{align*}
\] (24)

where \(\zeta\) is a small positive constant.

**Switching conditions in mode 2.** Additive inputs are: \(v = CR + CF = \alpha + \mu\), and plate
velocity remains positive until the sum of couples is zero. In this mode, the following conditions are satisfied:

\[
\begin{align*}
    x_1(k+1) &> 0 & (a) \\
    x_2(k+1) &> 0 & (b) \\
    CR + CF &= \alpha + \mu = \text{constant} & (c)
\end{align*}
\]

Switching conditions in function of \(x_{\text{test}}\) and \(v\) are given by (26):

\[
\begin{align*}
    x_{\text{test}} &> 0 \\
    v(k) &> \alpha + \mu
\end{align*}
\]

Switching conditions in mode 3. Electrical torque induces a lower torque than the resistor torque. The velocity remains negative until the sum of all couples is zero. In this operating mode, the system satisfies the conditions (27):

\[
\begin{align*}
    x_1(k+1) &\geq 0 & (a) \\
    x_2(k+1) &\leq 0 & (b) \\
    CR + CF &= \alpha - \mu = \text{constant} & (c)
\end{align*}
\]

We obtain the switching conditions (28), such that:

\[
\begin{align*}
    x_{\text{test}} &> 0 \\
    v(k) &< +\alpha - \mu
\end{align*}
\]

Switching conditions in mode 4. The system satisfies the following conditions:

\[
\begin{align*}
    x_1(k+1) &= x_1(k) & (a) \\
    x_2(k+1) &= 0 & (b) \\
    \alpha - \mu &< CR + CF < \alpha + \mu & (c)
\end{align*}
\]

As long as \(v(k)\) satisfies condition (29c), the system remains in mode 4. Thus, the switching conditions are:

\[
\begin{align*}
    x_{\text{test}} &> 0 \\
    \alpha - \mu &< v(k) < \alpha + \mu
\end{align*}
\]

**Step 4.** Using the set of models composed by (13), (15), (17) and (19) with the switching conditions (24), (26), (28) and (30), we can present the valve behavior with an automaton. Let us define transitions, such that: \(T_1=\alpha + \mu\), \(T_2=\alpha - \mu\), \(T_3=-\alpha + \mu\) and \(T_4=-\alpha - \mu\). The automaton is presented in figure 10.
3 Identification

In this section, we propose an identification approach using multimodel aspect explained in section 2, data classification and linear identification techniques.

3.1 Problem statement and principle

The basic idea of parameters identification is to use input-output data and also input-output description. We consider PieceWise Affine (PWA) systems composed by \( m \) submodels. With system order \( na, nb \) and disturbance order \( nc \), the PWA model can be defined by equation (31).

\[
y(k) = \begin{cases}
  \sum_{i=1}^{na} a_{1,i} y(k-i) + \sum_{i=1}^{nb} b_{1,i} u(k-i) + \sum_{i=1}^{nc} c_{1,i} e(k-i) & \text{for} \quad v(k) > T_1 \\
  \vdots \\
  \sum_{i=1}^{na} a_{m,i} y(k-i) + \sum_{i=1}^{nb} b_{m,i} u(k-i) + \sum_{i=1}^{nc} c_{m,i} e(k-i) & \text{for} \quad v(k) < T_4
\end{cases}
\]

(31)

where \( u, y \) and \( e \) are respectively inputs, output and noise with \( c_0 = 1 \), zero mean and \( \sigma \) variance. The parameter vectors for submodels \( M_i \) \( (i = 1, \ldots, m) \) are denoted by:

\[
\theta_i = \begin{bmatrix}
  a_{1,i} & \cdots & a_{na,i} & b_{1,i} & \cdots & b_{nb,i} & c_1 & \cdots & c_{nc}
\end{bmatrix}^T
\]

(32)

In order to estimate the parameter vectors, we assume that \( N \) output-input measurements \( (y(k), u(k)), k = 0, \ldots, N \), are available. Orders \( na, nb, nc \) and number of submodels \( m \) are assumed to be known. The parameter vectors estimated for submodels \( M_i \) \( (i = 1, \ldots, m) \) are denoted by:

\[
\hat{\theta}_i = \begin{bmatrix}
  \hat{a}_{1,i} & \cdots & \hat{a}_{na,i} & \hat{b}_{1,i} & \cdots & \hat{b}_{nb,i} & \hat{c}_1 & \cdots & \hat{c}_{nc}
\end{bmatrix}^T
\]

(33)

Thus the estimation of model (31) is given by (34).
Let $e(k) = y(k) - \hat{y}(k)$ be the error and the regressor vector be denoted by:

$$\phi(k) = [y(k-1) \cdots y(k-na) u(k-1) \cdots u(k-nb) e(k-1) \cdots e(k-nc)]$$

The aim of identification is to process input–output data so that identification leads to $\hat{\theta}_i = \theta_i$ and $e(k) = e(k)$. Let us assume that $C_i$ is the set of measurements $(y(k), u(k))$ with respect to model $M_i$, and $\Psi_i$ is the set of couples $(\phi(k), y(k))$ with respect to $M_i$. If $(y(k), u(k)) \in C_i$ then $(\phi(k), y(k)) \in \Psi_i, i=1,\ldots,m$. Estimator $\hat{y}(k)$ of $y(k)$ is obtained with regressor vector $\phi(k)$ and parameters vector $\hat{\theta}_i$ so that $\hat{y}(k) = \phi(k)\hat{\theta}_i$. Estimation is obtained according to an optimization process that minimises quadratic error $J$:

$$J = \sum_{C_i} e(k)^2 = \sum_{C_i} (y(k) - \hat{y}(k))^2$$ (36)

The main difficulty in solving identification problem is to classify data in appropriate clusters $C_i$ and $\Psi_i$. The linear model $M_i$ can be identified by using data in set $\Psi_i$ and linear identification techniques (ARX, ARMAX…) [9]. Our algorithm exploits a combined use of classification and linear regression techniques and consists in 3 steps (fig.11):

1. Collect measurements $(u(k), y(k))$ with sampling period $T_e$. Measurements must be collected in all operating modes $M_i, i=1,\ldots,m$. PRBS (Pseudo-Random Binary Signal) can be used for this purpose.

2. Compute derivative $\dot{y}(k) = \dot{y}(t)|_{t=kT_e}$ with Legendre polynomials and use it with output $y(k)$ in order to classify measurements $(y(k), u(k))$ in classes $C_i$ and $(\phi(k), y(k))$ in classes $\Psi_i$.

3. Use $(\phi(k), y(k))$ in each class $\Psi_i$ and linear identification method in order to identify model $M_i$. 
The computing of instantaneous derivatives consists to estimate \( y^{(j)}(t) \) the \( j \)th derivative order of continuous signal \( y(t) \) by using a linear discrete filter. We assume that we have \( Y(t) \) a vector of samples of continuous signal \( y(t) \), collected on an observation window of width \( m=2p+1 \). It given by:

\[
Y(t) = \left[ y(t-p\tau), \ldots, y(t), \ldots, y(t+p\tau) \right]
\]  

(37)

\( \tau = T_c \) with \( T_c \) is sample time and \( p \) is an integer number. \( y^{(j)}(t) \), the \( j \)th derived of signal \( y(t) \) at the moment \( t=k\cdot T_c \) is given according to [4] by (38):

\[
y^{(j)}(k\cdot T_c \tau) = \frac{2^j j!}{E_j(p) \tau_w} w_j(k\cdot T_c \tau)
\]  

(38)

With \( \tau_w = 2p \tau \) and \( w_j(t) \) is discrete linear filter, computed as:

\[
w_j(k\cdot T_c \tau) = \sum_{n=-p}^{p} r_{j,p}(n)y(k\cdot T_c + n \tau)
\]  

(39)

The coefficients \( r_{j,p}(n) \) (called Legendre polynomial) of this filter are calculated as follows:
where, $E_j$ is a $p$ function computed when $r_{j,p}(n)$ satisfies the following condition:

$$\sum_{n=-p}^{p} r_{k,p}^2(n) = 1$$  \hspace{1cm} (41)

According to the equation (40) and (41) above, we have for the first derivative:

$$\begin{align*}
 r_0(n) &= \frac{1}{E_0(p)} \\
 r_1(n) &= \frac{n}{pE_1(p)}
\end{align*}$$  \hspace{1cm} (42)

Then, according to (38) and (39), we have at instants $kT_e$ the following relations:

$$\begin{align*}
 y(kT_e) &= \frac{1}{E_0(p)} w_0(kT_e) \\
 y(kT_e) &= \frac{2}{E_1(p)\tau_w} w_1(kT_e)
\end{align*}$$  \hspace{1cm} (43)

After computing values of $y(k)$, we start classification by building all couples $(y(k), \dot{y}(k))$. Each couple is affected to one region of phases diagram, so that we can classify $(u(k), y(k)) \in \mathcal{C}_i$ and thereafter $(\phi(k), y(k)) \in \Psi_i$.

After determining of classes $\Psi_i$ associated to models $M_i$ the last step is to use data in each class and a linear identification technique in order to identify parameters $\theta_i$ of model $M_i$. Let us define $N_i$ as the cardinality of $\Psi_i$. LS (least square) linear techniques or RLS (recursive least square) can be used according to equation (44):

$$\begin{align*}
 \hat{\theta}(k) &= \hat{\theta}(k-1) + \{P(k) - \varphi(k)^T \varphi(k)\} \varepsilon^T(k) \\
 P(k) &= \frac{1}{\lambda_1} \left[ P(k-1) - \frac{P(k-1)\varphi(k)^T\varphi(k)P(k-1)}{\varphi(k)^T+\varphi(k)P(k-1)\varphi(k)} \right] \\
 \varepsilon^T(k) &= y(k) - \varphi(k)\hat{\theta}(k-1)
\end{align*}$$  \hspace{1cm} (44)

3.2 Application to throttle valve

The experimental device (fig.12) used for the throttle valve identification consists on the valve, the power circuit, the control panel, and PC equipped with 1103 DSpace interface, Matlab/Smulink and Desk control software.
Step 1. According to section 2 we can notice that there are 4 linear models ($M_i$) above the dead zone. Multimodel identification is reduced to identify both models $M_2$ and $M_3$. In fact in both operating modes, inputs $v_1$ and $v_2$ are constants.

To make it possible the system to visit all operating modes in particular $M_2$ and $M_3$, PRBS is used with minimal width equal to 0.01s and magnitude between 1.7 and 3.5 Volts. The PWM used has a frequency equal to 2kHz. The sampling is 0.001s.

Step 2. According to the preceding algorithm, Legendre polynomials and measured output $y(k)$ are used in order to compute derivative $\dot{y}(k)$ for each $k$. Fig. 13 shows input $u$, measured output $y$ and derivative $\dot{y}(k)$ computed from output samples. Derivative computation is realized using a window of width $2p+1=11$.

From fig. 13 one can see that it is possible to determine the time interval where velocity $\dot{y}(k)$ is positive or negative (fig. 14). Thus, one can classify ($y(k)$, $\dot{y}(k)$) with respect to operating modes $M_2$ and $M_3$. In fact the mode $M_2$ (resp. $M_3$) is characterized by $y(k)>0$ and $\dot{y}(k)>0$ (resp. $y(k)>0$ and $\dot{y}(k)<0$). In order to improve classification, we use a velocity threshold $\pm S$ ($S>0$) to compute derivative sign (fig. 15).

The velocity threshold makes it possible to separate effectively data with respect to modes $M_2$ and $M_3$ when plate is moving and also modes $M_1$ and $M_4$ when plate does not move:

\begin{equation}
\text{(45)}
\end{equation}

In order to classify ($\phi(k)$, $y(k)$) $\in \Psi_i$, $i=\{2,3\}$, one must know the values of $v_1$ and $v_2$ in both modes. According to throttle valve modelling, values of $v_1$ and $v_2$ can be deduced. Thus (45) leads to (46):

\begin{equation}
\text{(46)}
\end{equation}

Orders of system satisfy $na = nb = 3$. Thus regressor associated to the linear model (11) can be deduced. Measurements ($y(k)$, $\dot{y}(k)$) are classified according to (45) and values of $v_1$ and $v_2$ are deduced with respect to (46).
Fig 13. Input, output and output derivatives

Fig 14. Measured output and derivative
Step 3: Data \((\phi(k), y(k))\) and identification techniques (i.e. equation (44)) are used to identify parameter vectors \(\theta_2\) and \(\theta_3\) of model \(M_2\) and \(M_3\). As a conclusion, PWA model of the throttle valve, above the dead zone, is given by equation (47):

\[
\begin{cases}
  y(k) = y(k-1) = 0 & (M_1) \\
  y(k) = \phi(k) \theta_2 & (M_2) \\
  y(k) = \phi(k) \theta_3 & (M_3) \\
  y(k) = y(k-1) \neq 0 & (M_4)
\end{cases}
\]

(47)

3.3 Simulations and results

For simulation and discussion purposes, commutation conditions are determined with respect to state \(x_1\), input \(u\) and estimation of \(v\). Fig. 16 to 18 give the results between the identified multimodel and the system output.
Fig. 18 suggests that our identification approach leads to small error magnitude. In fact, dynamics of systems and multimodel are quite similar for positive and negative velocity: relative error is about 2%.

4. Conclusions
The main contribution of the proposed paper was to propose a multimodel approach for the modelling of non-linear systems. The multimodel is composed of several linear continuous time or discrete time models that represent all operating modes. It is also composed of a set of commutations and has been represented using automaton formalism. Nonlinearities are replaced by bounded additive inputs. These inputs are estimated and compared with respect to thresholds. The multimodel approach has been also applied to the throttle valve in combustion engine. Identification of the valve model is then obtained according to the multimodel approach.

If multimodel appears to be an interesting alternative for the modelling of complex non-linear systems, the principal difficulty remains the determination of data used for parameters identification of each local model. To identify these parameters and to calculate the input and output derivatives, we have used non-linear properties in polyhedral partition and Legendre polynomials. Data are classified and then used to
identify local linear models. The outlook of this work is in one hand to extend our
modelling and identification approach to any non-linear system. In other hand, it is to use
Legendre polynomial in order to identify continuous form of model system, and for data
classification.

REFERENCES

2872-2877.
of automatic control Lund Institute of Technology, February.
Control Systems Magazine, pp. 64-81.
electronic throttle’, Proceeding of the American Control Conference, Arlington, VA June 25-27,
pp. 1310-1314.
degree of freedom structure’, Proceeding of the 35th IEEE CDC, Kobe, Japan USA, pp.
1785-1788.
a non linear throttle system using ARX models’, 17ème IMACS International Congress,
Paris, France 11 – 15, July.
throttle valve’, Proceeding of the 40th IEEE Conference on Decision and Control, Orlando,
Florida USA, pp. 1819-1824.
6, pp. 993-1002.
non linéarités non différentiable’, Thèse Laboratoire d’automatique de Grenoble (LAG).
affine and hybrid systems’, Proceeding of the American Control Conference, Arlington,
June, pp. 3521-3526.
servo controllers to electronic throttle control’, Proceeding of the 37th IEEE Conference on
Decision and Control, Tampa, Florida USA, December, pp. 1841-1545.
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